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A DISCRETE TIME EOQ MODEL FOR DELAYED DETERIORATING ITEMS WITH SHORTAGES

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Abstract

In this work, a discrete time Economic Order Quantity (EOQ) inventory model was developed for delayed deterioration items with shortages. The model consist of three stages in its cycle. In the first stage, the inventory depletes to a certain level due to market demand only while the second stage depletion occurs due to combined effect of market demand and deterioration. In the final stage, shortage time is assumed by the stockist and are fully backlogged. In the first two stages, the demand rates are different but are all constants. The model is used to determine the optimal ordering quantity and replenishment cycle. Numerical example is given to illustrate the application of the model and sensitivity analysis carried out to see the effect of parameter changes.

Keywords: Stockist; Backlogged; Lost sale; Deterioration; Discrete

1. INTRODUCTION

If an organization runs out of stock for an item and a customer seeks the item and finds the inventory empty or the amount needed exceeds the available stock, the demand can either go unfulfilled or be satisfied later when the product becomes available. The former case is called a lost sale while the latter is called a backorder. There are some opportunities and costs associated with backorders i.e. loss of goodwill, loss of future sales, cost of emergency orders due to the need for quick inventory replenishment and so on. However, there are situations where planned shortages are beneficial. These include the case where the cost of keeping an item in stock is higher than the profit from selling it. Also many retailers prefer to have a small amount of shortages in order not to have their capital tied in stocks. Retailers that are dealing with deteriorating items find it better to have shortages rather than to stock the items which are already deteriorating. In extreme cases an organization keeps no stock at all and meets all demand by backorders. As a result of which taking backorder into consideration is necessary. The inventory models with shortages under a backlogging condition have been studied by many researchers. A replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging was developed by Uthayakumar and Geetha [1] putting into consideration the effect of inflation, time value of money with zero lead time. Chang and Lin [2] developed a partial backlogging inventory model for non- instantaneous deteriorating items with stock-dependent demand rate under inflation and over a finite planning horizon. They proposed a mathematical model to find minimum total relevant cost and optimal order quantity and discussed the effect of inflation and time value of money with respect to backordering parameter and non-instantaneous deteriorating items. Debashis and Pavan [3] proposed an inventory model that incorporates partial backlogging and deterioration where holding cost and demand rate are time dependent. The paper was presented in two versions: the first deals with deterministic values of the parameters while the second one takes into consideration the interval of uncertainty of the parameters. Mishra *et al* [4] optimized fuzzified economic order quantity model allowing shortage and deterioration with full backlogging.

Deterioration is defined as a decay, damage or spoilage of items such that the items cannot be used for their original purpose. Items like photo films, farm products, chemicals, drugs are examples of items in which deterioration may occur during the period of storage. Some of the items have insignificant rate of deterioration hence the effect of such deterioration is neglected. For some items however, their rate of deterioration is high, hence the need for deterioration to be taken into consideration when analysing their inventory systems.

Recent study have been on the effect of deterioration on inventory in which deterioration was considered to begin as soon as the items was kept in stock [5-11]. In some situations however, items like yams, potatoes, beans and so on do not deteriorate

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until a later time. [12-15] in their study made time a continuous variable which is rare in practice. In real life problems however, time is mostly treated as a discrete variable with unit of days, weeks, months and so on like the works of [16-20]. This paper have considered a common case in which some stock is kept, but not enough to cater for the demand. In this case there is no lost sale because the backorder is assumed to be full. Our aim therefore is to determine the amount to be kept in stock and how much to be fully backordered so that we can optimise the profit and minimise the loss while considering time to be discrete.

2. The Mathematical Model

The following notation and assumptions are made in developing the model.

Notation

- D_1 The demand rate (units per unit time) during the period before deterioration sets in.
- D_2 The demand rate (units per unit time) after deteriorating sets in.
- T_1 The time when deterioration sets in.
- T_2 The time the whole stock finishes.
- T_3 The length of time in which there is deterioration
- **Q** The order quantity per order.
- **T** The inventory cycle length (time units).
- **C** The unit cost of the item.
- **K** The ordering cost per order.
- i The inventory carrying charge.
- λ The rate of deterioration in a unit time. ($0 < \lambda < 1$)
- I_d The inventory level at the time deterioration begins
- $I_d(t)$ The inventory level at any time t, after deterioration sets in.
- **b** The maximum backorder level (shortages) allowed
- B_c The backorder cost per cycle
- b_c The backorder cost per unit, per unit time (Naira per unit, per unit time)
- I_0 The initial inventory

Assumptions

(i) Replenishment is instantaneous.
(ii) The lead time is zero.
(iii) Shortages are allowed.
(iv) There is no replacement or repair of the deteriorated items during the period under consideration.
(v) Unconstrained suppliers capital (payment is made immediately the item is supplied)
(vi) Shortages only occur during the deterioration period

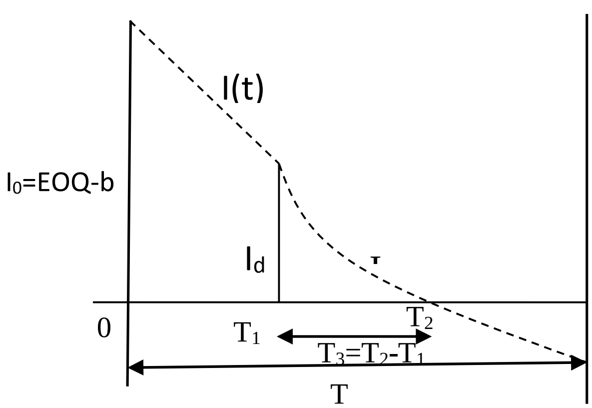

Fig 1: The position of inventory in every cycle with shortages.

Depletion of inventory from the beginning of the cycle up to the time deterioration sets in will occur only due to demand. Since time is taken to be discrete and I (t) is the inventory level at any time t, before deterioration sets in, the difference equation describing the inventory level at any time $t, (1, 2, ..., T_1)$ is given by

$$\Delta I(t) = -D_1 \qquad (1)$$

This can be solved as follows:

Since $\Delta f(x) = f(x+h) - f(x)$, where h is the step length,

Then $\Delta I(t) = I(t+1) - I(t)$

For $t = 0, 1, 2, ..., (\mathrm{m}-1), \mathrm{m} = T_1$,

$\therefore \Delta I(0) = I(1) - I(0) = -D_1$

$\Rightarrow I(1) = I(0) - D_1$

$\Delta I(1) = I(0) - 2D_1$

$\Delta I(2) = I(0) - 3D_1$ So that continuing up to t , we get

$$I(t) = I(0) - tD_1 \qquad (2)$$

Using equation (2) and applying the boundary conditions at $t = T_1,\ I(t) = I_d$ and we have

$I_d = I(0) - T_1 D_1$ This implies

$$I(0) = I_d + D_1 T_1 \qquad (3)$$

Substituting Equation (3) into (2) we get

$I(t) = I_d + D_1 T_1 - tD_1$ Or

$$I(t) = I_d + (T_1 - t)D_1 \qquad (4)$$

Since $I_d(t)$ is the Inventory level at any time t after deterioration sets in and D_2 is the demand rate at the time deterioration begins, the decrease in Inventory level will hence forth depend on both deterioration and demand. The difference equation which describes the instantaneous state of the Inventory over (T_1, T_2) is given by

$$\Delta I_d(t) = -\lambda I_d(t) - D_2 \qquad (5)$$

$T_1 \le t \le T_2$,

However, $\Delta I_d(T_1) = I_d(T_1+1) - I_d(T_1) = -\lambda I_d(T_1) - D_2$ from equation (5)

$\Rightarrow I_d(T_1+1) = (1-\lambda)I_d(T_1) - D_2$

In a similar way,

$I_d(T_1+2) = (1-\lambda)^2 I_d(T_1) + (\lambda - 2)D_2$

Similarly,

$I_d(\mathrm{T}_1+3) = (1-\lambda)^3 I_d(T_1) + \left(\frac{(1-\lambda)^3 - 1}{\lambda}\right)\mathrm{D}_2$

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$$\Rightarrow I_d(\mathrm{T}_1+s) = (1-\lambda)^s I_d(T_1) + \left(\frac{(1-\lambda)^s - 1}{\lambda}\right)\mathrm{D}_2 \qquad (6)$$

For S = 0, 1 ... (T_2-T_1)

If $T_1 + s = t \Rightarrow s = t - T_1$

$$\therefore I_d(t) = I_d(T_1)(1-\lambda)^{t-T_1} - \frac{D_2}{\lambda}(1-(1-\lambda)^{t-T_1}) \qquad (7)$$

At $t = T_2,\ \ I_d(T_2) = 0$ equation (7) becomes

$$0 = I_d(T_1)(1-\lambda)^{T_2-T_1} - \frac{D_2}{\lambda}(1-(1-\lambda)^{T_2-T_1}) \qquad (8)$$

However, $I_d(T_1) = I_d$

i.e. $I_d = -\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T_2})$ (9)

Hence, substituting equation (9) into (4), we get:

$$I(t) = -\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T_2}) + (T_1-t)D_1 \quad (10)$$

The total demand between T_1 and T_2 is equal to the demand rate at the onset of deterioration multiplied by the time period during which the item deteriorates.

$\therefore$ Total demand between T_1 and T_2 = D_2T_3. Let $d(T_3)$ be the number of items that deteriorate during the time interval $[T_1, T_2]$, then

$$d(T_3) = I_d - D_2T_3 \quad (11)$$

Substituting I_d from equation (9) into (11) we have

$$d(T_3) = -\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T_2}) - D_2T_3$$

i.e.
$$d(T_3) = -\frac{D_2}{\lambda}[1-(1-\lambda)^{T_1-T_2} + \lambda T_3] \quad (12)$$

The total number of units in inventory during a cycle is determined from the build-up of inventory as follows:
From equation (4),

$$I(0) = I_d + (T_1-0)D_1$$
$$I(2) = I_d + (T_1-2)D_1$$
$$I(3) = I_d + (T_1-3)D_1$$

Since $I_d = -\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T_2})$

Continuing up to $T_1 - 1$ we get

$$I(T_1-1) = I_d + D_1$$

So that at T_1 we get

$$I(T_1) = I_d$$

Thus, the total number of units in the period $[0, T_1]$ is

$$\sum_{t=0}^{T_1} I(t) = (T_1+1)I_d + (0+1+2+3+...+(T_1-1)+T_1)D_1$$
$$= \frac{T_1+1}{2}(2I_d + T_1D_1)$$

So that the average number of units per unit time in the period $[0, T_1]$ is given as

$$I_A(t) = \frac{1}{T_1+1}\sum_{t=0}^{T_1} I(t)$$
$$= \frac{[2I_d + T_1D_1]}{2} \quad (13)$$

In a Similar way, to obtain the total inventory during deterioration period i.e.

$\sum_{t=T_1+1}^{T_2} I_d(t)$ For $T_1+1 \le t \le T_2$ we proceed as follows:

For $t = T_1+1$ $\quad I_d(t) = (1-\lambda)I_d + \left(\frac{(1-\lambda)-1}{\lambda}\right)D_2$

For $t = T_1+2$ $\quad I_d(t) = (1-\lambda)^2 I_d + \left(\frac{(1-\lambda)^2-1}{\lambda}\right)D_2$

For $t = T_1+3$ $\quad I_d(t) = (1-\lambda)^3 I_d + \left(\frac{(1-\lambda)^3-1}{\lambda}\right)D_2$

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For $t = T_2 - 2$ $$I_d(t) = (1-\lambda)^{(T_2-T_1)-2}\, I_d + \left(\frac{(1-\lambda)^{(T_2-T_1)-2} - 1}{\lambda}\right) D_2$$

For $t = T_2 - 1$ $$I_d(t) = (1-\lambda)^{(T_2-T_1)-1}\, I_d + \left(\frac{(1-\lambda)^{(T_2-T_1)-1} - 1}{\lambda}\right) D_2$$

For $t = T_2$ $$I_d(t) = (1-\lambda)^{(T_2-T_1)}\, I_d + \left(\frac{(1-\lambda)^{(T_2-T_1)} - 1}{\lambda}\right) D_2$$

Let $1-\lambda = q$

$$\therefore \sum_{t=T_1+1}^{T_2} I_d(t) = I_d\left[q+q^2+q^3+\ldots+q^{(T_2-T_1)}\right] + \frac{D_2}{\lambda}\left[q+q^2+q^3+\ldots+q^{(T_2-T_1)} - (T_2-T_1)\right]$$

$$= \frac{I_d\left[\lambda q(1-q^{(T_2-T_1)})\right] + D_2\left[q(1-q^{(T_2-T_1)}) - (1-q)(T_2-T_1)\right]}{(1-q)\lambda}$$

Since $1-\lambda = q \Rightarrow 1-q = \lambda$

This implies $$\sum_{t=T_1+1}^{T_2} I_d(t) = \frac{I_d\left[\lambda(1-\lambda)(1-(1-\lambda)^{(T_2-T_1)})\right] + D_2\left[(1-\lambda)(1-(1-\lambda)^{(T_2-T_1)}) - \lambda(T_2-T_1)\right]}{\lambda^2}$$

$$= \frac{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}{\lambda^2}\left[\lambda I_d + D_2 - \frac{\lambda(T_2-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}\right] \quad (14)$$

Hence $$\sum_{t=0}^{T_2} I(t) = \frac{T_1+1}{2}\left[2I_d + T_1 D_1\right] + \frac{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}{\lambda^2}\left[\lambda I_d + D_2 - \frac{\lambda(T_2-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}\right]$$

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The backorder cost per cycle $B_C$ is $b_c \sum_{t=0}^{t=T-T_2} D_2 t = b_c D_2 \sum_{t=0}^{t=T-T_2} t = \frac{b_c D_2 (T-T_2)(T-T_2+1)}{2}$

Therefore, average inventory per unit time will be $\frac{1}{T_1+1}\sum_{t=0}^{T_1} I(t) + \frac{1}{T_2-T_1}\sum_{t=T_1+1}^{T_2} I_d(t)$ =

$$\frac{\left[2I_d + T_1 D_1\right]}{2} + \frac{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}{\lambda^2 (T_2-T_1)}\left[\lambda I_d + D_2 - \frac{\lambda(T_2-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}\right] \quad (16)$$

**2.1 Inventory Carrying Cost (or Holding Cost)**

Average holding Cost ($H_A$) = unit cost of item multiplied by inventory carrying charge ( i%) multiplied by average inventory

$$\therefore \quad H_A = i\% \text{ x unit cost x} \left[\frac{1}{T_1+1}\sum_{t=0}^{T_1} I(t) + \frac{1}{T_2-T_1}\sum_{t=T_1+1}^{T_2} I_d(t)\right] \quad (17)$$

$$= ic\left(\frac{\left[2I_d + T_1 D_1\right]}{2} + \frac{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}{\lambda^2 (T_2-T_1)}\left[\lambda I_d + D_2 - \frac{\lambda(T_2-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}\right]\right)$$

$$= ic\left(\frac{\left[2I_d + T_1 D_1\right]}{2} + \frac{D_2(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}{\lambda^2 (T_2-T_1)}\left[(1-\lambda)^{(T_1-T_2)} - \frac{\lambda(T_2-T_1)}{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}\right]\right) \quad (18)$$
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2.2 Total Variable Cost

The total variable cost per unit time in a cycle is given by

$$C(T) = \frac{K}{T} + C\frac{d(T_3)}{T} + H_A + \frac{B_C}{T}$$

$$= \frac{K}{T} + \frac{C}{T}\left(\frac{-D_2}{\lambda}\left[1-(1-\lambda)^{(T_1-T_2)} + (T_2-T_1)\lambda\right]\right)$$

$$+ic\left[\frac{-D_2}{\lambda} + \frac{D_2}{\lambda}(1-\lambda)^{(T_1-T_2)} + \frac{T_1D_1}{2} + \frac{D_2(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}{\lambda^2(T_2-T_1)}\left[(1-\lambda)^{(T_1-T_2)} - \frac{\lambda(T_2-T_1)}{(1-\lambda)\left[1-(1-\lambda)^{(T_2-T_1)}\right]}\right]\right]$$

$$+\frac{1}{T}\frac{\left[b_cD_2(T-T_2)(T-T_2+1)\right]}{2}$$

(19) Since T must be a non-negative integer the conditions for $C(T)$ to have a minimum at $T = T^*$ are

$$C(T^*) \le C(T^*-1) \text{ and } C(T^*) \le C(T^*+1)$$

$$\Rightarrow C(T^*) - C(T^*-1) \le 0 \text{ and } C(T^*+1) - C(T^*) \ge 0$$

$$C(T^*) - C(T^*-1) \le 0 \le C(T^*+1) - C(T^*) \qquad (20)$$

which simplifies to

$$\frac{-K}{D_2} + \frac{C}{\lambda} - \frac{C(1-\lambda)^{T_1-T_2}}{\lambda} + C(T_2-T_1) + \frac{b_c(T-T_2)(T_2-1+T)}{2} \le 0 \le \frac{-K}{D_2} + \frac{C}{\lambda} - \frac{C(1-\lambda)^{T_1-T_2}}{\lambda}$$

$$+C(T_2-T_1) + \frac{b_c(T+1-T_2)(T+T_2)}{2}$$

Provided $T \ge 1$.

This implies

$$\frac{b_c(T-T_2)(T_2-1+T)}{2} \le \frac{K}{D_2} - \frac{C}{\lambda} + \frac{C(1-\lambda)^{T_1-T_2}}{\lambda} - C(T_2-T_1) \le \frac{b_c(T+1-T_2)(T+T_2)}{2} \qquad (21)$$

Provided $T \ge 1$

The EOQ Corresponding to T is determined as follows:

EOQ = $D_1T_1 + D_2T_3 + d(T_3) + b$ where b= D_2 (T-T_2) is the maximum backorder level allowed.

$$= D_1T_1 + D_2(T_2-T_1) + d(T_3) + D_2(T-T_2)$$

From Equation (12)

$$EOQ = D_1T_1 + D_2(T-T_1) - \frac{D_2}{\lambda}[1-(1-\lambda)^{T_1-T_2} + \lambda T_3]$$

$$= D_1T_1 + D_2(T-T_2) - \frac{D_2}{\lambda}[1-(1-\lambda)^{T_1-T_2}] \qquad (22)$$

2.3 NUMERICAL EXAMPLE

Using equation (19), (21) and (22) the optimal values of T, EOQ and total variable cost C (T) are calculated for the following example.

D_1 = 13units, D_2 = 7units, C = # 65, K = 1300, i = 0.16/365, λ= 0.11, T_1 = 8days, T_2 = 10days, b_c = #5 (per unit time), b = 21units.

Substituting in the equations we have T = 13days, EOQ = 141.7 units, and C (T) = # 131.74.

2.4 SENSITIVITY ANALYSIS

There is possibility of uncertainties in any decision–making problem which can lead to changes in the values of parameters involved. For this reason sensitivity analysis is usually carried out to see the effect of the change. We use the example above to carry out a sensitivity analysis on the total cost C (T), cycle length (T) and the economic order quantity (EOQ) with respect to changes in other parameters. A computer software (Python 3.0) was used for the analysis. To see any change that occurs to the three decision variables as a result while keeping the remaining parameters unchanged. Table 1 gives the full result of the analysis.

Table: 1 Sensitivity Analysis carried out to see the effect of parameter changes

Parameters	% Change	% Change in EOQ	% Change in T	% Change in C(T)
D_1	-5	-3.66966	0	-0.05624
	-4	-2.93573	0	-0.04499
	-3	-2.20180	0	-0.03374
	-2	-1.46786	0	-0.02249
	-1	-0.73393	0	-0.01125
	0	0	0	0
	1	0.73393	0	0.01125
	2	1.46786	0	0.02249
	3	2.20180	0	0.03374
	4	2.93573	0	0.04499
	5	3.66966	0	0.05624
D_2	-5	-1.33034	0	-1.14828
	-4	-1.06427	0	-0.91863
	-3	-0.79820	0	-0.68897
	-2	-0.53214	0	-0.45931
	-1	-0.26607	0	-0.22966
	0	0	0	0
	1	0.26607	0	0.22966
	2	0.53214	0	0.45931
	3	0.79820	0	0.68897
	4	1.06427	0	0.91863
	5	1.33034	0	1.14828
C	-5	0	0	-0.59140
	-4	0	0	-0.47312
	-3	0	0	-0.35484
	-2	0	0	-0.23656
	-1	0	0	-0.11828
	0	0	0	0
	1	0	0	0.11828
	2	0	0	0.23656
	3	0	0	0.35484
	4	0	0	0.47312
	5	0	0	0.59140
K	-5	0	0	-3.79548
	-4	0	0	-3.03638
	-3	0	0	-2.27729
	-2	0	0	-1.51819
	-1	0	0	-0.75910
	0	0	0	0
	1	0	0	0.75910
	2	0	0	1.51819
	3	0	0	2.27729
	4	0	0	3.03638
	5	0	0	3.79548

i	-5	0	0	-0.07855
	-4	0	0	-0.06284
	-3	0	0	-0.04713
	-2	0	0	-0.03142
	-1	0	0	-0.01571
	0	0	0	0
	1	0	0	0.01571
	2	0	0	0.03142
	3	0	0	0.04713
	4	0	0	0.06284
	5	0	0	0.07855
λ	-5	-0.11046	0	-0.59800
	-4	-0.08852	0	-0.47919
	-3	-0.06650	0	-0.35999
	-2	-0.04440	0	-0.24039
	-1	-0.02224	0	-0.12040
	0	0	0	0
	1	0.02231	0	0.12080
	2	0.04470	0	0.24200
	3	0.06716	0	0.36360
	4	0.08970	0	0.48561
	5	0.11231	0	0.60803
T_1	-5	-0.96433	0	3.98537
	-4	-0.78160	0	3.13338
	-3	-0.59375	0	2.30911
	-2	-0.40084	0	1.51229
	-1	-0.20291	0	0.74267
	0	0	0	0
	1	0.20783	0	-0.71597
	2	0.42055	0	-1.40549
	3	0.63810	0	-2.06881
	4	0.86045	0	-2.70617
	5	1.08754	0	-3.31780
T_2	-5	-0.73910	0	-0.28486
	-4	-0.60615	0	-0.34933
	-3	-0.46585	0	-0.35355
	-2	-0.31812	0	-0.29705
	-1	-0.16287	0	-0.17936
	0	0	0	0
	1	0.17057	0	0.24151
	2	0.34892	0	0.54567
	3	5.47508	0	0.84509
	4	5.66929	0	1.05761
	5	5.87157	0	1.32973
B_C	-5	0	0	-0.61312
	-4	0	0	-0.49049
	-3	0	0	-0.36787
	-2	0	0	-0.24525
	-1	0	0	-0.12265
	0	0	0	0
	1	0	0	0.12262
	2	0	0	0.24525
	3	0	0	0.36787
	4	0	0	0.49049
	5	0	0	0.61312

3. DISCUSSION OF RESULTS

The following is the summary of what can be deduced from the results of the sensitivity analysis:

(1) An increase or decrease in D_1 and D_2 will increase or decrease the EOQ and C(T) respectively while the period (T) remains unchanged. This is expected since more stock is needed to take care of the increasing demand and vice-versa. Hence the EOQ increases and so the C(T) will also increase. In the case of D_2 however, if the decrease goes below 10% or the increase goes above 33%, the T changes.

(2) An increase or decrease in C (cost of the item) will increase or decrease respectively, the total variable cost C(T). This is expected since the item's cost has direct effect on the total variable cost. The other two decision variables however remain unchanged.

(3) An increase or decrease in ordering cost K will increase or decrease respectively the total variable cost since ordering cost has direct effect on the variable cost. However, if the increase goes above 10% or the decrease below 24%, the T changes.

(4) An increase or decrease in inventory carrying charge will also increase or decrease respectively the total variable cost. This is also expected as the inventory carrying charge has direct effect on the total variable cost.

(5) When the deterioration rate increases, the total cost and EOQ also increases. The increase in EOQ when deterioration is high is to take care of the deteriorated items. The total cost increases due to increase in deterioration cost.

λ_1

(6) When T_1 is higher, it results in a higher value of EOQ and lower value of the total cost. This is expected since if T_1 is high it will take longer time before deterioration sets in and so the model will choose a bigger EOQ to reduce cost. Hence the EOQ will be high. The cost is reduced because less stock will deteriorate due to increase in T_1.

(7) When T_2 is high, it results in a higher value of EOQ and C(T) which is clear since the higher T_2 is, then more stock will deteriorate and so EOQ should increase to take care of that thereby resulting in higher cost. The period T however remains unchanged until when T_2 increases by 3% or it decreases by 8%.

(8) An increase or decrease in B_c increases or decreases C(T) respectively. This is also expected since an increase or decrease in the backorder cost will definitely affect the total variable cost. However, if the increase goes beyond 27% or the decrease below 8%, the T changes.

4. CONCLUSION

In this work, a discrete time EOQ model on inventory of deteriorating items which exhibit delay in deterioration with shortages is presented. Demand before deterioration sets in is different from demand after deterioration has set in but they are both constants. The model allows shortages which are fully backlogged but the shortage time is assumed by the stockist and the demand from the shortages is fulfilled first whenever an order arrives. The depletion of the items before deterioration sets in is dependent on demand only, but when deterioration sets in, depletion now depends on both demand and deterioration. Items that exhibit this property include farm produce like tomatoes, potatoes, yams, beans and so on.

A numerical example has been presented to illustrate the application of the model developed and a sensitivity analysis is carried out to study the effect of various changes in the parameters on the decision variables.

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