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NUMERICAL ANALYSIS OF TEMPERATURE DISTRIBUTION AND SOLIDIFICATION THICKNESS OF LIQUID CAST IN THE MOULD OF CONTINUOUS SLAB CASTING

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Abstract

In continuous casting, it is difficult to determine the temperature distribution and solidification thickness using experimental techniques. This paper aim at analysing the temperature distribution and solidification thickness of the liquid cast in the mould numerically with respect to the pouring temperature of 1546ºC and mould preheat temperature of 320ºC. The Garlerkin Finite element method was used to discretize and analyse the temperature distribution. The pouring temperature dropped from 1546ºC to 1532ºC for 50sec, after a time interval of 100sec droped to 1525ºC and further dropped to 1513ºC at the outlet of the mould zone. This result corresponds with the results obtained from literature which had a mould outlet temperature of about 1515ºC. The thickness was observed to increase after some time to about 2.4375cm which aid the downward movement of the liquid cast into the solidifying steel shell.

Keywords: *Liquid cast, Copper mould, Temperature distribution, Solidification thickness*

1.0 Introduction

A continuous casting is the process of casting metals into endless lengths of strips or slabs which can be further shaped into suitable products, using continuous casting machines and continuous casting furnaces. Though the process may sound simple, it a complex metallurgical process [1]. The continuous solidification and withdrawal from a shaping mould involves thermal and mechanical interactions between the mould and the moving solidifying metal. Continuous casting is grouped under special process of casting [2].

A 1-D transient finite-difference calculation of heat conduction within the solidifying steel shell coupled with 2-D steady-state heat conduction within the mould wall was modelled by [3]. The model features a detailed treatment of the interfacial gap between the shell and mould, including mass and momentum balances on the solid and liquid interfacial slag layers, and the effect of oscillation marks. The model predicts shell thickness, temperature distributions in the mould and shell, thickness of the re-solidified and liquid powder layers, heat flux profiles down the wide and narrow faces, mould water temperature rise, ideal taper of the mould walls, and other related phenomena.

A computational simulation system for modeling the solidification process in a continuous casting facility for steel slabs was developed [4]. The system couples a module for solving the direct problem (the calculation of temperatures in the steel strand) with an inverse analysis module that was developed for evaluating the steel/mould heat fluxes from the information provided by thermocouples installed in the continuous casting mould copper plates.

A steady state, two dimensional mathematical model for continuous casting of steel was developed by [5]. The governing partial differential equations of fluid flow and thermal energy transport together with the appropriate set of boundary conditions were derived and a procedure for their non-dimensional representation outlined [5].

In this paper, we are looking at the mould region of the continuous slab casting using the transient for of the 1-D Fourier heat transfer equation applying the Garlerkin finite element method. The aim of this research is to determine numerically the temperature distribution and the solidification thickness of the liquid cast in the mould for about 200sec after pouring.

2.0 Methodology

The mathematical models used in continuous slab casting are based on the Fourier heat conduction equation and solidification of the liquid cast. The Fourier heat conduction equation is analysed using the Galerkin Finite Element method

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to determine the temperature distribution of the liquid cast in the mould of the continuous casting. In the analysis involving Finite Element method, the governing equation can only be solved if it is in order one but the governing equation for the temperature distribution in continuous casting is in order two [6]. In this paper we weaken equation (1) which known as the governing equation.

$$C\rho\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) \tag{1}$$

where C = specific heat capacity of the material, ρ = density of material (kg/m^3)

k = thermal conductivity (j/kgK), T = temperature (^{o}C), t = time (sec)

The boundary condition is specified in the form of heat flux transferred from the molten metal surface [7] to the mould shown in Equation (2)

$$q_{sk}=\alpha_{sk}\left(T_s-T_k\right) \tag{2}$$

$T_s =$ strand surface temperature

$T_k =$ mould surface temperature from the side of strand

$\alpha_{sk} =$ heat transfer coefficient for water spray cooling

In the development of the weak form, we assumed a linear mesh and placed it over the mould domain. This was achieved by multiplying equation (1) by the weight function (w) and integrate by part over the mould domain which resulted to equation (3)

$$-\int_{x_A}^{x_A+h} w\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)\right]dx=-\left[wk\frac{\partial T}{\partial x}\right]_{x_A}^{x_A+h}+\int_{x_A}^{x_A+h} k\frac{\partial w}{\partial x}\frac{\partial T}{\partial x}dx \tag{3}$$

Further simplification yield equation (4) and (5)

$$\int_{x_A}^{x_A+h} k\frac{\partial w}{\partial x}\frac{\partial T}{\partial x}dx+\int_{x_A}^{x_A+h} wC\rho\frac{\partial T}{\partial t}dx-wQ_A-wQ_B=0 \tag{4}$$

where $-Q_A=\left(k\frac{\partial T}{\partial x}\right)\Big|_{x_A}$ and $Q_B=\left(k\frac{\partial T}{\partial x}\right)\Big|_{x_A+h}$ (5)

Equation 4 is known as the weak form of the governing equation, since the primary variable is a function of itself, the Langrange interpolation function is admissible, which can be expressed in equation (6)

$T^e=\sum_{j=1}^{n}T_j^e\psi_j^e(t)$ and $w=\psi_i^e(t)$ (6)

Equation (6) is substituted into equation (4) to obtain the finite element based model in equation (7)

$$k\left[K_{ij}^e\right]\left\{T_j^e\right\}+C\rho\dot{T}\left\{M_{ij}^e\right\}=\left\{Q_i^e\right\} \tag{7}$$

Where K_{ij}^e is known as the thermal conductivity matrix and M_{ij}^e is also known as the enthalpy matrix and Q_i^e is known as the heat flux matrix.

Solving equation (7) further, we introduce the time approximation function of the α family of interpolation in which a weighted average of the time derivative of the dependent variable is approximated to two consecutive time steps by linear interpolation of the variables [6], which yields equation (8)

$$\left[C\rho\left[M_{ij}^e\right]+\alpha k\Delta t_{s+1}\left[K_{ij}^e\right]\right]\left\{T_j\right\}_{s+1}=\left[C\rho\left[M_{ij}^e\right]-(1-\alpha)k\Delta t_{s+1}\left[K_{ij}^e\right]\right]\left\{T_j\right\}_s+\alpha k\Delta t_{s+1}\left[\left\{Q_i^e\right\}_s+\left\{Q_i^e\right\}_{s+1}\right] \tag{8}$$

Using the Crank Nicholson scheme, where $\alpha=0.5$, equation (8) is reduced to equation (9)

$$\left\{T_j\right\}_1=\left[C\rho\left[M_{ij}^e\right]+k\frac{\Delta t_1}{2}\left[K_{ij}^e\right]\right]^{-1}\left[C\rho\left[\left[M_{ij}^e\right]-k\frac{\Delta t_1}{2}\left[K_{ij}^e\right]\right]\left\{T_j\right\}_0+\frac{\Delta t_{s+1}}{2}\left\{Q_i^e\right\}_{s+1}\right] \tag{9}$$

Equation (8) becomes the Finite Element model used for the temperature distribution of the liquid cast in the mould.

2.1 Evaluating the elemental matrices

The thermal conductivity matrix can be derived by substituting the Lagrange interpolation functions [6,8] into K_{ij}^{e} matrix to obtain the various elements in the matrix. Equation (10) shows the element obtained in the first roll, first column.

$$K_{11}^{e}=\int_{x_A}^{x_A+h_e}\frac{\partial}{\partial x}\left[\left(1-\frac{x}{h_e}\right)\left(1-\frac{2x}{h_e}\right)\right]\frac{\partial}{\partial x}\left[\left(1-\frac{x}{h_e}\right)\left(1-\frac{2x}{h_e}\right)\right]dx \tag{10}$$

$$K_{11}^{e}=\frac{7h_e^2-24h_e x+48x^2}{3h_e^3}$$

The Enthalpy matrix can also be derived by substituting the Lagrange interpolation functions [6,8] into M_{ij}^{e} matrix to obtain the various elements. Equation (11) shows element obtained in the first roll, first column.

$$\left[M_{11}^{e}\right]=\int_{x}^{x+h_e}\psi_1\psi_1 dx$$

$$\left[M_{11}^{e}\right]=\int_{x}^{x+h_e}\left[\left(1-\frac{x}{h_e}\right)\left(1-\frac{2x}{h_e}\right)\right]\left[\left(1-\frac{x}{h_e}\right)\left(1-\frac{2x}{h_e}\right)\right]dx$$

$$\left[M_{11}^{e}\right]=\frac{\frac{2h^4}{15}-h_e{}^3x+3h_e{}^2x^2-4h_e x^3+4x^4}{h_e{}^3} \tag{11}$$

The process is repeated for four (4) elements and assembled for both the K_i^e and M_i^e equations as shown in equation (12) and (13).

The assembled K^e thermal conductivity matrix is given as

$$\left[K^e\right]=\frac{1}{3h_e}\begin{bmatrix} 7 & -8 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ -8 & 16 & -8 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & -8 & 38 & -80 & 49 & 0 & 0 & 0 & 0\\ 0 & 0 & -80 & 208 & -128 & 0 & 0 & 0 & 0\\ 0 & 0 & 49 & -128 & 230 & -344 & 193 & 0 & 0\\ 0 & 0 & 0 & 0 & -344 & 784 & -440 & 0 & 0\\ 0 & 0 & 0 & 0 & 193 & -440 & 614 & -800 & 433\\ 0 & 0 & 0 & 0 & 0 & 0 & -800 & 1744 & -944\\ 0 & 0 & 0 & 0 & 0 & 0 & 433 & -944 & 511 \end{bmatrix} \tag{12}$$

The assembled M^e Enthalpy matrix is given as

$$\left[M^e\right]=\frac{h_e}{30}\begin{bmatrix} 4 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\ 2 & 16 & 2 & 0 & 0 & 0 & 0 & 0 & 0\\ -1 & 2 & 68 & -178 & 149 & 0 & 0 & 0 & 0\\ 0 & 0 & -178 & 496 & -418 & 0 & 0 & 0 & 0\\ 0 & 0 & 149 & -418 & 1628 & -3118 & 2039 & 0 & 0\\ 0 & 0 & 0 & 0 & -3118 & 7696 & -5038 & 0 & 0\\ 0 & 0 & 0 & 0 & 2039 & -5038 & 10508 & -16738 & 9989\\ 0 & 0 & 0 & 0 & 0 & 0 & -16738 & 38896 & -23218\\ 0 & 0 & 0 & 0 & 0 & 0 & 9989 & -23218 & 13864 \end{bmatrix} \tag{13}$$

3.0 Result and Discussion

Using Equation (9), the one Dimensional (1D) heat transfer equation and with the data presented in Table 1, analysis of the liquid cast in the mould region was carried out. It was observed that there was a decrease in temperature in the liquid metal cast from 1546ºC to 1513ºC. This is due to the fact that the heat in the system is conducted away from the liquid cast into the surrounding [9].

Table 1: Thermo physical properties of casting material [10]

Thermo Physical Properties	Grade l Steel
$K_d\left(cal/cm\sec^0 C\right)$ at $0^0 C$	0.025
$\rho\left(gr/cm^3\right)$	7.3
$\rho\left(cal/gm^0 C\right)$	0.118
$T_s\left({}^0C\right)$	1494
$T_L\left({}^0C\right)$	1546
$T_f\left({}^0C\right)$	158
$\Delta H\left(cal/gr\right)$	60
Melting Heat (J/kg)	244000

This decrease in temperature was very pronounced in the region close to the liquid cast / mould interface. The reason for this sudden change in temperature is that as the liquid cast is poured into the mould, the mould quickly conducts the heat away from the liquid cast / mould interface. Also, at the interface between the cast, there is also a decrease in temperature as time increases. This is because the heat in the liquid cast is released into the atmosphere.

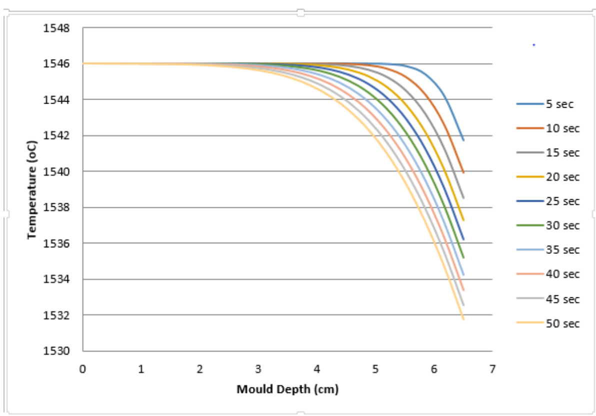

Figure 1: A graph of Temperature against mould depth (after 5sec)

Figure 1 shows the temperature distribution between 5sec and 50sec, as the liquid cast is poured the temperature remains constant at a depth of about 6.0cm for a temperature of 1546 ºC and at 50sec the temperature drops to 1532 ºC and at a depth of 2.5cm.

Furthermore there is a gradual decrease of temperature from 1532 ºC to 1525 ºC for a time of 55sec to 100sec, this is shown in Figure 2 while the depth remains 2.5cm.

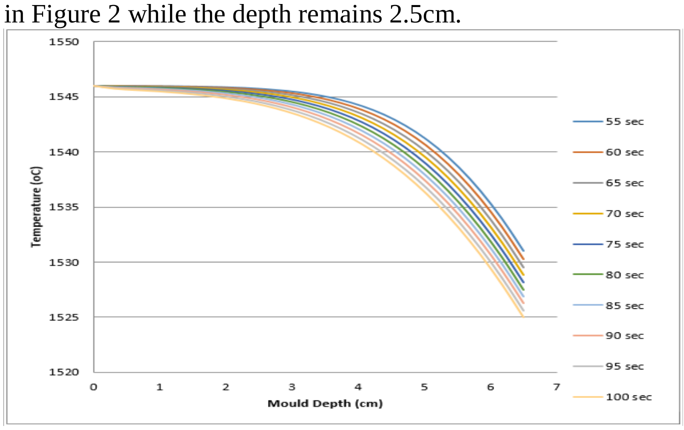

Figure 2: A graph of Temperature against mould depth (after 50sec)

Figure 3 shows the temperature distribution between 105sec and 150sec, in this region the liquid cast remains in the mould with a temperature drop of 1525ºC to 1520ºC at a depth of about 1.5cm to 0.4cm respectively.

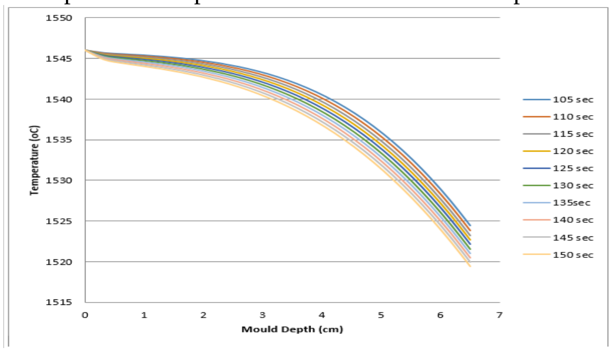

Figure 3: A graph of Temperature against mould depth (after 100sec)

Figure 4 shows a steep drop in temperature from 155sec to 200sec at a depth of 6.50cm. However the temperature was steady at a depth of 0.4cm before the sudden change of temperature from 1520ºC to 1513ºC.

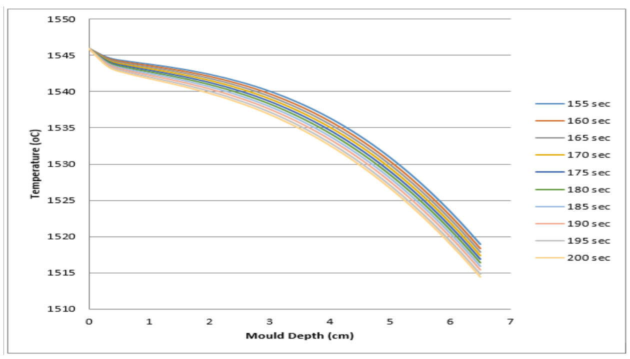

Figure 4: A graph of Temperature against mould depth (after 150sec)

Using the parameters presented in Table 1 heat transfer in the mould region is controlled by the convection of liquid superheat to the shell surface, latent heat evolution in the mushy zone, heat conduction through the solid shell, the thickness and other properties of the interface between the shell and the mould, heat conduction through the copper mould, and heat convection to the mould-cooling water.

3.1 Solidification in the Mould

It is evident from Figure 5 that as the time increases, the molten metal tend to solidify from the liquidous state to solidus state due to heat loss in the mould. This is as a result of the fact that, as the time increases, more heat withdrawn from the liquid cast into the mould and finally into the atmosphere. It was observed that at time t=0, the thickness was 0 cm since none of the liquid metal solidifies at the time of pouring the molten metal into the mould. At about 250sec from when the molten metal was poured into the mould, the solidified thickness of the cast was about 0.4063 cm. At about 250 sec from when the molten metal was poured into the mould, the solidified thickness increases to about 2.4375 cm at 500 sec.

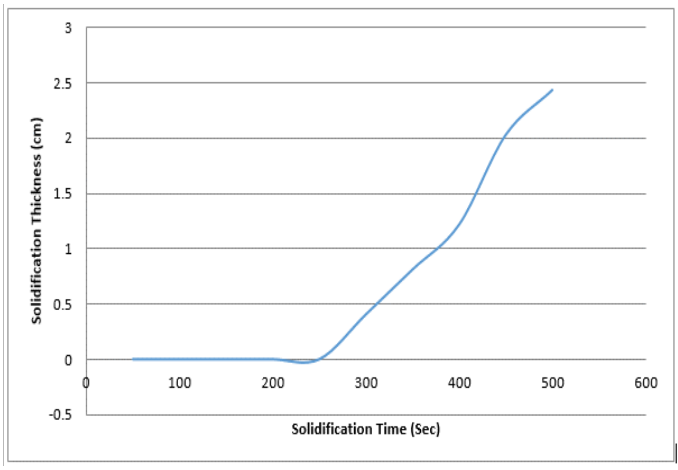

Figure 5: A graph of Solidification thickness against Solidification time (mould)

4.0 Conclusion

In this research, the Galerkin finite element method has been used to obtain the temperature distribution and the solidification thickness of the liquid cast in the mould in continuous slab casting using the time approximation equation. The copper mould was preheated to a temperature of 320ºC and the pouring temperature of the liquid cast was 1546ºC. The results obtained from this work were compared with the results from finite volume method [10] and it was observed that both results were in agreement. The results obtained shows that the Garlerkin Finite Element method is an efficient and accurate method of predicting the temperature distribution of the cast in the mould while the thickness was observed also to increase after some time to about 2.4375cm which aid the downward movement of the liquid cast into the solidifying steel shell.

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