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NUMERICAL SOLUTIONS OF HEAT AND MASS TRANSFER OF MAGNETOHYDRODYNAMIC FLOW OVER A VERTICAL PLATE IN THE PRESENCE OF HEAT DISSIPATION AND THERMAL RADIATION

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ABSTRACT

The similarity solution is used to transform the model in form of system of partial differential equations(PDE), describing the problem into a system of coupled ordinary differential equations. The resulting governing system of ODE along with the boundary conditions are solved by fourth order Runga-Kutta shooting method implemented on Maple. The results are presented graphically and in tabular form and the conclusion is drawn that the embedded physical flow parameters such as magnetic parameter, chemical reactions, thermal radiations, Schmitdt number, Soret number, permeability, heat source and Prandtl have significantly influenced on flow velocity, temperature and concentration profiles.

Introduction

The study of MHD flow over a vertical plate with convective surface boundary conditions has attracted much interest in recent years due to its significance in many engineering industrial processes. The industrial applications include glass-fiber production, condensation process of metallic plate in a cooling bath and glass and aerodynamic extrusion of plastic sheets. These among other applications led researchers to investigate various aspect of the problem. Thermal radiation effects are of great significance in the field of space technology and in processes involving high temperatures. For instance, some devices for space applications are designed to operate at high temperature levels in order to achieve high thermal efficiency. Radiation is also considered especially when calculating thermal effects in devices such as a nuclear power plant, gaseous nuclear rockets and rocket nozzles. Recent advancement in hypersonic flights, gas cooled nuclear reactors, rocket combustion chambers and missile reentry, have perhaps shifted researchers' attention to thermal radiation as a mode of energy transfer, and emphasize the need for improved understanding of radiative transfer in these processes. In a pioneering work, Sakiadis [1,2] investigated the boundary layer flow induced by a moving plate in a quiescent ambient fluid. Thereafter, various scholars and

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researchers have investigated many aspects of the problem. Recently, Aroloye and Oluwalana [3] thoroughly investigated thermal radiative MHD flow, heat and mass transfer over vertical plate with internal heat generation and chemical reaction. Deka and Neog [4] analyzed MHD flow past an impulsively started infinite vertical plate with thermal stratification and radiation. Srinivas and Muthuraj [5] examined the effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel using homotopy analysis method. Laksmi et al. [6] solved numerically the MHD flow over a moving vertical porous plate with heat and mass transfer. MHD free convective boundary layer flow through porous medium past a moving vertical plate with heat source and chemical reaction was analyzed by Krishna and Reddy [7]. Balamurugan et al. [8] studied about unsteady MHD free convective flow past a moving vertical plate with time dependent suction and chemical reaction in a slip flow regime. Makinde [9] investigated on MHD heat and mass transfer over a moving vertical plate with a convective surface boundary conditions. Seini and Makinde [10] investigated the hydromagnetic flow with dufour and soret effects past a vertical plate embedded in porous media. The effect of thermal radiation on heat and mass transfer of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field was examined by Makinde and Ogulu [11]. The similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media was stufied by Chamkha and Khaled [12]. Seddeek[13] studied the thermal radiation and buoyancy effects on MHD free convection heat generation flow over an accelerating permeable surface with temperature-dependent viscosity. Postelnicu[14] numerically studied the influence of magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media by considering the Soret and Dufour effects. Makinde[15] carried out a numerical study on the effect of thermal radiation on boundary layer flow with heat and mass transfer past a moving vertical porous plate. Cortell [16] theoretically examined the flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species. The numerical results show that the effect of destructive chemical reaction is to diminish the concentration boundary layer and this phenomenon is quite the opposite when a generative reaction is present. Bataller[17] investigated the effect of thermal radiation on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Rajeswari et al. [18] studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through a stationary vertical porous surface in the presence of suction with power law surface temperature and concentration. Aziz [19] reported a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition.

In the present study, we comprehensively presented the Numerical solutions of heat and mass transfer of magnetohydrodynamic flow over a vertical plate in the presence of heat dissipation and thermal radiation by by fourth order Runga-Kutta shooting method implemented on Maple**.** It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies. Also, our results were validated by comparing with related existing work in literature and there is excellent agreement

MATHEMATICAL MODEL

We consider heat and mass transfer of magnetohydrodynamic flow over a vertical plate in the presence of heat dissipation and thermal radiation. The plate is porous thereby there in and out flow of fluid across the plate. Magnetic field strength B_0 is applied perpendicular to the plate. The fluid is considered to be conducting and incomprehensible, *x*-axis is taken along the direction of plate and *y* axis normal to it. If *u*, *v*, *T*, and *C* are the fluid *x*-component of velocity, *y*-component of velocity, temperature and concentration, respectively, then under the Boussinesq and boundary layer approximations, the governing equations for this problem can be written as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(U_\infty-u)+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty)-\frac{vu}{k_p} \tag{2}$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\sigma\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho C_p}(U_\infty-u)^2+\frac{Q_0}{\rho C_p}(T-T_\infty)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kr(C-C_\infty)+D_T\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where v is the kinematic viscosity, C∞ is the free stream concentration, U_∞ is the plate velocity, ꞓ is the thermal diffusivity and D is the mass diffusivity, β_T is the thermal expansion coefficient, β_C is the solutal expansion coefficient, C_p is the heat capacity, ρ is the fluid density, g is gravitational acceleration, and σ is the fluid electrical conductivity, q_r is the radiative heat flux, Q_0 is the heat generation coefficient, chemical reaction. The boundary conditions at the plate surface and far into the cold fluid may be written as:

$$u(x,0)=U_0 \qquad v(x,0)=0, \qquad -k\frac{\partial T}{\partial y}(x,0)=h_1[T_1-T(x,0)] \tag{5}$$
$$C_w(x,0)=Ax^b+C_\infty,\quad u(x,\infty)=0,\quad T(x,\infty)=T_\infty,\, C(x,\infty)=C_\infty,$$

where L is the plate characteristic length, C_w is the species concentration at the plate surface, is the plate surface concentration exponent, and k is the thermal conductivity coefficient. The stream function ψ, satisfies the continuity Equation (1) automatically with

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ \ v=-\frac{\partial\psi}{\partial x}$$

A similarity solution of Equations (1)–(6) is obtained by defining an independent variable and a dependent variable f in terms of the stream function as:

$$\eta=y\sqrt{\frac{U_0}{vx}}, \qquad \phi(\eta)=\frac{C-C_w}{C_w-C_\infty} \tag{6}$$

The dimensionless temperature and concentration are given as:

$$\phi(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \qquad \phi(\eta)=\frac{C-C_w}{C_w-C_\infty} \tag{7}$$

After introducing Equations (6)–(8) into Equations (1)–(5), we obtain:

$$f'''+\frac{1}{2}ff''-(1-f')M+Gr\theta+Gr\phi+\frac{u}{K}=0 \tag{8}$$

$$\theta''+\frac{1}{2}\Pr f\theta'+(1-f')M+Q\theta-Ra\theta'=0 \tag{10}$$

$$\phi''+\frac{1}{2}Scf\phi'+Kc\phi+Sr\theta''=0 \tag{11}$$

$$f(0)=0, \quad f(0)=1, \quad \theta'=Bi[\theta(0)-1], \quad \phi(0)=1 \tag{12}$$

$$f'(\infty)=0, \qquad \theta(0)=0, \qquad \phi(0)=0 \tag{13}$$

Where the prime symbol represents the derivative with respect to η

$$M=\frac{\sigma B_0^2 x}{\rho U_0}, \quad Gc=\frac{gB_T\left(T_f-T_\infty\right)}{U_0^2}, \quad Gc=\frac{gB_T\left(C_w-C_\infty\right)}{U_0^2}, \quad Sr=\frac{D_T\left(T_f-T_\infty\right)}{\nu\left(C_w-C_\infty\right)}$$

$$Bi=\frac{h_f}{k}\sqrt{\frac{vx}{U_0}}, \ \Pr=\frac{v}{\alpha}, \ \Pr=\frac{v}{D} \ Ra=\frac{16\sigma T_\infty^3}{3k^* Cp}, \quad Q=\frac{v^2 Q_0}{\sigma\rho C_p}, \quad K=\frac{vkr}{U_0^2}$$

Where M is the magnetic parameter, Gr is the local thermal Grashof number , Gr is the local solutal Grashof number , Bi is the local convective heat transfer parameter , Pr is the Prandtl and Sc is the Schmidt number, Ra is the thermal radition, Qis the internal heat generation and kc is the chemical reaction, Sr is the Soret number and K is the permeability of the porous medium

The set of Equations (9)–(11) under the boundary conditions (12) and (13) have been solved numerically by applying the Nachtsheim and Swigert (1965) shooting iteration technique together with Runge–Kutta sixth-order integration scheme. From the process of numerical computation, the plate surface temperature, the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are, respectively, proportional to, are also worked out and their numerical values are presented in a tabular form

Result and discussion

Table 1. Computation showing, $f''(0)$, $\theta'(0)$, and $\phi'(0)$ for various values of embedded parameter

Pr	Q	Ra	Gr	Gc	M	Pr	Sc	Sr	Kr	Kc	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.4	0.3	0.1	0.1	0.1	0.1	0.72	0.62	0.5	0.2	0.2	0.789093	0.078531	0.071531
0.4	0.3	1.0	0.1	0.1	0.1	0.72	0.62	0.5	0.2	0.2	0.729131	0.089435	0.086433
0.4	0.3	10	0.1	0.1	0.1	0.72	0.62	0.5	0.2	0.2	0.612032	0.091236	0.078123
0.76	0.4	0.1	0.5	0.1	0.1	0.72	0.62	0.6	0.3	0.3	0.568181	0.095679	0.078125
0.76	0.4	0.1	1.0	0.1	0.1	0.72	0.62	0.6	0.3	0.3	0.453267	0.096783	0.098865
0.76	0.4	0.1	0.1	0.5	0.1	0.72	0.62	0.7	0.3	0.3	0.380716	0.123891	0.076120
0.8	0.5	0.1	0.1	1.0	0.1	0.72	0.62	0.7	0.4	0.4	0.332042	0.123900	0.024561
0.8	0.5	0.1	0.1	0.1	1.0	0.72	0.62	0.7	0.4	0.5	0.267891	0.178936	0.098571
0.8	0.5	0.1	0.1	0.1	5.0	0.72	0.62	0.8	0.4	0.5	0.217678	0.768901	0.086721
0.9	0.6	0.1	0.1	0.1	0.1	1.00	0.62	0.8	0.5	0.6	0.208915	0.078561	0.098467
0.9	0.6	0.1	0.1	0.1	0.1	7.10	0.62	0.8	0.5	0.6	0.194565	0.067543	0.061234
0.9	0.7	0.1	0.1	0.1	0.1	0.72	0.78	0.9	0.6	0.9	0.196745	0.112345	0.078710
0.9	0.7	0.1	0.1	0.1	0.1	0.72	2.63	0.9	0.6	0.9	0.134678	0.230896	0.075861

Table 1. showing Computation of local Skin friction $f''(0)$, local Nusselt number $\theta'(0)$ and local Sherwood number $\phi'(0)$ when compared with makinde(2010).

Effects of Parameter Variation on Velocity Profiles

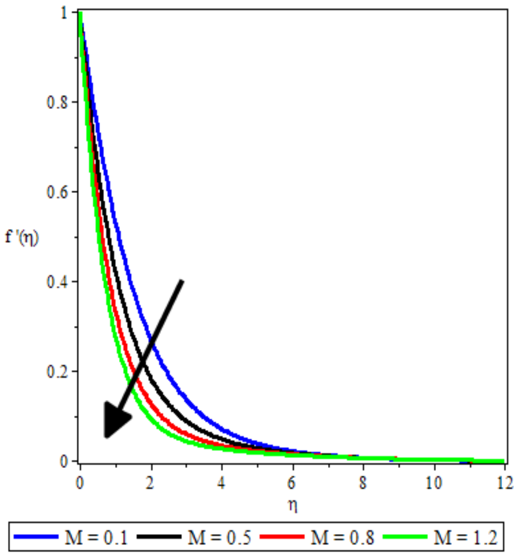


Fig 1: Influence of M on velocity profile

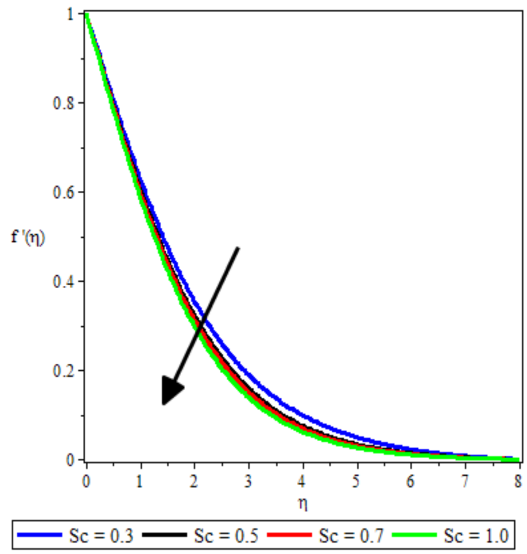


Fig 2: Influence of on Sc on velocity profile

The numerical results for the velocity profiles are displayed in Figures 1–8. The effects of magnetic field parameter (M) on the velocity field are shown in Figure 1. It is seen from this figure that the velocity profiles decrease monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition. Moreover, it is interesting to note in Figure 1 that the effect of increasing magnetic field parameter (M) is to decrease the value of the velocity profiles throughout the boundary layer. The presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. This result

qualitatively agrees with the results obtain in Makinde (2010), since the magnetic field exerts retardisng force on the free convection flow. Similar trend of decrease in fluid velocity profile is observed in fig 2 with increase in Schmitdt number.

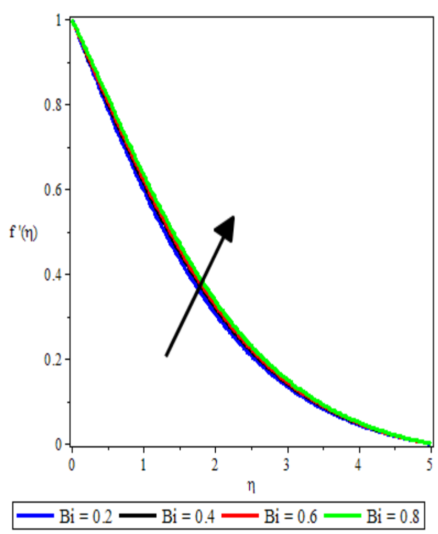


Fig 3:Influence of Bi on velocity profile

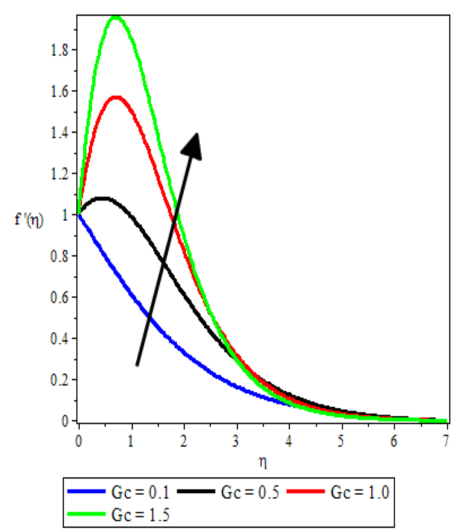


Fig 4: Influence of Gc on velocity profile

In Figure 3, a slight increase in the velocity profiles within the boundary layer is observed with and increase in the convective heat exchange parameter (Bi). Since the fluid on the right surface of the plate is heated up by the hot fluid on the left side of the plate, making it to become lighter

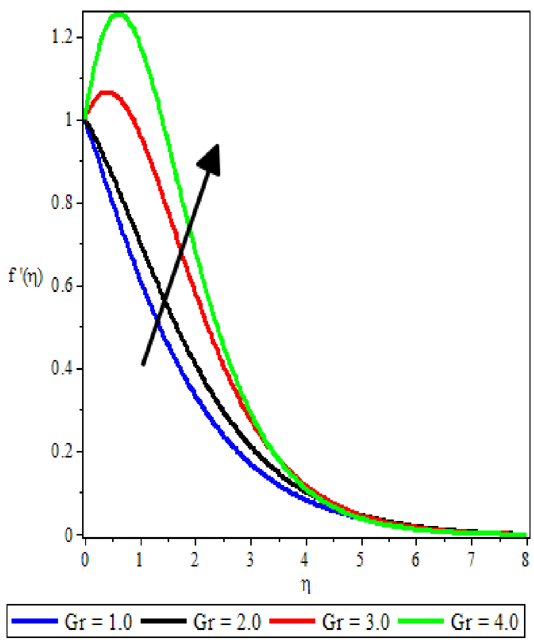


Fig 5: Influence of *Gr* on velocity profile

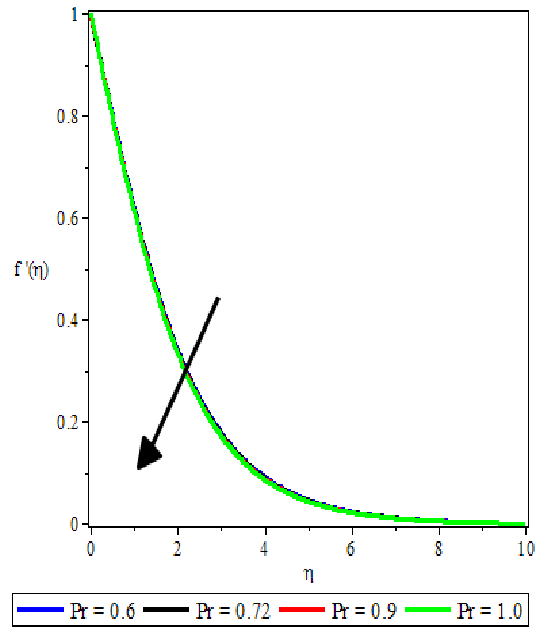


Fig 6:Influence of Pr on velocity profile

and flow faster. The effects of both thermal Grashof Gr and solutal Grashof Gc numbers are shown in Figures 4 and 5. As the Grashof number increases, the fluid velocity increases, reaching its peak value within the boundary layer and then decreases monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition. From these figures, it is

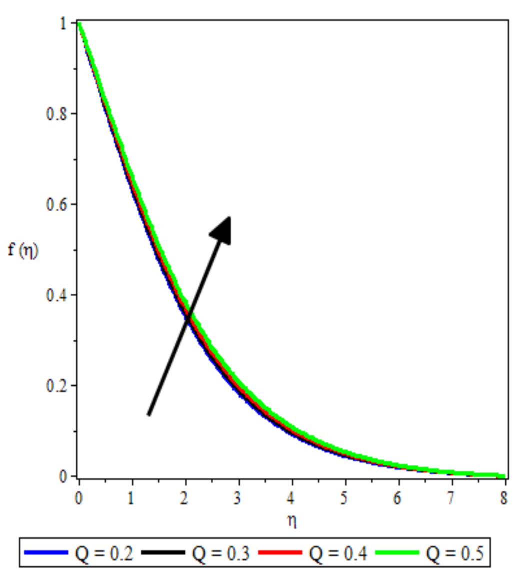


Fig 7: Influence of *Q* on velocity profile

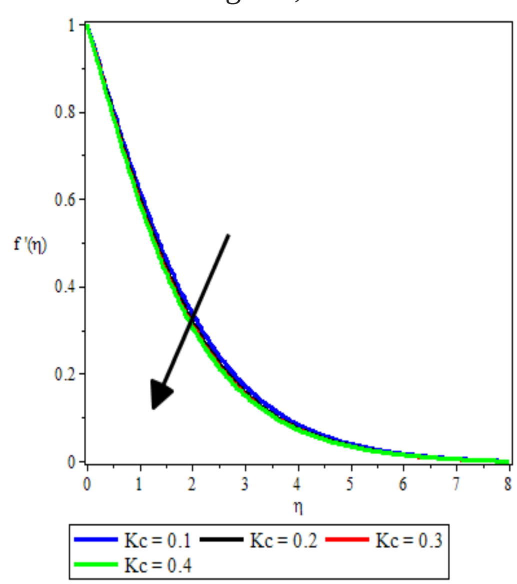
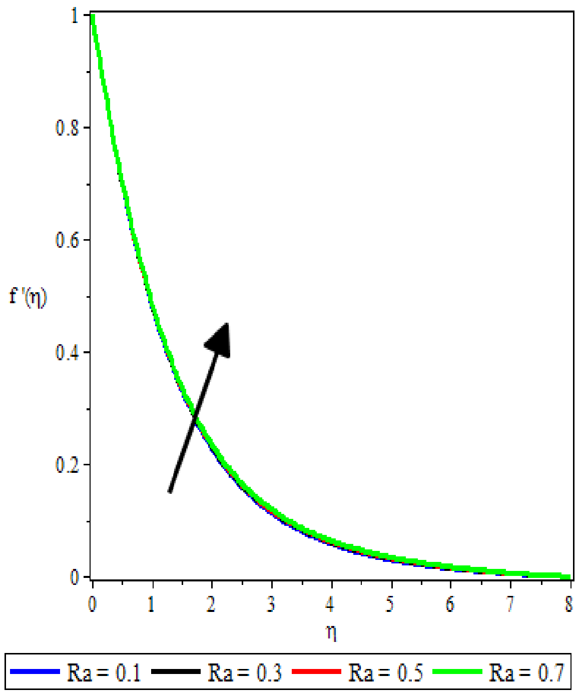


Fig 8: Influence of *Kc* on velocity profile



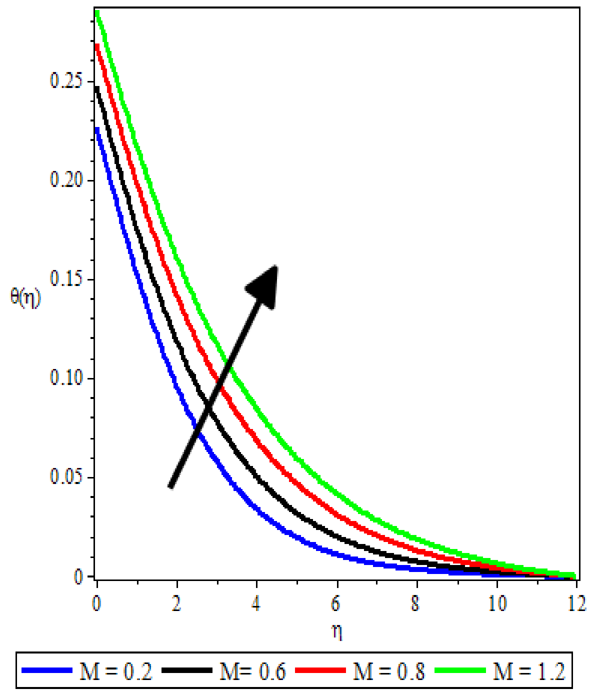
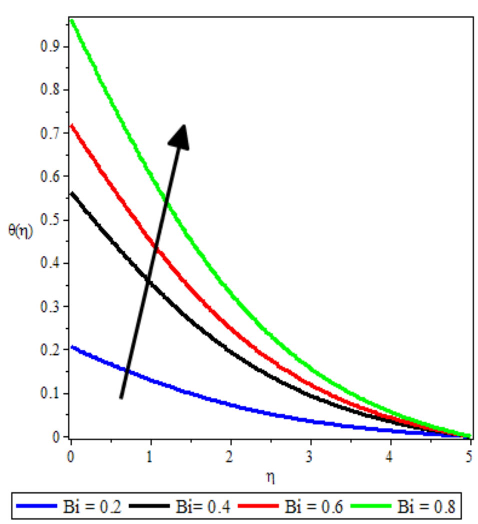


Fig 9: Influence of *Ra* on velocity profile

Fig 10: Influence of *M* on temperature profile

obvious that the buoyancy. This result also in agreement with Makinde(2010). In figure 6, increase in Prandtl number resulted in small decrease in velocity profiles while opposite effect is observed is observed in Figure 6 in which increase in heat absorption lead to increase in velocity profile. In figure 9, increase in chemical reaction correspond to decrease in fluid velocity because the particle in the fluid tend to closely packed together and resulted in slow down the fluid movement In figure 9, increase in thermal radiation result in increase in thermal radiation.

Effects of Parameter Variation on Temperature Profiles



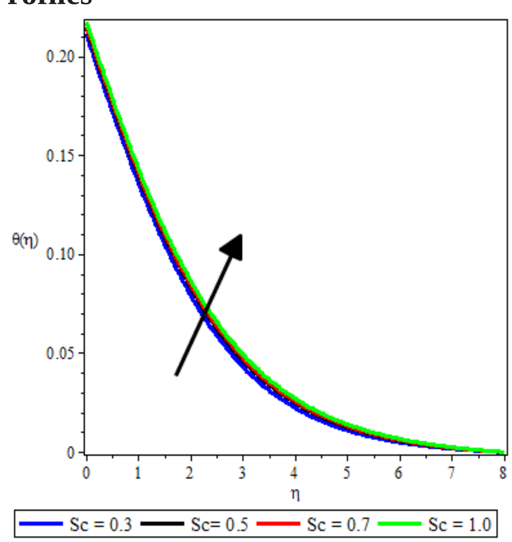
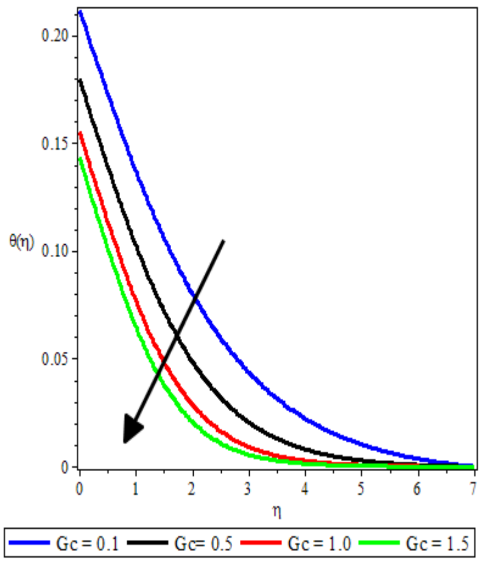


Fig 11: Influence of *Sc* on temperature profile

Fig 12: Influence of *Bi* on temperature profile

The numerical results for the temperature profiles are shown in Figures 10–18. It is seen from these figures that the fluid temperature attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary conditions. It is noteworthy that the thermal boundary layer thickness increases with an increase in the intensity of the magnetic field parameter (*M*), plate surface convective heat exchange parameter (*Bi*) and Schmidt number (*Sc*). Increase in the intensity of buoyancy forces (*Gr*, *Gc*) and the Prandtl number (*Pr*) causes a decrease in the fluid temperature leading to leading to a decaying thermal boundary layer thickness. The results in here agreed with the one reported recently in Makinde (2010). Increase in thermal radiation parameter Ra, heat source parameter Q and chemical reaction parameter increases and enhance fluid temperature.



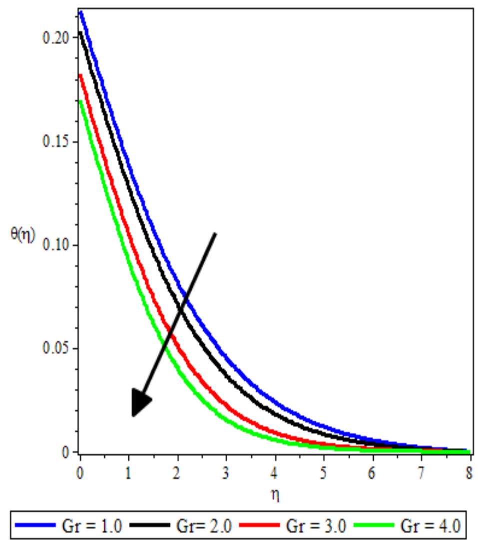
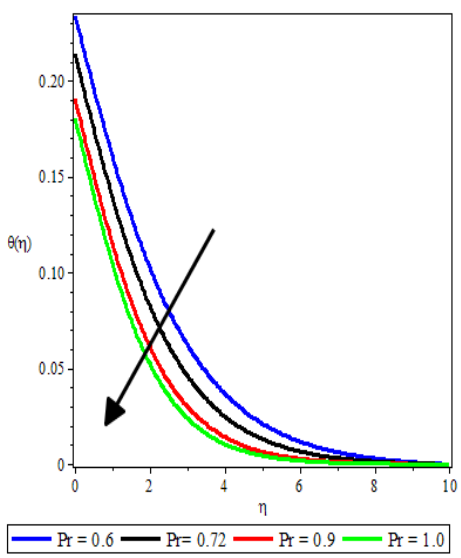


Fig 13: Influence of Gr on temperature profile

Fig 14: Influence of Gc on temperature profile



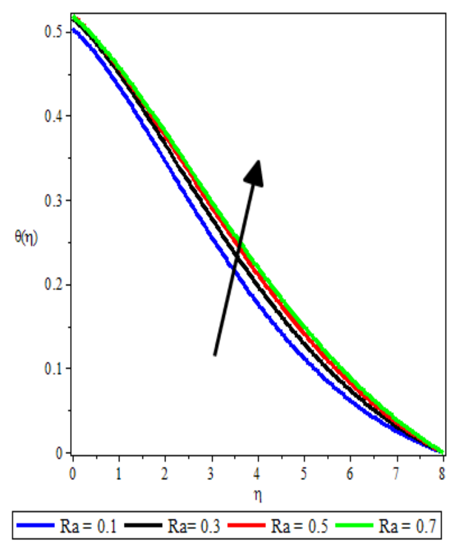


Fig15:Influence of Ra on temperature profile

Fig16:Influence of Ra on temperature profile

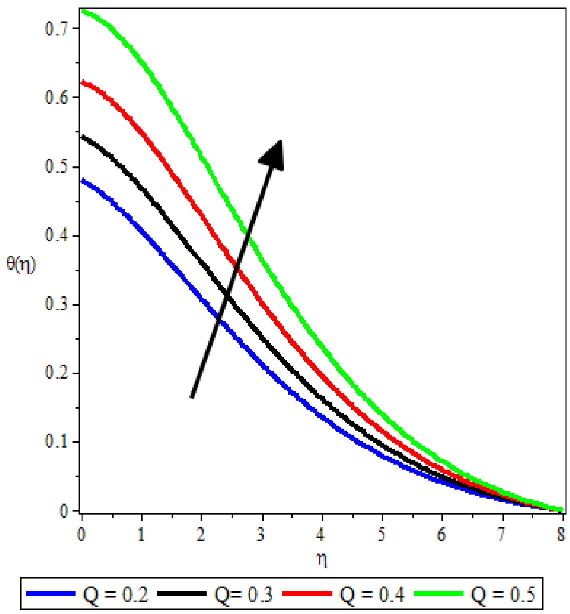


Fig17:Influence of Q on temperature profile

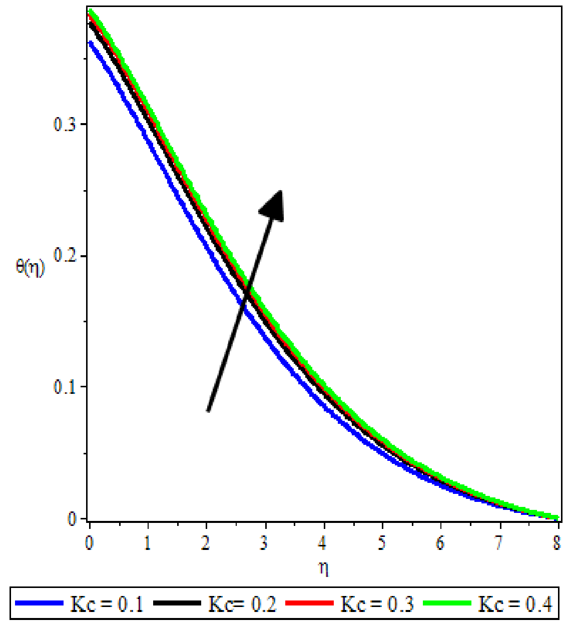


Fig18:Influence of Kc on temperature profile

Effects of Parameter Variation on Concentration Profiles

Concentration profiles for different values physical parameters in the boundary layer are shown in Figures 19–22. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition. From these figures, it is noteworthy that the

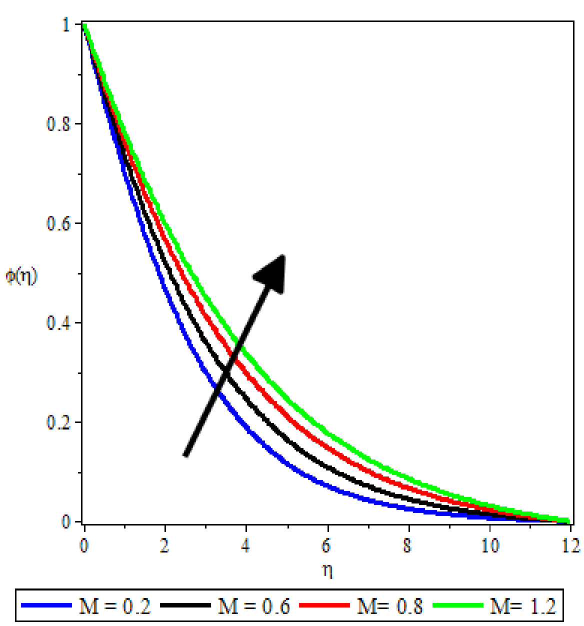


Fig 19: Influence of M on concentration profile

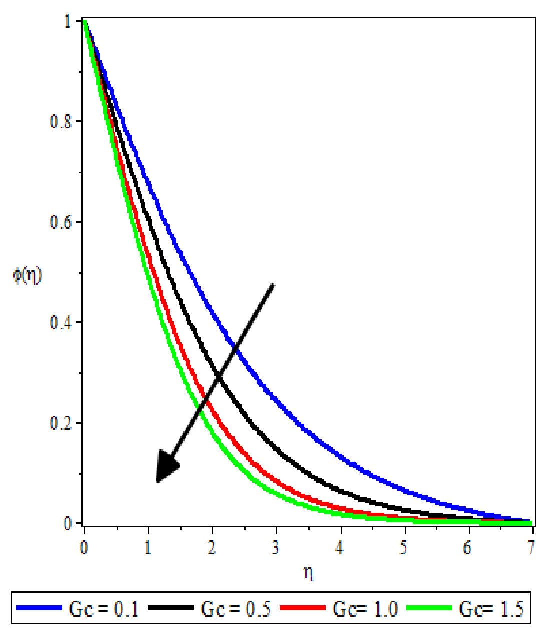
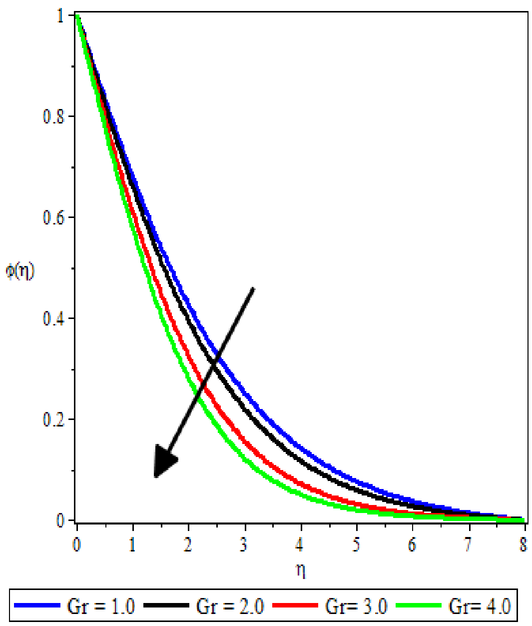


Fig 20: Influence of Gc on concentration profile

concentration boundary layer thickness increases with an increase in the magnetic field intensity (M). An increase in the values of thermal and solutal Grashof numbers (Gr, Gc) due to buoyancy forces causes a decrease in the concentration boundary layer thickness. Moreover, as the Schmidt



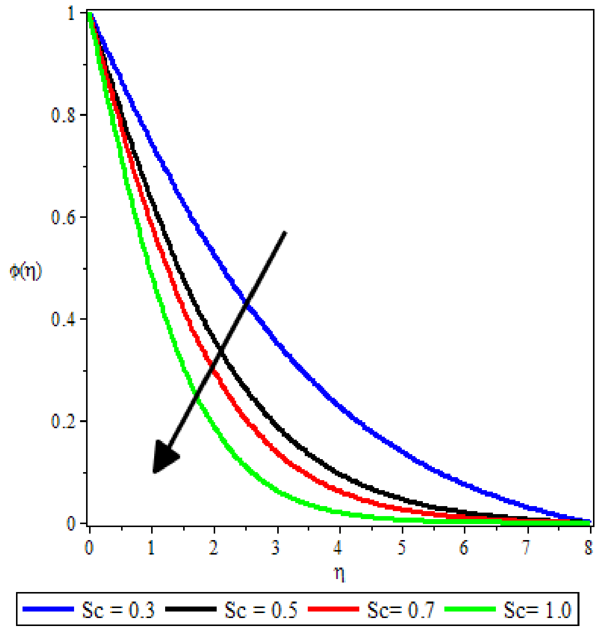


Fig 21: Influence of *Sc* on concentration profile   Fig 22: Influence of Pr on concentration profile

number (Sc) increases due to a decrease in the chemical species molecular diffusivity, the thickness of the concentration boundary layer decreases. The result are also in agreement with Makinde (2010).

Conclusion

In this paper, heat and mass transfer of magnetohydrodynamic flow over a vertical plate in the presence of heat dissipation and thermal radiation has been investigated The governing equations are transformed via similarity variable to a system of nonlinear ordinary differential equations. The resulting dimensionless odinary differential equations are solved by using fourth order Runge Kutta shooting method with aid of maple software. Numerical results for embedded flow parameters, such as the Soret number (Sr), Grashof number (Gr) for heat and mass transfer, the Schmidt number (Sc), Prandtl number (Pr), chemical reaction parameter (Kr), permeability parameter (K), magnetic parameter (M), skin friction (τ), Nusselt number (Nu) and Sherwood number (Sh) on the velocity, temperature and concentration profiles are presented graphically and discussed qualitatively

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