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CONSEQUENTIAL DYNAMICS OF VIOLENCE: MATHEMATICAL INSIGHTS FROM VICTIMS FOR EFFECTIVE CONTROL

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Abstract

We proposed, developed, and analysed a mathematical model of intergroup behavioural dynamics in a community that is divided between two distinguishable social groups. The model, built using a system of simple ordinary differential equations, is based on intuitively conceived assumptions on violence that are verifiable and conformable to findings in sociological studies. We obtained and performed sensitivity analyses on the basic threshold quantity to have a better understanding of the dynamics of violence in the society. We established correctional conditions that suggest rehabilitation possibilities and potentials.

***Keywords*:** Terrorism, violence, victim, rehabilitation, mathematical model

1. Introduction

The unlawful and/or unconstitutional expression of internalised radical ideology or thinking has resulted in dire psychological harm, resource deprivation and even collateral damages and losses. The World Health Organisation has defined violence as the intentional use of physical force or power, threatened or actual, against oneself, another person, or against a group or community, that either results in or has a high likelihood of resulting in injury, death, psychological harm, maldevelopment or deprivation [1]. Arguably, violence has been a part of the human experience. And like the continual complication of these experiences, it has undergone consequential modification, transformation, and sophistication [1 – 5]. The impact of violence, to say the least, is enormous and dire [1]. Due to its peculiarity and spread, it is difficult to obtain a precise global incidence of the impact violence, however, an annual average estimate puts over a million human fatality, with even more instances of non-fatal injuries translating into billions of US dollars in annual health care expenditures worldwide, and billions more for national economies in terms of days lost from work, law enforcement and lost investment [1]. Indeed, violence has stretched individuals, families, and communities beyond tolerance limits. It is therefore of extreme importance to identify, isolate and understand the influencing determinants that contribute to aggressive predispositions [1, 4, 5, 7]. Broadly speaking, violence, on the one hand, is categorised along the corridors of intense self-directed, interpersonal and collective physical aggression; and on the other hand, characterised into physical, sexual, psychological and deprivation or neglect [1]. Scores of lives and enormous resources have been destroyed and are still being destroyed because of escalating ideological and cultural differences, authoritarianism, perceived marginalisation and injustices in resource allocation and distribution imbalances [6, 7]. Incidences of violence and crisis are easily sparked by the deterioration of moral uprightness and the internalisation of extreme beliefs and ideologies [2, 4, 5], and this is often widely perceived as consequences of societal failures, excesses, and injustices. The rise in crisis and violence, together with the attendant effect on victims is, in recent times, very worrisome [6]. It is expedient, therefore, to correct this dysfunctional mind-set through a systematic capacity building initiative which primarily focuses on assisting criminals to reconstruct personally meaningful and socially acceptable identities. Victims of criminality go through varying degrees of traumatic

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experiences [4 – 7]. The programme for their rehabilitation is often either nonexistence or poorly managed [6] as could easily be observed in internally displaced persons' (IDPs) camps spread across the country. The combined effect of the impact of insecurity on the country's growth and development has been colossal and dire [2, 3]. The paper aims to contribute to the ongoing insecurity management processes in the country by proposing a new mathematical model that will suggest a paradigm for studying terrorism and its consequences.

2. Model formulation

The formation of our model follows the natural change pattern of opinions due to the interactional dynamics in a society. Therefore, for the model system (1), we make the following assumptions. The total population $N(t)$, is divided into the following subgroups $S(t)$ to represent a susceptible group of individuals in the population who, though not yet radicalised, are at risk of adopting extreme ideologies, $H(t)$ represents the subgroup of individuals in the community that have no propensity for radicalisation because of their strong will power to resist being easily swayed to adopting violent predisposition, $I_V(t)$ to represent individuals in the community who have had sustained and radicalised social interaction with violence predisposed individuals and have recently become fanatical; $V(t)$ to represent individuals who have become extremely aggressive and are therefore completely disposed to violence and violent acts in the community, and finally, $T(t)$, to denote the group of individuals who are being deradicalized through some organised rehabilitation programme. The total population is therefore given as $N(t)=S(t)+H(t)+I_V(t)+V(t)+T(t)$, and the model is described by the system of ordinary differential equations below.

$$
\begin{aligned}
\frac{dS}{dt} &= (1-k)\Delta + \alpha_3(1-h)T - \beta(I_V + \eta_1 V + \eta_2 T)S - (\psi + \mu)S, \\
\frac{dH}{dt} &= k\Delta + \psi S + \alpha_1 I_V + \alpha_2 V + \alpha_3 hT - \nu\beta(I_V + \eta_1 V + \eta_2 T)H - (\pi_1 + \mu)H, \\
\frac{dI_V}{dt} &= \beta(I_V + \eta_1 V + \eta_2 T)S + \nu\beta(I_V + \eta_1 V + \eta_2 T)H - (\pi_2 + \phi + \alpha_1 + \mu)I_V, \\
\frac{dV}{dt} &= \phi I_V + (\alpha_3 + \delta_1 + \alpha_2 + \mu)V, \\
\frac{dT}{dt} &= \pi_1 H + \pi_2 I_V + \pi_3 V - (\alpha_3 + \delta_2 + \mu)T.
\end{aligned}
\tag{1}
$$

The model (1) is built on the assumptions that a fraction $1-k$, where $0 \le k < 1$, of all recruitees, Δ, into the population are added to *S* while the remaining *k* are added to the subpopulation *H* at time *t* and each individual in the population has the same natural death rate of μ, individuals in *V* and *T* have, in addition, a violence induced death rate δ_1 and δ_2, where it is assumed that $\delta_2 < \delta_1$. The remaining parameter terms of the model system and their specific roles and functions in the model are presented in Table 2.

Table 1: Description of variables for the model of the dynamics of violence

le	**›tion**
	ible individuals
	ıals with no propensity for radicalisation (non-susceptible individuals)
	ıals who have recently become fanatical
	ıals who are predisposed to violence
	ıals who are being deradicalised

Table 2: Description of parameters for the model of the dynamics of violence

le	**ιtion**	
	ment rate into the population	ıals km^{-2} month^{-1}
	death rate	
y, a_1, a_2, a_3	ιity rate coefficients	
	isation exposure rate	ıals km^{-2} month^{-1}
h_1, h_2, n	ation parameters	
	ι of recruited individuals who are vulnerable to violence	
	ι of recruited individuals who are not easily swayed to violence	
p_1, p_2, p_3	alization rates for *H*, I_V and *V* classes	
	sion rate of I_V class to *V* class	
	e induced death rate	

Basic properties of the model

By using various theorems and performing some algebraic computation, we can analyse the model system.

Invariant set

We establish the feasible region and bound of the model (1) by stating and proving the theorem below.

Theorem 1. *The region* Ω *given by*

$$\Omega = \left\{ \left(S(t), H(t), I_V(t), V(t), T(t)\right) \in \mathbb{R}^5_+ \,\middle|\, N \le \frac{\Lambda}{\mu} \right\}.$$

is positively invariant and attracting with respect to the model system (1)

Proof

Let $\left(S(t), H(t), I_V(t), V(t), T(t)\right)$ be any solution of the model (1) with nonnegative initial condition

$\left\{S(t) \geq 0, H(t) \geq 0, I_V(t) \geq 0, V(t) \geq 0, T(t) \geq 0\right\}$.

It can be seen from the first equation of the model (1) that

$$\frac{dS}{S} \geq -\left[\beta\left(I_V + \eta_1 V + \eta_2 T\right) + (\psi + \mu)\right]dt$$

Thus, on integrating it, we have

$$\ln S(t) \geq -\left[\beta\left(I_V + \eta_1 V + \eta_2 T\right) + (\psi + \mu)\right] + C_0.$$

That is $S(t) \geq C_0 e^{-\left[\beta\left(I_V + \eta_1 V + \eta_2 T\right) + (\psi + \mu)\right]t}$ where C_0 is a constant of integration. Further, by applying the initial conditions at $t = 0$, we get $C_0 = S(t)$.

Therefore, $S(t) > S(0) e^{-\left[\beta\left(I_V + \eta_1 V + \eta_2 T\right) + (\psi + \mu)\right]t} \geq 0.$

Obviously, $S(t)$ is a nonnegative function of t, thus, $S(t)$ is always positive.

Using the same procedure, with $\frac{dH}{H} \geq -\left[\beta\left(I_V + \eta_1 V + \eta_2 T\right) + (\psi + \mu)\right]dt$, then

$$E_V(t) \geq C_1 e^{-\left[\beta\left(I_V + \eta_1 V + \eta_2 T\right) + (\psi + \mu)\right]t},$$

where C_1 is a constant of integration. Thus, applying initial condition at $t = 0$, then $C_1 = H(0)$. It therefore follows that

$H(t) \geq H(0)e^{-[\beta(I_V+\eta_1 V+\eta_2 T)+(\psi+\mu)]t} \geq 0.$

Similarly, $I_V(t) \geq I_V(0)e^{-(\phi+\mu)t} \geq 0$, $V(t) \geq V(0)e^{-(\delta_1+\mu)t}$ and $T(t) \geq T(0)e^{-(\alpha+\delta_2+\mu)t} \geq 0.$

It can be noted that with the rate of change of the total population given by $\frac{dN}{dt} = \Delta - \mu N - \delta_1 R - \delta_2 T$, then $\frac{dN}{dt} + \mu N \leq \Delta.$ Therefore $N(t) \leq \frac{\Delta}{\mu}\left(1 + Ce^{-\mu t}\right)$, where C is the constant of integration.

It therefore follows that $\lim_{t\to\infty} N(t) \leq \frac{\Delta}{\mu}$, which proves the bound of the solutions in the domain Ω. hence, all feasible solution set of the population under consideration enter the region

$$\Omega = \left\{ (S(t), H(t), I_V(t), V(t), T(t)) \in \Re_+^5 \middle| S \geq 0, H \geq 0, I_V \geq 0, V \geq 0, T \geq 0, N \leq \frac{\Delta}{\mu} \right\}$$

Existence of violence free equilibrium (VFE)

The free equilibrium points, stationary points of the model system (1) that ensures the nonexistence of radicalisation in the population, are $I_V^0 = 0, V^0 = 0, T^0 = 0$, so that by equating the subsystems of the model (1) to zero, the system can be solved to obtain $S^0 = \frac{\Delta}{\mu+\psi}$, $H^0 = \frac{\Delta[\psi + k(\mu+\psi)]}{(\pi_1+\mu)(\mu+\psi)}$.

Thus, the DFE for the model system is $E^0 = \left(S^0, H^0, I_V^0, V^0, T^0\right) = \left\{\frac{\Delta}{\mu+\psi}, \frac{\Delta[\psi+k(\mu+\psi)]}{(\pi_1+\mu)(\mu+\psi)}, 0,0,0\right\}$

The basic reduction numbers

We apply the strategy of the next generation matrix to determine the control reproduction number, R_0, of the dynamics of violence model. Thus, the requisite matrices for the violence recruits' terms, F, and the remaining violence initiation terms, V for the model (1), are respectively

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta\left(S^0 + vH^0\right) & \eta_1\beta\left(S^0 + vH^0\right) & \eta_2\beta\left(S^0 + vH^0\right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} \pi_1 + \mu & 0 & 0 & 0 \\ 0 & \pi_2 + \phi + \alpha_1 + \mu & 0 & 0 \\ 0 & -\phi & \pi_3 + \delta_1 + \alpha_2 + \mu & -\alpha_3 h \\ -\pi_1 & -\pi_2 & -\pi_3 & \alpha_3 + \delta_2 + \mu \end{pmatrix}.$$

Thus,

$$R_0 = \frac{\beta\left(S^0 + vH^0\right)\left(a_2 + a_5\eta_1 + a_3\eta_2\right)}{a_2 K_0}, \qquad (3)$$

where $a_2 = K_1K_3K_4 - h\alpha_3[\pi_3 K_1 + \pi_1(\alpha_2 + K_3)]$, $a_3 = K_1(\phi\pi_3 + \pi_2 K_3) + \pi_1(\phi\alpha_2 + \alpha_1 K_3)$,
$a_5 = \phi K_1 K_4 + h\alpha_3[\pi_2 K_1 + \pi_1(\alpha_2 - \phi)]$.

The following result follows from Theorem 2 of [8].

Lemma 1. *The Violence Equilibrium (VFE) of the dynamics of violence model (1), is locally asymptotically stable (LAS) if* $R_0 < 1$ *and unstable if* $R_0 > 1$.

The threshold quantity R_0 is the recruitment number for violence [9]. It measures the average number of violence – recruits initiated by a single violence predisposed individual in a population where a certain fraction of violence predisposed individuals are rehabilitated

Analysis of R_0. We perform analysis on the threshold quantity R_0 to determine the most effective stage in the dynamics of violence to initiate and execute rehabilitation. In other words, we establish the consequential effect of executing rehabilitation on individuals who are victims of violence (modelled by the rate π_1) of those being initiated into violence (modelled by the rate π_2) or those perpetrating violence (modelled by the rate π_3) on controlling the escalation of violence in the community. It can easily be seen from (3) that

$$\lim_{\pi_1\to\infty} R_0 = \frac{\beta(S^0+vH^0)\{a_2+\eta_1 a_5+\eta_2[\phi(\pi_3K_1+\pi_1\alpha_2)+K_3(\pi_2K_1+\pi_1\alpha_1)]\}}{K_2[K_3K_4-h\alpha_3(\pi_3+\alpha_2+K_3)]}>0, \tag{4}$$

$$\lim_{\pi_2\to\infty} R_0 = \frac{\beta K_1(S^0+vH^0)(\eta_1\alpha_3 h+\eta_2K_3)}{K_1K_3K_4-h\alpha_3[\pi_3K_1+\pi_1(\alpha_2+K_3)]}>0, \tag{5}$$

and,

$$\lim_{\pi_3\to\infty} R_0 = \frac{\beta(S^0+vH^0)\{K_1K_4+\eta_2[\pi_1\alpha_1+K_1(\phi+\pi_2)]-h\alpha_3(\pi_1+K_1)\}}{K_2[K_1K_4-h\alpha_3(\pi_1+K_1)]}>0 \tag{6}$$

It therefore follows from section 3.1.2 of [10] that a sufficiently effective violence control programme that focuses on rehabilitating individuals who are still in the violence initiation stage (at rate $\pi_2\to\infty$), or even the violence predisposed individuals themselves (at rate $\pi_3\to\infty$), has the capacity to adequately control or substantially manage violence in the community once such efforts can ensure that the righthand sights of (4), (5) and (6) are both less than 1.

Vital information can further be obtained by computing the partial derivatives on R_0 with respect to the rehabilitation parameters π_1,π_2 and π_3.

$$\frac{\partial R_0}{\partial \pi_1} = \beta(S^0+vH^0)\frac{a_5\eta_1+a_3\eta_2+a_2(1+a_0\eta_2+a\eta_1+a_2)}{a_2^2K_2}>0, \tag{7}$$

$$\frac{\partial R_0}{\partial \pi_2}\ \beta(S^0+vH^0)\frac{K_1K_2(h\alpha_3\eta_1+\eta_2K_3)+\eta_2a_3+\eta_1a_5+a_2}{a_2K_2^2}>0, \tag{8}$$

and

$$\frac{\partial R_0}{\partial \pi_3} = \beta(S^0+vH^0)\frac{a_1a_2+(\eta_2a_3+a_4-\eta_1a_5)[K_1K_4-h\alpha_3(\pi_1+K_1)]}{a_2^2K_2} \tag{9}$$

where $a=\phi K_4+h\alpha_3(\pi_2+\alpha_1-\phi), a_0=\phi(\alpha_2-\pi_2)+(\pi_2+\alpha_1)K_3$,

$a_1=(\pi_1\alpha_1\eta_1-h\pi_1\alpha_3)+K_1[K_4+\eta_2(\phi+\pi_2)-h\alpha_3], a_2=K_1K_3K_4-h\alpha_3[\pi_3K_1+\pi_1(\alpha_2+K_3)]$,

$a_3=K_1(\phi\pi_3+\pi_2K_3)+\pi_1(\phi\alpha_2+\alpha_1K_3), a_4=K_1(K_3K_4-h\pi_3\alpha_3)-h\pi_1\alpha_3(\alpha_2+K_3)$,

$a_3=K_1(\phi\pi_3+\pi_2K_3)+\pi_1(\phi\alpha_2+\alpha_1K_3), a_4=K_1(K_3K_4-h\pi_3\alpha_3)-h\pi_1\alpha_3(\alpha_2+K_3)$,

$a_5=\phi K_1K_4+h\alpha_3[\pi_2K_1+\pi_1(\alpha_1-\phi)]$.

Thus, for the case $\pi_3=0$ (that is, when only violence predisposed individuals undergo the rehabilitation programme), then $\frac{\partial R_0}{\partial \pi_2}<0,$ provided

$$\eta_1<\Pi_1=\frac{h\alpha_3[\pi_3K_1+\pi_1(\alpha_2+K_3)]-[\eta_2(a_3+K_1K_2K_3)+K_1K_3K_4]}{a_5+h\alpha_3K_1K_2}. \tag{10}$$

It could therefore be understood that any measure aimed at tackling violence in the community where rehabilitation

programmes are specifically designed and carried out on identified violence contemplative individuals would only be effective in reducing the escalation of violence if $\eta_1 < \Pi_1$. Further, such programme would fail tackle the nurturing of violent individuals as well as the escalation of violence if $\eta_1 = \Pi_1$ and worst still such a programme regime would likely encourage recruitment of violent individuals and the aggravation of violence in the community if $\eta_1 > \Pi_1$.
It also be noted from (5) that

$$\eta_2 < \Pi_2 = \frac{h\alpha_3\left[K_1\pi_3 + \pi_1(\alpha_2 + K_3)\right] - \left[K_1K_3K_4 + \eta_1(\alpha_5 + h\alpha_3K_1K_2)\right]}{\alpha_3 + K_1K_2K_3}. \qquad (11)$$

Thus, we conclude that rehabilitating individuals who only contemplative individuals would only be effective in reducing the escalation of violence if $\eta_2 < \Pi_2$. Further, such programme would fail tackle the nurturing of violent individuals as well as the escalation of violence if $\eta_2 = \Pi_2$, and worst still such a programme regime would likely encourage recruitment of violent individuals and the aggravation of violence in the community if $\eta_2 > \Pi_2$.
The result is summarised below.
Lemma 2. Rehabilitation of violence contemplative individuals in the community (including individuals who are being radicalised to imbibe violent dispositions) will only be effective if $\eta_1 < \Pi_1$ or $\eta_2 < \Pi_2$, ineffective if $\eta_1 = \Pi_1$ or $\eta_2 = \Pi_2$, and detrimental if $\eta_1 > \Pi_1$ or $\eta_2 > \Pi_2$.
Global stability of VFE
Theorem. Let the following inequalities hold in Ω:

$$\left[q_3\phi + q_2\eta_1\beta\left(S^* + \nu H^*\right)\right] < \frac{1}{3}q_2a_1q_3\left(\pi_3 + \delta_1 + \mu\right)$$

and

$$\left[q_4^2\pi_2\pi_3 + q_3\alpha_3h + \frac{\eta_2\beta^2S^*}{\lambda}\left(S^* + \nu H^*\right)\right]^2 < \frac{1}{12\lambda}a_1\beta S^*q_4\left(\alpha_2 + \delta_2 + \mu\right)$$

where

$$q_3 > max\left\{\frac{3\left(\nu\eta_1\beta H^*\right)^2}{\left(\pi_3 + \delta_1 + \mu\right)\left[\nu\lambda + \left(\pi_1 + \mu\right)\right]}, \frac{\left(\eta_1\beta S^*\right)^2}{\left(\pi_3 + \delta_1 + \mu\right)\left[\lambda + \left(\psi + \mu\right)\right]}\right\} \text{ and}$$

$$q_4 > \frac{3\left[\alpha_3(1-h) - \eta_2\beta S^*\right]^2}{\left(\alpha_2 + \delta_2 + \mu\right)\left[\lambda + \left(\psi + \mu\right)\right]},$$

then E^* is globally asymptotically stable.
Proof
Consider the following positive definite function about E^*

$$U = \frac{1}{2}\left[q_0\left(S - S^*\right)^2 + q_1\left(H - H^*\right)^2 + q_2\left(I_V - I_V^*\right)^2 + q_3\left(V - V^*\right)^2 + q_4\left(T - T^*\right)^2\right] \qquad (12)$$

where q_0, q_1, q_2, q_3 and q_4 are positive constants that will be appropriately chosen.
Then, simplification of the corresponding time derivative of U using the equations in the right-hand sides of the model equation (1) gives

$$\dot{U} = q_0\left(S - S^*\right)\left\{-\left[\lambda + \left(\psi + \mu\right)\right]S_1 - \beta S^*I_{V1} - \eta_1\beta S^*V_1 + \left[\alpha_3(1-h) - \eta_2\beta S^*\right]T_1\right\}$$
$$+ q_1\left(H - H^*\right)\left\{\psi S_1 - \left[\nu\lambda + \left(\pi_1 + \mu\right)H_1 - \nu\beta H^*I_{V1} - \nu\eta_1\beta H^*V_1 - \nu\eta_2\beta H^*T_1\right]\right\}$$

$$+q_2\left(I_V - I_V^*\right)\left\{\lambda S_1 + \nu\lambda H_1 + a_1 I_{V1} + \eta_1\beta\left(S^* + \nu H^*\right)V_1 + \eta_2\beta\left(S^* + \nu H^*\right)T_1\right\}$$
$$+q_3\left(V - V^*\right)\left[\phi I_{V1} - \left(\pi_3 + \delta_1 + \mu\right)V_1 + \alpha_3 h T_1\right]$$
$$+q_4\left(T - T^*\right)\left[\pi_1\right]H_1 + \pi_2 I_{V1} + \pi_3 V_1 - \left(\alpha_2 + \delta_2 + \mu\right)T_1$$

Thus,

$$q_1\psi^2 < \frac{1}{4}q_0\left[\lambda + (\psi + \mu)\right]\left[\nu\lambda + (\pi_1 + \mu)\right]$$

$$\left(q_0\beta S^* - q_2\lambda\right)^2 < \frac{1}{4}q_0 q_2 a_1\left[\lambda + (\psi + \mu)\right]$$

$$q_0\left(\eta_1\beta S^*\right) < \frac{1}{3}q_3\left(\pi_3 + \delta_1 + \mu\right)\left[\lambda + (\psi + \mu)\right]$$

$$q_0\left[\alpha_3(1-h) - \eta_2\beta S^*\right]^2 < \frac{1}{3}q_4\left(\alpha_2 + \delta_2 + \mu\right)\left[\lambda + (\psi + \mu)\right]$$

$$\left(q_1\nu\beta H^* - q_2\nu\lambda\right)^2 < \frac{1}{4}a_1 q_1 q_2\left[\nu\lambda + (\pi_1 + \mu)\right]$$

$$q_1\left(\nu\eta_1\beta H^*\right)^2 < \frac{1}{3}q_3\left(\pi_3 + \delta_1 + \mu\right)\left[\nu\lambda + (\pi_1 + \mu)\right]$$

$$q_1\left(q_1\nu\eta_2\beta H^* - q_4\pi_1\right)^2 < \frac{1}{4}q_1 q_4\left(\alpha_1 + \delta_1 + \mu\right)\left[\nu\lambda + (\pi_1 + \mu)\right]$$

After minimising the lefthand side and maximising the righthand side of the inequalities, the stability conditions can be appropriately obtained as follows.

Let $q_0 = q_1 = 1.$ Then the following can be obtained

$$q_2 = \frac{\beta S^*}{\lambda},$$

$$q_3 > max\left\{\frac{3\left(\nu\eta_1\beta H^*\right)^2}{\left(\pi_3 + \delta_1 + \mu\right)\left[\nu\lambda + (\pi_1 + \mu)\right]}, \frac{\left(\eta_1\beta S^*\right)^2}{\left(\pi_3 + \delta_1 + \mu\right)\left[\nu\lambda + (\pi_1 + \mu)\right]}\right\}$$ and

$$q_4 > \frac{3\left[\alpha_3(1-h) - \eta_2\beta S^*\right]}{\left(\alpha_2 + \delta_2 + \mu\right)\left[\lambda + (\psi + \mu)\right]}$$

Then the conditions

$$q_3\phi + q_2\eta_1\beta\left(S^* + \nu H^*\right) < \frac{a_1 q_2 q_3\left(\pi_3 + \delta_1 + \mu\right)}{3}$$

and

$$\left[q_4^2\pi_2\pi_3 + q_3\alpha_3 h + q_2\eta_2\beta\left(S^* + \nu H^*\right)\right] < \frac{a_1 q_2 q_4\left(\alpha_2 + \delta_2 + \mu\right)}{12}$$

will guarantee the negative definiteness of dU/dt, which shows that *U* is indeed a Lyapunov function and hence the theorem is proved.

3. **Discussion**

The rapid spread of radical ideologies has exposed the vulnerability of individuals to radicalisation and extremisms thereby leading deviant behavioural patterns with observable consequential significances world-wide. To contribute to ongoing effort at managing violence, we have proposed and analysed a simple nonlinear model for the dynamics of violence in a community. Our model assumed that the antisocial behaviour of a proportion of individuals in the community, that are predisposed to violence, exerts a significant influence on the expected long term behaviour outcomes of, especially, the vulnerable members of the community. The sensitivity analysis we performed on the threshold quantity R_0 suggests promising correctional pathways for rehabilitation. We are therefore optimistic that our model can provide important mathematical insight to combating violence in the society.

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