

**EFFECT OF INSECURITY ON FOOD SECURITY: MATHEMATICAL INSIGHTS FROM  
FARMER-HERDER CLASHES IN NIGERIA**

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*Abstract*

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*The pressure of feeding the estimated 3<sup>rd</sup> most populous country by 2100 would have been herculean but for the struggling contribution of subsistent farming and pastoralist to Nigeria's economic growth. Evidence of insecurity abound in the country: ethno-religious crisis, deepening levels of moral values, falling standard of family and communal life, rising antisocial behaviours, heightening atmosphere of mistrust, fear, and anxiety; rising levels of poverty, hunger, hostility, and criminality. The cooperate existence of the country has come under extreme threat. To attempt to provide a way out, we proposed, developed, and analysed a mathematical model of Farmer – Herder conflict using a system of ordinary differential equations. We obtained and performed qualitative analysis which indicates the condition for harmonious coexistence in the society.*

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**Keywords:** Insecurity, herders-farmers conflict, mathematical model, ordinary differential equations

**1. Introduction**

Nigeria's growth rate of 3.2% will translate to a population more than 401 million by 2050. Yet, the country could not leverage on the potentials of diversity to entrench national consciousness and patriotism. Sadly, the clutches of divisiveness have stuck her resilience, ingenuity, and nationalistic zeal to trudge along corridors of development and nationhood. Over the past decade, Nigeria has had to contend with overwhelming multifaceted security challenges that have undermined her unity and cooperate existence [1, 2]. In recent times, the rise in Fulani herders and sedentary crop farmers conflict has added an unfortunate dimension to the Nigerian insecurity challenge. This is against the historical symbiotic relationship that existed where herders' animals fertilised farmers' land. Factors responsible for the conflicts are varied. The nomadic nature of Fulani herders, insecurity in the Northeast and Northwest because of the Boko Haram and banditry activities respectively and cattle rustling in the Northcentral [2]. The many incidences of herders-farmers' clashes reported across the country have created a substantial humanitarian crisis. It has been reported that no fewer than 62,000 people were displaced in Kaduna, Benue, and Plateau states [3, 4]; resulting in attacks and reprisal attacks, with at least 3,641 people killed between January 2016 and October 2018, 57 percent of them in 2018 alone [3]. Other consequences of farmers and Fulani herdsmen conflict include destruction of properties, theft of cattle and goats, destruction of crops, physical fight with machetes and sticks, pollution of drinkable water, destruction of reservoirs and sources of drinkable water, burning of rangelands and houses; and damages to irrigational facilities [1, 2, 4], thereby resulting in impoverishment, displacements, abuses, rapes, and wanton and gruesome killings [1 – 4]. The causes of these are varied: the internalisation of extreme beliefs and ideologies due to perceived societal failures, excesses and injustices, destruction of farms by grazing livestock, rustling [1 – 4]. Concern is yet to be channelled towards the recurring incidences of destruction of lives and properties, especially smallholder-farms and the effort to identify and alleviate their most basic challenges, like nourishment, a

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accommodation, and rehabilitation. The present research hopes to contribute to the ongoing insecurity management processes in the country by proposing a new mathematical model that will suggest a paradigm for studying terrorism and its consequences.

**2. Model formulation**

$$\begin{aligned}
 \frac{dF_S}{dt} &= \Pi_F - \mu F_S - \beta_1(V_F + \eta_1 V_H)F_S, \\
 \frac{dF_V}{dt} &= \beta_1(V_F + \eta_1 V_H)F_S - (v_1 + d_1 + \mu)F_V, \\
 \frac{dV_F}{dt} &= v_1 F_V - (\alpha_1 d_1 + \mu)V_F, \\
 \frac{dH_S}{dt} &= \Pi_H - \mu H_S - \beta_2(V_H + \eta_2 V_F)H_S, \\
 \frac{dH_V}{dt} &= \beta_2(V_H + \eta_2 V_F)H_S - (v_2 + d_2 + \mu)H_V, \\
 \frac{dV_H}{dt} &= v_2 H_V - (\alpha_2 d_2 + \mu)V_H.
 \end{aligned}
 \tag{1}$$

Table 1: Description of variables ad for the model of the dynamics of violence

Variable	Description
$F_S(t)$	Farmers who are susceptible to Herder irresponsible activities
$F_V(t)$	Farmers who are victims of Herder irresponsible actions and behaviours
$V_F(t)$	Farmers with propensity for irresponsible behaviours and provocative farming practices
$H_S(t)$	Herders who are vulnerable to Farmer irresponsible activities
$H_V(t)$	Herders who are victims of Farmer-irresponsible behaviours and actions
$V_H(t)$	Herders with propensity for irresponsible behaviours and provocative farming practices

Variable	Description	Units
$\Pi_F, \Pi_H$	Recruitment rate into the population	Individuals km <sup>-2</sup> month <sup>-1</sup>
$\mu$	Natural death rate	month <sup>-1</sup>
$\alpha_1, \alpha_2$	Propensity rate coefficients	Month <sup>-1</sup>
$\beta_1, \beta_2$	Effective clash rates for Farmers ad Herders	Individuals km <sup>-2</sup> month <sup>-1</sup>
$\eta_1, \eta_2$	Modification parameters	Month <sup>-1</sup>
$\eta_1, \eta_2$	Progression rates to defiant Farmer and Herder classes respectively	Month <sup>-1</sup>
$d_1, d_2$	Clash-induced mortality for Farmers ad Herder	Month <sup>-1</sup>

Since the model (1) monitors human populations, all its variables and parameters are nonnegative. Thus, consider the criminal feasible region

$$\Omega = \left\{ (F_S, F_V, V_F, H_S, H_V, V_H) \in \mathbb{R}_+^6 \mid N \leq \Pi / \mu \right\}$$

We proceed as follows to prove that all solutions originating from the region  $\Omega$  remains in  $\Omega$  for all time (that is  $\Omega$  is a positive invariance set).

The rate of change of the total population is obtained by adding the equations in model (1) as

$$\frac{dN}{dt} = \Pi - \mu N - d_1 F_V - \alpha_1 d_1 V_F - d_2 H_V - \alpha_2 d_2 V_H,
 \tag{2}$$

where  $P = P_F + P_H$ .

obviously, for  $N > \Pi / \mu$ , then  $\frac{dN}{dt} < 0$ . Following  $\frac{dN}{dt}$  is bounded by  $\Pi - \mu N$  then

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Pi}{\mu}(1 - e^{-\mu t})$$

Furthermore,  $N(t) \leq \frac{\Pi}{\mu}$ , if  $N(0) \leq \frac{\Pi}{\mu}$ . Thus, the  $\omega$ -limit sets of the system (1) are contained in the region  $\Omega$ . In

other words, all solutions of the model (1) originating in  $\Omega$  will remain there for all time. This proves that the region  $\Omega$  is definitely a positive invariant set and attracts all its solutions. Thus, it is sufficient to consider the dynamics of the model (1) in  $\Omega$ .

#### Analysis of the model.

##### Local stability of the clash – free equilibrium (CFE)

Clashes – free equilibrium for the model system (1) is understood as an equilibrium point of the model system (1) where there are no clashing activities in the system. This equilibrium point can easily be obtained by setting the right-hand sides of the equations of the model (1) to zero we obtain its CFE as

$$E^0 = (F_S^0, F_V^0, V_F^0, H_S^0, H_V^0, V_H^0) = \left( \frac{\Pi_F}{\mu}, 0, 0, \frac{\Pi_H}{\mu}, 0, 0 \right). \quad (3)$$

We proceed with the investigation of the dynamics of the model (1) by considering a threshold quantity  $R_0$ , which technically corresponds to the basic reproduction number in epidemiological models. This quantity is obtained, as outlined in [6] as

$$R_0 = \frac{v_1 \beta_1 K_3 K_4 + v_2 \beta_2 K_1 K_2 + \sqrt{v_1^2 \beta_1^2 K_3^2 K_4^2 + v_2^2 \beta_2^2 K_1^2 K_2^2 + 2v_1 v_2 \beta_1 \beta_2 (2\eta_1 \eta_2 - 1) K_1 K_2 K_3 K_4}}{2K_1 K_2 K_3 K_4}$$

where  $K_1 = v_1 + d_1 + \mu$ ,  $K_2 = \alpha_1 d_1 + \mu$ ,  $K_3 = v_2 + d_2 + \mu$  and  $K_4 = \alpha_2 d_2 + \mu$ .  $K_4 = a_2 d_2 + m$

The following results follow again from [6].

**Lemma 1.** *The CFE of the clashes – model (1), given by (3), is locally asymptotically stable if  $R_0 < 1$  and unstable if  $R_0 > 1$ .*

By evaluating the Jacobian of the model (1) about the CFE the statement can be proven once all the corresponding eigenvalues are shown to be negative. Therefore, we conclude that Farmer – Herder clashes could be securely addressed in the community if  $R_0 < 1$ , while on the other hand it can be understood that Farmer – Herder clashes would continue in the community if  $R_0 > 1$ .

Finally, the short- and long-term dynamics of model (1) under some form of effective control measures can be summarised as follows.

**Theorem 1.** *The CFE of the Farmer – Herder clashes is globally asymptotically stable in  $\Omega$  if  $R_0 < 1$ .*

**Theorem 2.** *The CFE of the Farmer – Herder clashes is globally asymptotically stable in  $\Omega$  if  $R_0 > 1$ .*

The foregoing claims can easily be proven using the method of Lyapunov functions of the Goh – Volterra type (nonlinear)

#### CONCLUDING REMARKS

Our analysis of the Farmer – Herder clashes model has established the regions for the varied occurrence of extreme outcomes, each with consequential implications. It is easy to infer that persistence of the conflict would dangerously impact the hitherto symbiotic relationship between farmers and herdsmen. Our model noted the significant role that the non-implementation of necessary control measure plays in the vicious circling of crises in the agricultural sector. It is paramount therefore that another model that incorporates workable control/management measures be built and analysed. These management/control measures should have the capacity to reduce the incidence rate of the clashes. Needless to say, improving the robustness of this model could suggest cost effective and result oriented pathways that will provide valuable support to the security situation in the sector.

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