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AREA OF SEMI-CIRCLE FUZZY REGRESSION VIA TRIANGULAR FUZZY NUMBER

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ABSTRACT

In this paper, we presented a structural, informative and theoretical computation of Area of the Semi-circle fuzzy regression via the triangular fuzzy regression method. The length and the midpoint of the inner triangular and outer triangular fuzzy regression were the required tool applied on the semi-circle fuzzy regression. The resultant regression for the area of semi-circle were obtained for two instances.

1. Introduction

Semi-circle fuzzy regression analysis is an aspect of the convectional fuzzy regression originally developed by Tanaka, which can be used to model a relationship between the dependent and independent variable in a fuzzy environment.

Fuzzy regression was first proposed by Tanaka, Uejima and Asai [14]. The concept of fuzzy regression has been applied to various aspect of science and engineering such as in [8] and [9] which deals with a multi-objective and bridge regression approaches respectively. Regression analysis is a tool that evaluates the function relationship between the dependent and independent variable.

Fuzzy regression analysis is an extension of the classical regression when the following condition(s) holds (the data set is small, there is difficulties verifying distribution assumption, vagueness in relationship between input and output variable, ambiguity and distortion introduced by linearization [5, 13]

The three important types of fuzzy regression are the triangular, trapezoid and the semi-circle fuzzy regression. So many people have worked on different types of fuzzy regression such as fuzzy goal programming [3,15]. The conventional fuzzy regression is as defined in [11] below.

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The aspect of the trapezoidal fuzzy regression, the distance of intuitionistic trapezoidal fuzzy number has been studied [2, 6, 7, 12, 16, 17].

On the part of the semi-circle fuzzy regression, the application of Half-circle fuzzy numbers and development under the influence of triangular fuzzy number has been proposed in [11]. A necessary shortcoming was ascertained in using triangular fuzzy regression to address the area of Semi-circle fuzzy regression. The above triangular fuzzy regression literature only addresses the issues of triangular fuzzy regression. Hence, this paper proposed the use of inner and outer triangular fuzzy regression in solving the problem of a Semi-circle fuzzy regression problem; using the area of the Semi-circle as an instant.

The remaining content of this paper are summarized as follows: Section two discusses the formulation and methodology of the standard semi-circle fuzzy regression via the triangular fuzzy regression. Section three discusses the operation and mechanism of the proposed analytical approaches, which attempt to overcome the limitation above.

2. The standard fuzzy regression model

The model is define as:

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}x_1 + \cdots + \tilde{A}_n x_n \qquad (1)$$

$$\text{Or } \tilde{Y} = \tilde{A}x_1 \oplus \ldots \oplus \tilde{A}_n x_n$$

where $\tilde{Y}$ is the fuzzy output, $\tilde{A}_j$ = 1, 2, …, n is a fuzzy coefficient, x = (x_1, …, x_n) is an n-dimensional non – fuzzy input vector. The fuzzy component can assume triangular, trapezoidal, semi-circle etc. However, we are interested in the semi-circle fuzzy numbers.

The model equation have the parameter $\tilde{A}_1$ with membership function as shown below

$$\mu_A(x) = \begin{cases} \sqrt{1-(x-h)^2}, & h-1 \le x \le h+1 \\ 0, & otherwise \end{cases} \text{[11]} \qquad (2)$$

The membership function of the semi-circle fuzzy as represented below in Figure 1

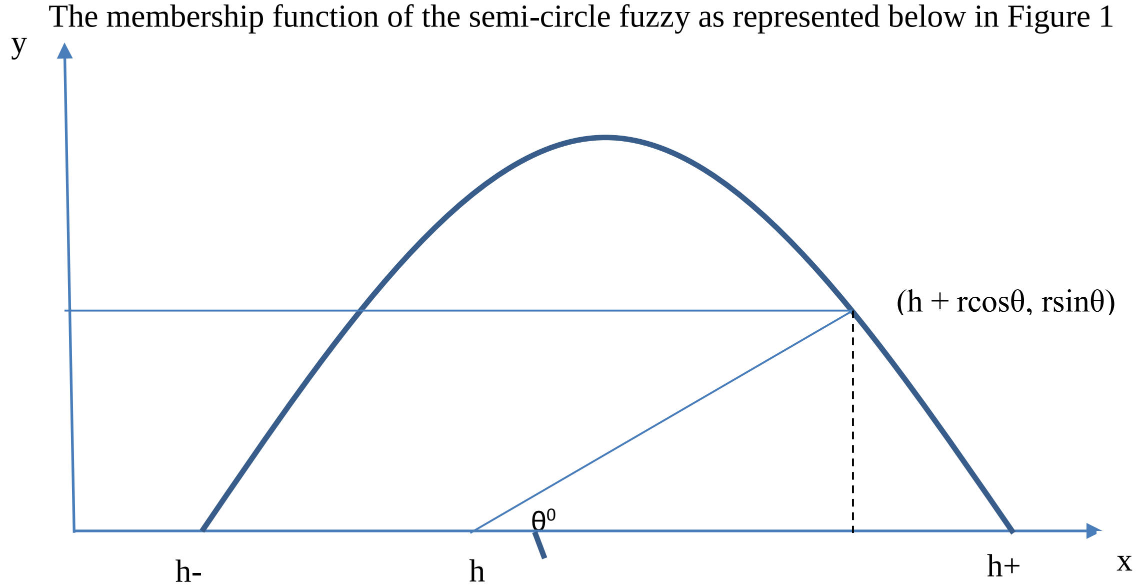


Figure 1: Polar representation of membership function of semi-circle fuzzy number.

Given Semi-circle parameter $\tilde{A}_j$ with membership as in (1). Let it the semi-circle linear function

$$\tilde{Y} = \tilde{A}x_1 \oplus +\tilde{A}_2x_2 \oplus \ldots \oplus \ \ldots \oplus \tilde{A}_nx_n$$

be obtained as follows:

$$h = \sum_{mode}^{n} h_jx_j, \quad h - 1 = \sum_{j=1}^{n} a_jx_j, \quad h + 1 = \sum_{j=1}^{n} b_jx_j.$$

It can be deduced from [1] that:

Example (1):

If a semi-circle fuzzy number $\tilde{A} = (h-1, h, h+1)$, then

$$A' \otimes x = [(h-1)x, hx, (h+1)x] = (hx - x, hx, hx + x) \quad (4)$$

Example (2):

If

$$\tilde{A} = (h-1, h, h+1) \text{ and } \tilde{B} = (g-1, g, g+1) \text{ then}$$

$$\tilde{C} = \tilde{A} \oplus \tilde{B} = (h + g - 2, \quad h + g, h + g + 2).$$

The two examples above show that the membership function for a semi-circle fuzzy linear function exists. However, from (3) we can deduce the membership function of the semi-circle fuzzy linear regression $Y^* = \beta_1^* x_1 \oplus \beta_1^* x_2 \oplus \ldots \oplus \beta_n^* x_n$ given as:

$$\mu_{y^*}(y) = \begin{cases} 1 \pm \sqrt{1 - (y - \sum \alpha_j x_i)}, & c_1 \le y \le c_3,\ c_3 \le y \le c_2 \\ 0, & otherwise \end{cases} \quad (6)$$

where

$$b_j = d_j, a_j = b_j - l_j, c_j = b_j + l_j$$

$$c_1 = \sum_{j=1}^{n} (\alpha_j - l_j), c_2 = \sum_{j=1}^{n} (\alpha_j + l_j) x_j, c_3 = \sum_{j=1}^{n} \alpha_j x_j$$

The output y membership function can be written as:

$$\mu_{y^*}(y) = \begin{cases} 1 \pm \sqrt{1 - (y - d)}, & m - d \le y \le m \\ 0, & otherwise \end{cases}$$

where m = center(mode) and d= spread.

On the other hand, the polar coordinate used to develop the circle function is semi-circle fuzzy number given as:

$$f(r,\theta): r^2 + r_0^2 - 2r_0 r cos(\theta - \theta_0) = a^2, a \epsilon \mathcal{R} \quad (8)$$

where:

$(r\theta_0, \theta\theta_0)$

is the center of the circle such that its membership function is given as:

$$\mu(r,\theta) = \begin{cases} r^2 + r_0^2 - 2r_0 r cos(\theta - \theta_0) - 1, & r_0 - 1 \le r_0 \le r_0 + 1 \\ 0, & otherwise \end{cases} \quad (9)$$

The degree of fitness for given data set Y = (m) is defined in $\tilde{h}$ and vagueness in
$V = \sum_{j=1}^{n}(m-d)_i = \sum_{j-1}^{n} l_j$.

To obtain Semi-circle parameter A^* which minimizes V subject to $\tilde{h} > H \; \forall \, data$ in
y = (m, d)

where H = degree of the Semi-circle linear model by decision maker.

Thus,

$Min\, V = l_1 + l_2 + \cdots + l_n$

Subject to

$\sum \alpha_j x + (1-H)\sum l_j x \ge m + (1-H)d, \;\; m \le c+1$

$-\sum \alpha_j x_j + (1-H)\sum l_j x_j \ge -m + (1-H)d, \;\; m \ge c+1$

$l_j \ge 0 \; for \; j = 1, 2, \ldots, n$

$d \ge 0.$

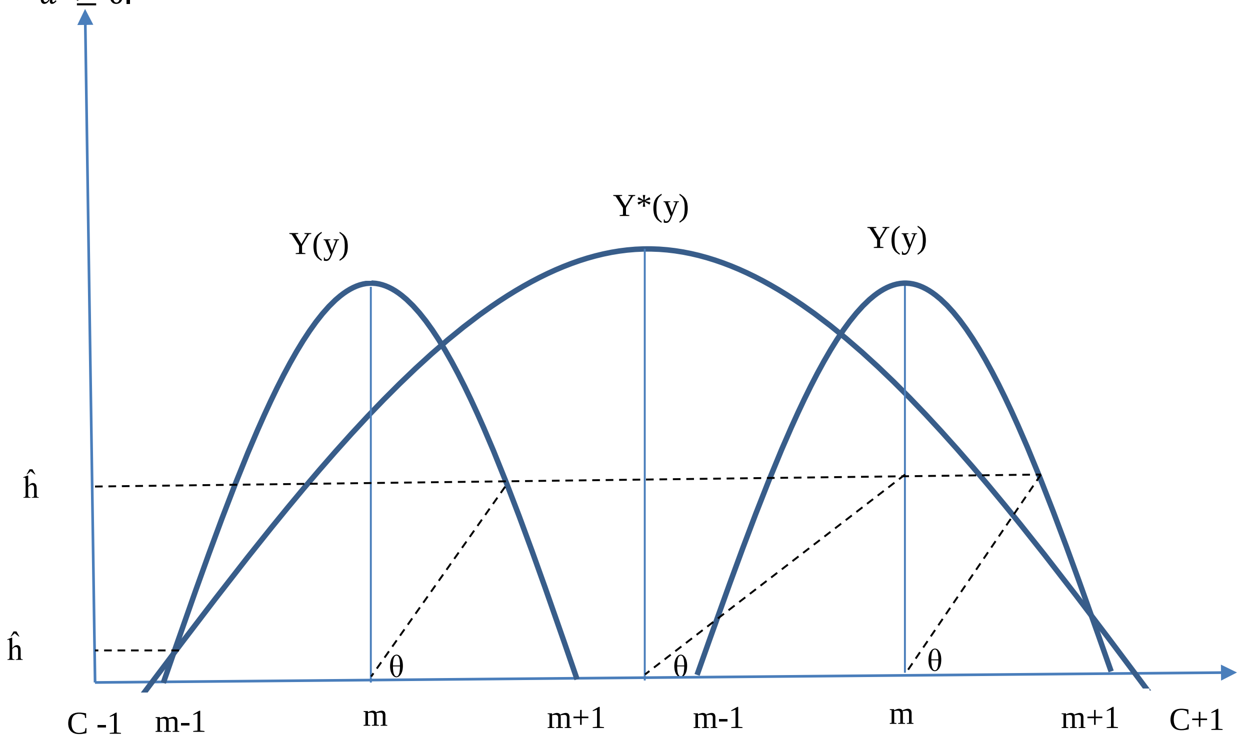


Figure 2: Degree of fitting y* to s given semi-circle fuzzy data y.

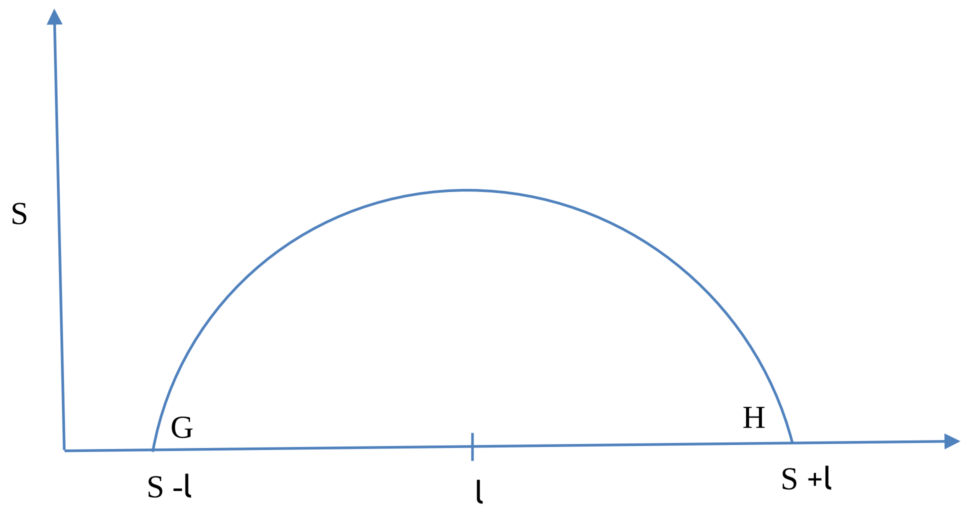


Figure 3: Membership function of a half-circle fuzzy number

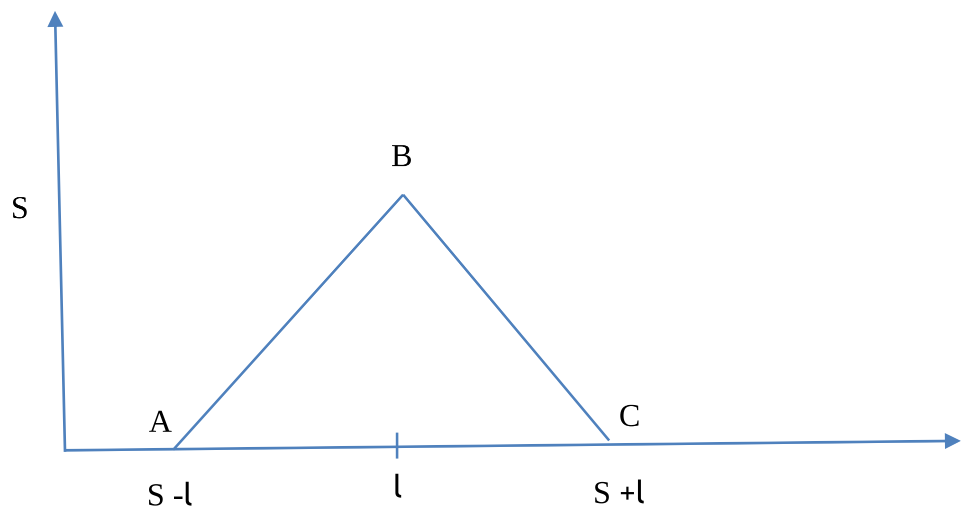


Figure 4: Membership function of inner triangular fuzzy number

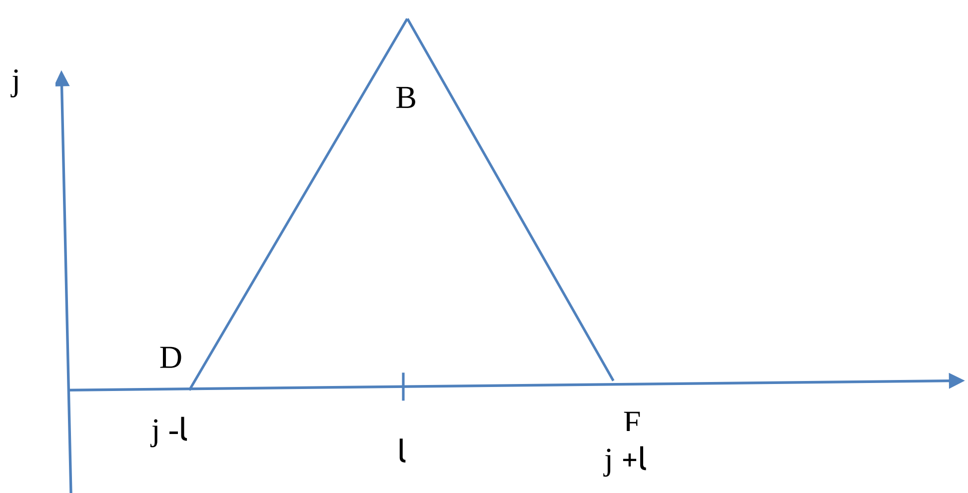


Figure 5: Membership function of outer triangular fuzzy number

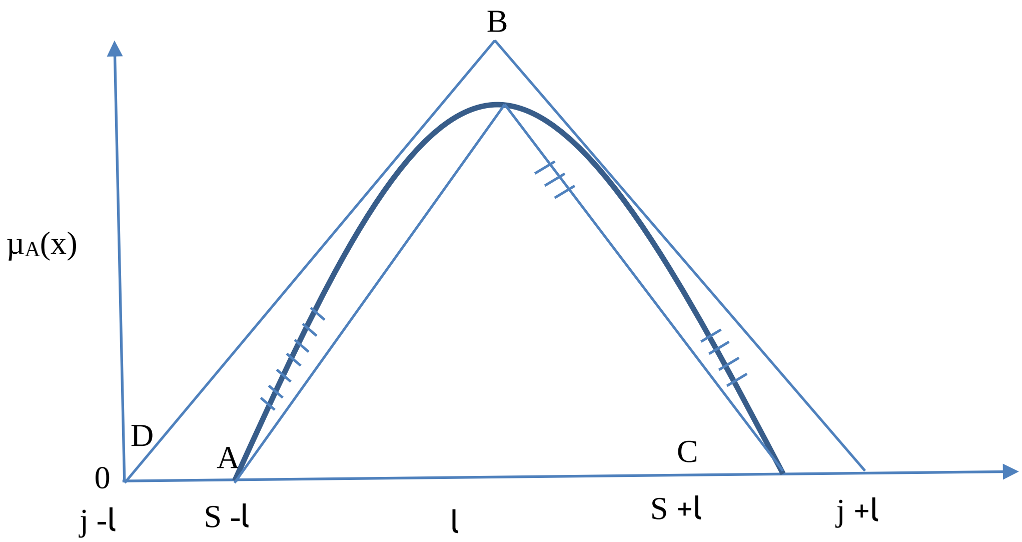


Figure 6: Fuzzy representation for triangular and Semi-circle number

Key:

 = Semi- circle

= = Inner triangle

__ = Outer triangle

$\mu_A(x)$ = Membership function

3. The concept of Semi-circle fuzzy number

The square of membership function of the half circle number is defined in terms of Pythagorean identity as:

$$1 = \mu_A^2(x) + (x - s)^2 \qquad (11)$$

where:

$\mu_A^2 = (0, 1)$ and $x \epsilon (s_1 - 1, s_1 + 1)$

Thus, we can always find two natural numbers such that the gcd$[(x - s)_1,\ (x - s)_2] = 1$

where:

$c = (x - s)_2^2 - (x - s)_1^2,\ S = 2(x - s)_1(x - s)_2$ and

$a = (x - s)_2^2 + (x - s)_1^2$ such that $(x - s)_2 > (x - s)_1 > 0$

In term of polar coordinates, we have:

$$f(r, \theta): (x - s)_1^2 + (x - s)_2^2 - 2(x - s)_1(x - s)_2 \cos(\theta_2 - \theta_1) = a \qquad (12)$$

which clearly follows the term in [1]

The properties of the Semi-circle fuzzy number is exactly the same as that of the properties of triangular fuzzy number, Hence,

$(c, s, a) \not\equiv [\ \mu_A(x)c, \mu_A(x)j, \mu_A(x)a]$

are both Pythagorean fuzzy number.

The issue of a common divisor can be shown. Hence, we have it that all Pythagorean Semi-circle fuzzy number are multiple of other cases.

To determine the area of Semi-circle fuzzy regression using at least two triangular fuzzy regressions. We use Figures 3, 4 and 5.

Figure 6 is obtained by merging Figures 3 -5 together.

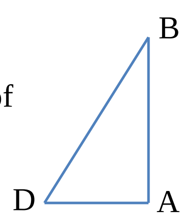
To obtain the area of the semi-circle in Figure 3, we impose, figure 3, figure 4 and figure 5 to obtain figure 6.

Note that the area of $\Delta\text{ABC} = \ \Delta\text{BCF}$, since we are working on symmetric triangle and symmetric semi-circle fuzzy regression.

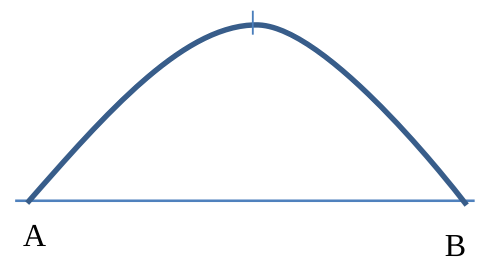
Therefore, using the basic ideas of [4, 11], we have

$\Delta\text{ABC} =$ ½ the area of the semi-circle

Area of



= ½ the area of



Hence, the area of Figure (3) logically can be define as area of the non-symmetric triangular fuzzy regression $\Delta\text{ABF} = \ \Delta\text{ADB}$ since they are equal.
The area of Figure (3) can also be defined with respect to symmetric triangular fuzzy regression with the following steps:

i. Calculate the area of ΔABD
ii. Calculate the area of ΔABC
iii. Compute the area of Figure (3) = $\Delta\text{ABD} + \ \Delta\text{ABC}$

Next, having obtained the area of the half circle i.e. Area of figure, every other analysis such as fuzzy least square, sum of squares and the distance still follows the conventional method [4,13]

4. Conclusion

Triangular fuzzy regression are often used by researchers because, the calculations are simple and easy. Hence, in this study, we obtained a standard method of calculating the area of Semi-circle fuzzy regression using the idea of the conventional triangular fuzzy regression method. In addition, the other statistical component such as sum of squares, fuzzy least squares and distance measure also follow. Finally, we were able to get more information about the Semi-circle fuzzy regression.

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