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MULTISWITCHING DOUBLE COMPOUND COMBINATION SYNCHRONISATION OF 5-DIMENSIONAL HYPERCHAOTIC SYSTEMS IN APPLICATION TO MAGNETOHYDRODYNAMIC SYSTEMS

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Abstract

In this paper we proposed an integrator backstepping technique for the realization of multiswitching and synchronization of double compound combination of 5-dimensional hyperchaotic systems with application to 5-dimensional hyperchaotic magnetohydrodynamic systems to verify our analytical method. Using the Runge-Kutta algorithm, our numerical results confirm the effectiveness of the proposed analytical technique.

Keywords: *Multiswitching, Double Compound Combination, Synchronisation,5-Dimensional, Hyperchaotic Systems, Magnetohydrodynamic Systems*

1.0 Introduction

It has been shown in [1-5] that deterministic dynamical systems exhibit sensitive dependence on initial conditions with proofs in the fields of sciences (physical and natural), medicine, and engineering Various attributes of nonlinear dynamical systems such as chaos, bifurcation, multistability, pattern formation, control, and synchronization have been investigated due to their potential applications in many disciplines. Due to its applications in information processing, secure communication, chemical reactions, and modeling brain activity, it was noted in [6] that there is an increasing interest in the study of synchronization of chaotic systems which has led to the discovery of various types of synchronization including complete synchronization, lag synchronization, phase synchronization, generalized synchronization, measure synchronization, projective synchronization, anticipated synchronization, reduced-order synchronization, compound and double compound as mentioned in [7,8]

To achieve stable synchronization between two or more chaotic systems, researchers have used several methods, including adaptive control and active control, sliding mode control, impulsive control, linear feedback control, backstepping control, open plus close loop control, adaptive fuzzy feedback and passive control [9-16] respectively. Notable among these methods is the backstepping control technique which has outstanding performance in the synchronization of identical and non identical chaotic systems as mentioned in [17] and [18].

Further to our works on Multiswitching Combination Synchronization in High Dimensional Hyperchaotic Systems as noted in [19,20], in this paper, we present Multiswitching Double Compound Combination Synchronisation of 5-Dimensional Hyperchaotic Systems with application to 5-dimensional Hyperchaotic magnetohydrodynamic systems via integrator backstepping technique, with an intention that the result will ensure better security when employed in communications applications. We used the Runge-Kutta algorithm and our numerical results confirmed the effectiveness of the proposed analytical technique, the synchronization was achieved.

2.0 Definition, formulation and design of controllers for the multiswitching double compound combination synchronisation of 5-Dimensional hyperchaotic systems in application to 5-Dimensional hyperchaotic magnetohydrodynamic systems

The compound-combination synchronisationscheme for five chaotic systems as proposed in [21] and the double compound synchronisation scheme for six systems proposed in [22] serve as the guide in this work. Consider systems (1), (2), (3) and (4) as drive systems and systems (5) and (6) as two response systems

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$\dot{x} = f(x)$ (1)
$\dot{y} = f(y)$ (2)
$\dot{z} = f(z)$ (3)
$\dot{p} = f(p)$ (4)
$\dot{q} = f(q) + U_1$ (5)
$\dot{w} = f(w) + U_2$ (6)

where $x(x_1, x_2, x_3 \dots x_i)^T$, $y = (y_1, y_2, y_3 \dots y_i)^T$, $z = (z_1, z_2, z_3 \dots z_i)^T$, $p = p_1, p_2, p_3 \dots p_i)^T$, $q = (q_1, q_2, q_3 \dots q_i)^T$ and $w = (w_1, w_2, w_3 \dots w_i)^T$, are the state variables Of systems (1) − (6) respectively, $f(x)\epsilon\Re^l, f(y)\epsilon\Re^m, f(z)\epsilon\Re^n, f(p)\epsilon\Re^o$, $f(q)\epsilon\Re^s$ and $f(w)\epsilon\Re^t$ are continous functions of the systems, $U_1 = (u_1, u_2, u_3 \dots u_q)^T \epsilon\Re^q, U_2 = (u_1, u_2, u_3 \dots u_w)^T \epsilon\Re^w$ are the controllers to be designed.Suppose $x = diag(x_1, x_2 \dots x_n)$, $y = diag(y_1, y_2 \dots y_n), z = diag(z_1, z_2 \dots z_n)$, $p = Diag(p_1, p_2 \dots p_n), q \; diag(q_1, q_2 \dots q_n)$ and $w \; diag(w_1, w_2 \dots w_n)$ are n−dimensional diagonal matrices Zhang and Deng (2014) gave an error definition of the synchronisationfor double compound as

Definition 1:If there exist six constant matrices $A, B, C, D, M, N\epsilon\Re^n X\Re^n$ such that

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|(Ax + By)(Cz + Dp) - Mq - Nw\| = 0 \quad (7)$$

then the drive systems (1) − (4) are said to be in double compound synchronisationwith the response systems (5) and (6),where $\|\cdot\|$ expresses the matrix norm, the driver systems (1) and (2) are called the scaling driver systems and the driver systems (3) and (4) are called the base driver systems and in one of their remarks, Zhang and Deng (2014) explained that (7) could be written as

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|Mq + Nw - (Ax + By)(Cz + Dp)\| = 0 \quad (8)$$

Comment 1: Following our definitions and comments in Ogundipe (2017), one can write (8) as

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|Mq_{nr} + Nw_{nr} - (Ax_{nd1} + By_{nd2})(Cz_{nd3} + Dp_{nd4})\| = 0 \quad (9)$$

This represents error dynamics for six indices being the number of systems in consideration. The error dynamics is

$$\lim_{t\to\infty}\|e_{\alpha\beta\gamma\delta\lambda\mu}\| = \lim_{t\to\infty}\|Mq_{\alpha r} + Nw_{\beta r} - (Ax_{d1} + By_{d2})(Cz_{d3} + Dp_{\mu d4})\| = 0 \quad (10)$$

so that the indices are now members taken from the dimension n of the systems. For easy identification of the mathematics function, assume that the maximum variable state space is five (5), each denoted by dimensions $1, 2, 3, 4, 5 = i, j, k, l, m$ for the five (5) dimensional systems in consideration.

Definition 2:If the error states in relation to definition 1 and the comments above are redefined such that for $e_{\alpha\beta\gamma\delta\lambda\mu}$, any, combination of, or all of the equality signs as described in comment 1 is changed, different from the dimension of the corresponding response sub-system, in at least one of the sub-systems, and

$$\lim_{t\to\infty}\|e_{\alpha\beta\gamma\delta\lambda\mu}\| = \lim_{t\to\infty}\|e\| \, Mq_{\alpha r} + Nw_{\beta r} - (Ax_{\gamma d1} + By_{\delta d2})(Cz_{\lambda d3} + Dp_{\mu d4})k = 0 \quad (11)$$

then, systems (1), (2), (3), (4), (5) and (6) are said to be in double compound multiswitching combination synchronisation state.

Comment 2: The conditions in definition 2 is referred to as generic conditions that must be met and which are dependent on the choice of the dimension, as the indices of the error system and (a) It follows that for a complete set of the 5D system, we have five 5 sets of 6-indices $\alpha, \beta, \gamma, \delta, \lambda$ and μ made up of choices from i, j, k, l, m. (b) This means that one determining factor for a complete set mentioned in comment 2(b) is the arrangement of the dimensions in the six 6 indices of the 5D system and (c) It is notable also that in synchronisation, the arrangement of the response system is kept in order and that the arrangements of the driver systems can be varied for varieties, each driver to be treated on its own merit.

In line with the above definitions and comments, we generate all possible arrangement, henceforth referred to as switches, for the first driver system as 3125 switches. It is notable also that the same number and type of switches exist for the second, third and fourth driver systems. This is because the systems are identical. The number of switches and groups are as presented in section Ogundipe (2017)

Now applying the above on the following 5D hyperchaotic magnetohydrodynamics system in Bekki (2001)

$$\dot{a} = \sigma(-a + rb - qd(1 + \frac{w(3 - w)}{\xi^2(4 - w)}e))$$
$$\dot{b} = -b + a - ac$$
$$\dot{c} = w(-c + ab)$$
$$\dot{d} = -\xi(d - a) - \frac{w}{\xi(4 - w)}ae$$

$\dot{e} = -\xi(4 - w)(e - ad)$ (12)

Let the parameters be described as $\sigma = a, r = b, q = c$ and $\xi = d$. Also, let $a_1 = (1 + \frac{w(3-w)}{\xi^2(4-w)}e)$, $a_2 = \frac{w}{\xi(4-w)}ae$ and $a_3 = (4 - w)$ and redefine the variables of system (3.144) as follows, $a = y(1), b = y(2), c = y(3), d = y(4), e = y(5)$ for the master system 1, $a = y(6), b = y(7), c = y(8), d = y(9), e = y(10)$ for themaster system 2, $a = y(11), b = y(12), c = y(13), d = y(14), e = y(15)$ for themaster system 3, $a = y(16), b = y(17), c = y(18), d = y(19), e = y(20)$ for themaster system 4, $a = y(21), b = y(22), c = y(23), d = y(24), e = y(25)$ for theslave system 1 and $a = y(26), b = y(27), c = y(28), d = y(29)$ and $e = y(30)$

for the slave system 2.Consequently, one can write the master systems as follows, $\dot{a} = \dot{y}(1), \dot{b} = \dot{y}(2), \dot{c} = \dot{y}(3), \dot{d} = \dot{y}(4), \dot{e} = \dot{y}(5)$ for master system 1, $\dot{a} = \dot{y}(6)$, $\dot{b} = \dot{y}(7), \dot{c} = \dot{y}(8), \dot{d} = \dot{y}(9), \dot{e} = \dot{y}(10)$ for the master system 2, $\dot{a} = \dot{y}(11), \dot{b} = \dot{y}(12), \dot{c} = \dot{y}(13), \dot{d} = \dot{y}(14), \dot{e} = \dot{y}(15)$ for the master system 3, $\dot{a} = \dot{y}(16), \dot{b} = \dot{y}(17), \dot{c} = \dot{y}(18), \dot{d} = \dot{y}(19), \dot{e} = \dot{y}(20)$ for the master system 4, $\dot{a} = \dot{y}(21), \dot{b} = \dot{y}(22), \dot{c} = \dot{y}(23)$, $\dot{d} = \dot{y}(24), \dot{e} = \dot{y}(25)$ for the slave system 1, $\dot{a} = \dot{y}(26), \dot{b} = \dot{y}(27), \dot{c} = \dot{y}(28), \dot{d} = \dot{y}(29)$ and $\dot{e} = \dot{y}(30)$ for the slave system 2.

Thus, for the double compound situation of the five dimensional magneto-hydrodynamicsystem defined in (12), the scaling driver systems are given by

$$\begin{aligned}
\dot{y}(1) &= a(-y(1) + by(2) - cy(4)a_1y(5)) \\
\dot{y}(2) &= -y(2) + y(1) - y(1)y(3) \\
\dot{y}(3) &= d(-y(3) + y(1)y(2)) \\
\dot{y}(4) &= -e(y(4) - y(1)) - a_2(y(1)y(5)) \\
\dot{y}(5) &= -ea_3(y(5) - y(1)y(4))
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\dot{y}(6) &= a(-y(6) + by(7) - cy(9)a_1y(10)) \\
\dot{y}(7) &= -y(7) + y(6) - y(6)y(8) \\
\dot{y}(8) &= d(-y(8) + y(6)y(7)) \\
\dot{y}(9) &= -e(y(9) - y(6)) - a_2(y(6)y(10)) \\
\dot{y}(10) &= -ea_3(y(10) - y(6)y(9)),
\end{aligned} \tag{14}$$

the base driver systems are

$$\begin{aligned}
\dot{y}(11) &= a(-y(11) + by(12) - cy(14)a_1y(15)) \\
\dot{y}(12) &= -y(12) + y(11) - y(11)y(13) \\
\dot{y}(13) &= d(-y(13) + y(11)y(12)) \\
\dot{y}(14) &= -e(y(14) - y(11)) - a_2(y(11)y(15)) \\
\dot{y}(15) &= -ea_3(y(15) - y(11).* y(14))
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
\dot{y}(16) &= a(-y(16) + by(17) - cy(19)a_1y(20)) \\
\dot{y}(17) &= -y(17) + y(16) - y(16)y(18) \\
\dot{y}(18) &= d(-y(18) + y(16)y(17)) \ (3.148) \\
\dot{y}(19) &= -e(y(19) - y(16)) - a_2(y(16)y(20)) \\
\dot{y}(20) &= -ea_3(y(20) - y(16)y(19))
\end{aligned} \tag{16}$$

while the response systems are given by

$$\begin{aligned}
\dot{y}(21) &= a(-y(21) + by(22) - cy(24)a_1y(25)) \\
\dot{y}(22) &= -y(22) + y(21) - y(21)y(23) \\
\dot{y}(23) &= d(-y(23) + y(21)y(22)) \\
\dot{y}(24) &= -e(y(24) - y(21)) - a_2(y(21)y(25)) \\
\dot{y}(25) &= -ea_3(y(25) - y(21)y(24))
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\dot{y}(26) &= a(-y(26) + by(27) - cy(29)a_1y(30)) + u_1 \\
\dot{y}(27) &= -y(27) + y(26) - y(26)y(28) + u_2 \\
\dot{y}(28) &= d(-y(28) + y(26)y(27)) + u_3 \\
\dot{y}(29) &= -e(y(29) - y(26)) - a_2(y(26)y(30)) + u_4 \\
\dot{y}(30) &= -ea_3(y(30) - y(26)y(29)) + u_5
\end{aligned} \tag{18}$$

Where u_1, u_2, u_3, u_4 and u_5 are the set of nonlinear controllers. From Ogundipe (2017) the switching combinations are chosen as follows:

$Group\ 1: i = j = k = l = m, switch$ (1,1,1,1,1),
$Group\ 49: i \neq j \neq k \neq l \neq m., switch$ (1,2,3,4,5)
We can write the error dynamics as

$$e_{111111} = y(21) + y(26) + \alpha(t)[y(1) + y(6)][y(11) + y(16)] + u_1;$$
$$e_{221122} = y(22) + y(27) + \alpha(t)[y(1) + y(6)][y(12) + y(17)] + u_2;$$
$$e_{331133} = y(23) + y(28) + \alpha(t)[y(1) + y(6)][y(13) + y(18)] + u_3; \quad (19)$$
$$e_{441144} = y(24) + y(29) + \alpha(t)[y(1) + y(6)][y(14) + y(19)] + u_4;$$
$$e_{551155} = y(25) + y(30) + \alpha(t)[y(1) + y(6)][y(15) + y(20)] + u_5.$$

Using the back stepping method of synchronisation as presented in Vincent *et al*. (2015) and considering (19) with the appropriate notations. Differentiating the error variables of (19), $\dot{e}_{111111} = A1 - B1A2 - e_{221122}(1 - C1) + D1 + u_1;$

$$\dot{e}_{221122} = A2 - B2A1 - e_{111111}(1 - B2) - C2 + u_2;$$
$$\dot{e}_{331133} = A3 - B3A4 + e_{441144}(1 + B3) - C3 + u_3; \quad (20)$$
$$\dot{e}_{441144} = A4 - B4A5 + e_{551155}(1 + B4) - C4 + u_4;$$
$$\dot{e}_{551155} = A5 - B5A3 + e_{331133}(1 + B5) - C5 + u_5.$$

Where$A1 = \dot{y}(21) + \dot{y}(26); A2 = \dot{y}(22) + \dot{y}(27); A3 = \dot{y}(23) + \dot{y}(28); A4 = \dot{y}(24) + \dot{y}(29); A5 = \dot{y}(25) + \dot{y}(30); B1 = A2(k1 * (y(11) + y(16)) - k2((\dot{y}(11) + \dot{y}(16))))/k1(\dot{y}(12) + \dot{y}(17)); C1 = e2(k1(y(11) + y(16)) - k2(\dot{y}(11) + \dot{y}(16)))/k1(\dot{y}(12) + \dot{y}(17)); D1 = k2(y(11) + y(16))(\dot{y}(1) + \dot{y}(6)); B2 = A1(k1(y(12) + y(17)) + k2(\dot{y}(12) + \dot{y}(17)))/k1(\dot{y}(11) + \dot{y}(16)); C2 = k2(y(12) + y(17))(\dot{y}(1) + \dot{y}(6)); B3 = A4(k1(y(13) + y(18)) + k2(y^{\cdot}(13) + \dot{y}(18)))/k1(\dot{y}(14) + \dot{y}(19)); C3 = k2(y(13) + y(18))(\dot{y}(1) + \dot{y}(6)); B4 = A5(k1(y(14) + y(19)) + k2(\dot{y}(14) + \dot{y}(19)))/k1(\dot{y}(15) + \dot{y}(20)); C4 = k2(y(14) + y(19))(\dot{y}(1) + \dot{y}(6)); B5 = A3(k1(y(15) + y(20)) + k2(\dot{y}(15) + \dot{y}(20)))/k1(\dot{y}(13) + \dot{y}(18)); C5 = k2(y(15) + y(20))(\dot{y}(1) + \dot{y}(6)); k1 = \alpha(t); k2 = \alpha^{\cdot}(t);$

With the error dynamics (20), if appropriate u_1, u_2, u_3, u_4 and u_5 are chosen suchthat equilibrium $(0, 0, 0, 0, 0)$ of the error system is stable and unchanged thenstabilization would be realized leading to stable synchronisation of the system. If $\eta_1 = e_{111111}$, its time derivative is $\dot{e}_{111111}$ and we can write the first part of (20) as

$$\dot{\eta}_1 = A1 - B1A2 - e_{221122}(1 - C1) + D1 + u_1, \quad (21)$$

Stabilise (21) using the Lyapunov function

$$v1 = \frac{1}{2}\eta_1^2 \quad (22)$$

By substituting for $\dot{\eta}_1$ in the derivative of (22), choosing $e_{221112} = \alpha_1(\eta_1) = 0$ as a virtual controller and$u_1 = -e_{111111} - A1 + B1A2 + e_{221122}(1 - C1) - D1 + e_{111111}k$, to have

$$\dot{v}_1 = -(1 - k)\eta_1^2 \leq 0. \quad (23)$$

Thus, $\dot{v}_1$ is negative definite if $k \leq 0$ showing that the subsystem $(\dot{\eta}_1)$ is asymptotically stable. Since the error between e_{221122} and $\alpha_1(\eta_1)$ is estimative as $\eta_2 = e_{221122}$and its derivative is written as$\dot{\eta}_2 = e_{221122}$, the $(\dot{\eta}_1, \dot{\eta}_2)$ subsystems is

$$\dot{\eta}_1 = -\eta_1(1 - k) + \eta_2,$$
$$\dot{\eta}_2 = A4 - B2A1 + e_{111111}(1 - B2) - C4 + u_2; \quad (24)$$

Stabilise (24) by choosing the second Lyapunov function given as

$$v_2 = v_1 + \frac{1}{2}\eta_2^2 \quad (25)$$

By substituting for $\dot{\eta}_2$ in the derivative of (25) choosing $e_{111111} = \alpha_2(\eta_2) = 0$ as a virtual controller and choosing $u_2 = -e_2 - A2 + B2 * A1 + e_{111111}(1 - B2) + C2 + e_{221122}k; \dot{v}_2 = -(1 - k)(\eta_1^2 + \eta_2^2) \leq 0,$ (26)

Thus, $\dot{v}_2$ is negative definite if $k \leq 0$ showing that the subsystem (η_1, η_2) is assymptotically stable. $Let\ \eta_3 = e_{331133}$ and its derivative is written as $= \eta_2 e_{331133}$,the (η_1, η_2, η_3) subsystem is

$$\dot{\eta}_1 = -\eta_1(1 - k) + \eta_2,$$
$$\dot{\eta}_2 = -\eta_2(1 - k) + \eta_1$$
$$\dot{\eta}_3 = A3 - B3A4 + e_{441144}(1 + B3) - C3 + u_3; \quad (27)$$

Stabilise (27) by choosing the third Lyapunov function given as

$$v_3 = v_2 + \frac{1}{2}\eta_3^2 \quad (28)$$

By substituting for $\dot{\eta}_3$in the derivative of (28) choosing $\eta_4 = \alpha_3(\eta_4) = 0$ as avirtual controller $and\ u_3 = -e_{331133} - A3 + B3A4 - e_{441144}(1 + B3) + C3 + e_{331133}k$;,

$$\dot{v}_3 = -(1 - k)(\eta_1^2 + \eta_2^2 + \eta_3^2) \leq 0, \quad (29)$$

Thus, $\dot{v}_3$ is negative definite if k ≤ 0 showing that the subsystem (η_1, $\eta_2\eta_3$) is assymptotically stable. Letη_4= $e_{441144}$and its derivative$\dot{e}_{412}$. The(η_1, $\eta_2\eta_3$,η_4) subsystem Is

$$\begin{aligned}\dot{\eta_1} &= -\eta_1(1-k)+\eta_2,\\ \dot{\eta_2} &= -\eta_2(1-k)+\eta_1,\\ \dot{\eta_3} &= -\eta_3(1-k)-2\eta_4,\\ \dot{\eta_4} &= A4-B4A5+e_{551155}(1+B5)-C4+u_4; \end{aligned} \quad (30)$$

Stabilise (30) by defining the fourth Lyapunov function given as

$$u_4 = u_3+\frac{1}{2}\eta_4^2 \quad (31)$$

By substituting for $\dot{\eta}_4$ in the derivative of (31) and choosing u_4=−e_{441144}−A4+ $B4A5 - e_{551155}(1+B4)+C4+e_{441144}k$; to have

$$\dot{v}_4 = -(1-k)(\eta_1^2+\eta_2^2+\eta_4^2) \le 0, \quad (32)$$

Thus, $\dot{v}_4$is negative definite if k ≤ 0, showing that the subsystem (η_1,η_2,η_3,η_4) is assymptotically stable. Let η_5 = $e_{551155}$and its derivative be e_{551155}, the whole system is

$$\begin{aligned}\dot{\eta_1} &= -\eta_1(1-k)+\eta_2,\\ \dot{\eta_2} &= -\eta_2(1-k)+\eta_1,\\ \dot{\eta_3} &= -\eta_3(1-k)-\eta_4,\\ \dot{\eta_4} &= -\eta_4(1-k)-\eta_5,\\ \dot{\eta_5} &= A5-B5A3+e_{331133}(1+B5)-C5+u5 \ldots \end{aligned} \quad (33)$$

Stabilise (33) by defining the fifth Lyapunov function given as

$$u_5 = u_4+\frac{1}{2}\eta_5^2 \quad (34)$$

By substituting for $\dot{\eta_5}$in the derivative of (34) and choosing $u_5 = -e_{551155} - A5 + B5A3 - e_{331133}(1+B5)+C5+e_{551155}k$,to have

$$\dot{v_5} = -(1-k)(\eta_1^2+\eta_2^2+\eta_3^2+\eta_4^2+\eta_5^2) \le 0, \quad (35)$$

Thus, $\dot{v_5}$is negative definite if k ≤ 0. The whole system is expressed as

$$\begin{aligned}\dot{\eta_1} &= -\eta_1(1-k)+\eta_2,\\ \dot{\eta_2} &= -\eta_2(1-k)+\eta_1,\\ \dot{\eta_3} &= -\eta_3(1-k)-\eta_4,\\ \dot{\eta_4} &= -\eta_4(1-k)-\eta_5,\\ \dot{\eta_5} &= -\eta_5(1-k)-\eta_3. \end{aligned} \quad (36)$$

Summarily, the controllers for multiswitching combination synchronisation of the hyperchaotic magneto-hydrodynamical system is

$$\begin{aligned} u_1 &= -e_{111111}-A1+B1A2+e_{221122}(1-C1)-D1+e_{111111}k,\\ u_2 &= -e2 \;-A2+B1*A1+e_{111111}(1-B2)+C2+e_{221122}k,\\ u_3 &= -e_{331133}-A3+B3A4-e_{441144}(1+B3)+C3+e_{331133}k,\\ u_4 &= -e_{441144}-A1+B1A5-e_{551155}(1+B4)+C4+e_{441144}k,\\ u_5 &= -e_{551155}-A5+B5A3-e_{331133}(1+B5)+C5+e_{551155}k \end{aligned} \quad (37)$$

3. Numerical simulation and results

The numerical simulations are presented here in order to verify the effectiveness of the controllers u_1, u_2, u_3, u_4 and u_5 for this study are presented in (37). Using the Matlab at ode45 for the numerical simulation and the system parameters chosen asa = 1.0,b = 14.47, c = 5.0,d = 0.1081,e = 0.0108 when the initial conditions were y(1) = −0.1, y(2) = 0.0, y(3) = 0.0, y(4) = 0.0, y(5) = 0.0, y(6) = −0.1, y(7) = 0.0, y(8) = 0.0, y(9) = 0.0, y(10) = 0.0, y(11) = −0.1, y(12) = 0.0, y(13) = 0.0, y(14) = 0.0, y(15) = 0.0, y(16) = 0.1, y(17) = 0.0, y(18)= 0.0, y(19) = 0.0, y(20) = 0.0, y(21) = 0.1, y(22) = 0.0, y(23) = 0.0, y(24) = 0.0, y(25) = 0.0, y(26) = 0.1, y(27) = 0.0, y(28) = 0.0, y(29) = 0.0 and y(30) = 0.0. The controllers u_i ($i = 1, 2, \ldots 5$) were activated at t ≥ 200. Theresult for multi-switching combinationsynchronised states e_{111111} and $e_{221122}$are shown in figure 4.16, for$e_{331133}$and e_{441144} in figure 4.17, the result fore_{551155} and a combined result for the whole system are shown in figure 4.18. The choice of t ≥ 200s wasto allow an appreciable transient of thetime series before the controllers were activated. This results signify thatmulti-switching combination double compound synchronisation of the 5Dhyperchaotic magnetohydrodynamic system has been achieved.

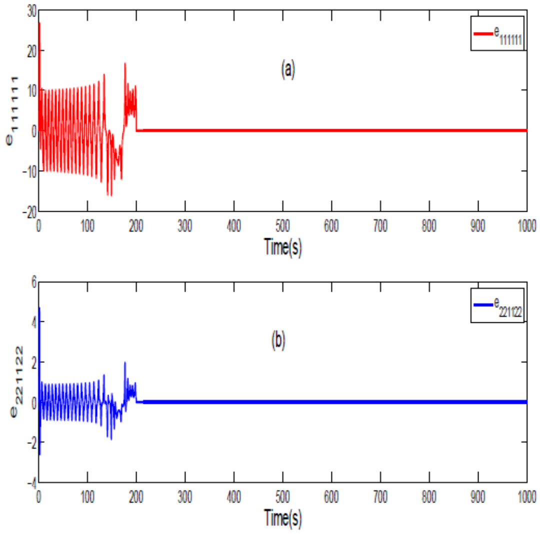

Figure 1 (a) Multi switched double compound combination synchronisation for state e_{111111} (b) Multi switched double compound combination synchronisation for state e_{221122} of the 5d magnetohydrodynamic system. when c = 5.0, d = 0.1081, e = 0.0108 when the initial conditions were y(1) = −0.1, y(2) = 0.0, y(3) = 0.0, y(4) = 0.0, y(5) = 0.0, y(6) = −0.1, y(7) = 0.0, y(8) = 0.0, y(9) = 0.0, y(10) = 0.0, y(11) = −0.1, y(12) = 0.0, y(13) = 0.0, y(14) = 0.0, y(15) = 0.0, y(16) = 0.1, y(17) = 0.0, y(18)= 0.0, y(19) = 0.0, y(20) = 0.0, y(21) = 0.1, y(22) = 0.0, y(23) = 0.0, y(24) = 0.0, y(25) = 0.0, y(26) = 0.1, y(27) = 0.0, y(28) = 0.0, y(29) = 0.0 and y(30) = 0.0.

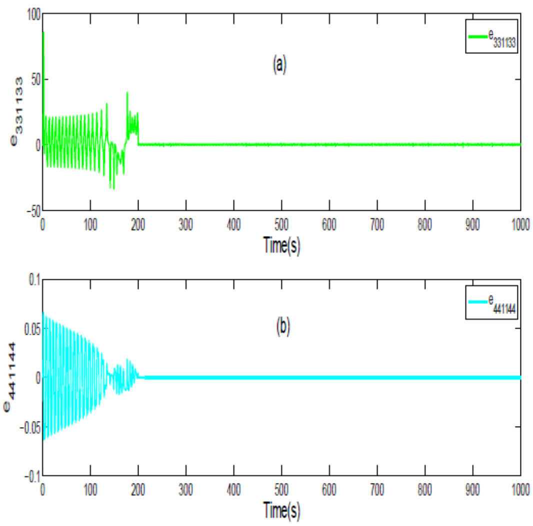

Figure 2 (a) Multi switched double compound combination synchronisation for state ee_{331133} (b) Multi switched double compound combination synchronisation for state e_{441144} of the 5d magnetohydrodynamic system. when c = 5.0, d = 0.1081, e = 0.0108 when the initial conditions were y(1) = −0.1, y(2) = 0.0, y(3) = 0.0, y(4) = 0.0, y(5) = 0.0, y(6) = −0.1, y(7) = 0.0, y(8) = 0.0, y(9) = 0.0, y(10) = 0.0, y(11) = −0.1, y(12) = 0.0, y(13) = 0.0, y(14) = 0.0, y(15) = 0.0, y(16) = 0.1, y(17) = 0.0, y(18)= 0.0, y(19) = 0.0, y(20) = 0.0, y(21) = 0.1, y(22) = 0.0, y(23) = 0.0, y(24) = 0.0, y(25) = 0.0, y(26) = 0.1, y(27) = 0.0, y(28) = 0.0, y(29) = 0.0 and y(30) = 0.0.

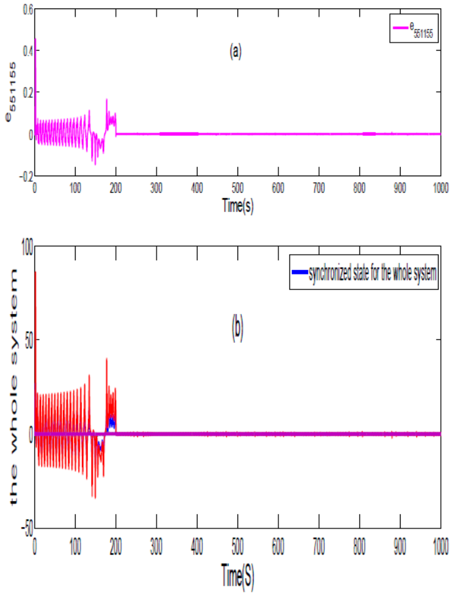

Figure 3 (a) Multi switched double compound combination synchronisation for state e_{551155} ,(b) Multi switched double compound combination synchronisation for the whole system of the 5d magnetohydrodynamic system. when c = 5.0, d = 0.1081, e = 0.0108 when the initial conditions were y(1) = −0.1, y(2) = 0.0, y(3) = 0.0, y(4) = 0.0, y(5) = 0.0, y(6) = −0.1, y(7) = 0.0, y(8) = 0.0, y(9) = 0.0, y(10) = 0.0, y(11) = −0.1, y(12) = 0.0, y(13) = 0.0, y(14) = 0.0, y(15) = 0.0, y(16) = 0.1, y(17) = 0.0, y(18)= 0.0, y(19) = 0.0, y(20) = 0.0, y(21) = 0.1, y(22) = 0.0, y(23) = 0.0, y(24) = 0.0, y(25) = 0.0, y(26) = 0.1, y(27) = 0.0, y(28) = 0.0, y(29) = 0.0 and y(30) = 0.0.

4. Conclusion

In this paper, we presented the results of Multiswitching Double Compound Combination Synchronisation of 5-Dimensional Hyperchaotic Systems in Application to 5-Dimensional Hyperchaotic Magnetohydrodynamic Systems which uses the Lyapunov stability theory. We have illustrated numerically the effectiveness of the proposed method for the multiswitching and synchronization of the systems. The multiswitching and synchronization of the systems were achieved.

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