Journal of NAMP 66 (2024) 61 - 72

Journal of
The Nigerian Association of
Mathematical Physics
Journal homepage: https://nampjournals.org.ng

BINOMIAL EXPANSION – A SIMPLE COMPARATIVE ANALYSIS AND TIME COMPLEXITY OF CONVENTIONAL AND ALTERNATIVE METHODS

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ARTICLE INFO

Article history:
Received xxxxx
Revised xxxxx
Accepted xxxxx
Available online xxxxx

Keywords:
Computer-ready, Time complexity, Analysis, Binomial expansion, Science.

ABSTRACT

Alternative (or computer-ready) approach of learning Mathematics is exemplified and compared to the conventional methods using binomial expansion as a sample subtopic. The approach uses the tabular system for obtaining the terms of a binomial expansion. Three groups of randomly selected 100 level students are tested using one method for each group. We present a comparative analysis of the data obtained. Some advantages of the alternative method were observed. It can go a long way to enhance students' understanding of the concept. Moreover, it enables the programming of binomial expansion for computer aided solutions to problems whose solutions require binomial expansion as can be deduced from the time complexity of the methods.

1. INTRODUCTION

Innovation in teaching methods at all levels of education is the greatest contributing factor towards science and technology. This is more so when such innovation is geared towards changes in teaching methodology, such of which entails teaching outside the procedures strictly laid down by 'obsolete' texts otherwise known as conventional or traditional method. The biggest drawback of our educational system especially in the sciences is that the old or conventional method of teaching is still being used today. By this method subject matters are presented in a strict and inflexible manner, which most times are way too technical for an average learner ([2], [3] and [4]).

In this study, the term conventional method will be used to refer to the teaching of binomial expansion based on methods that cannot directly be implemented in a programming environment for computer aided solutions to mathematics problems. Obviously, every data that can be presented in a table can as well be implemented in a programming environment since all computer programs have a data structure (the array) which can handle tabular data with ease. For the sake of simplicity, we shall think of an array as a data structure arranged in tabular form.

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https://doi.org/10.60787/jnamp-v66-309

The more or less challenging task that students face in expanding a binomial expression lies in determining the coefficients of the terms of the expansion. Also, a nicely expanded binomial expression has the correct powers for the variables and in the right order. Some of the methods used in expanding binomial expressions involve observing some patterns. A good example is the pattern encountered where the Pascal's triangle method is used. The purpose of the analysis is to formally show that students understand the alternative (tabular or computer-ready) method of teaching binomial expansion than the conventional approach which vis-à-vis have been established in this article as far as possible. Thus, the idea is to suitably present binomial expansion in the alternative easier-to-understand form; and this is the goal of this paper.

2. METHODOLOGY

The Survey research design is employed in which a randomly selected number of students are examined in the subject matter presented. Thus, only a part of the population is studied. A sample of 132 out of a population of 5,671 male and female students of Federal University, Dutsin-ma is examined. The sample is collected across disciplines in pure, numerate and applied sciences. The students are grouped into three – "Paschal" group, "factorial" group and "tabular" group. Each group is tested based on the method that its name represents.

The target population is the first year students of higher institution irrespective of origin or residence's geographical location. Thus, a fairly equal representation of variables was made. A 20 marks question which students are to answer in 15 minutes was administered to them directly in a well-spaced and ventilated class. The students used their group's method as instructed. We also emphasize here that the students are knowledgeable in all three methods.

After the statistical analysis has been made and the results obtained, a further test was carried out using the MQL4 programming language [5]. This language is similar to C++ and has proven to have efficiency and reliability. The various methods of expanding the binomial expression was coded in the language. The run time of the codes of the three expansion methods were examined and compared using this programming language.

3 INTRODUCING BINOMIAL EXPANSION

Just as 5^3 means $5 \times 5 \times 5 = 125$ (i.e., to multiply 5 by itself and the result multiplied by 5), or just as a more general a^3 means $a \times a \times a$ where the letter a stands for any number, the expression $(a+b)^3$ means $(a+b) \times (a+b) \times (a+b)$. The multiplication symbol '$\times$'is usually omitted to tidy things up. Thus, $(a+b)^3$ equals $(a+b)(a+b)(a+b)$. To expand the expression, we first expand the product of the last two, $(a+b)(a+b)$. Now this means a times $(a+b)$ added to b times $(a+b)$. That is $(a+b)(a+b) = a \times (a+b) + b \times (a+b)$ or the more tidy $a(a+b) + b(a+b)$. Furthermore, each of $a(a+b)$ and $b(a+b)$ are expanded, respectively, as $a \times a + a \times b$ and $b \times a + b \times b$, that is $aa + ab$ and $ba + bb$ or in more tidy forms $a^2 + ab$ and $ba + b^2$. The two are now added together to produce $a^2 + ab + ba + b^2$ and since the two middle terms ab and ba are equal, their sum is $2ab$. Therefore, $(a+b)(a+b)$ equals $a^2 + 2ab + b^2$. That is not all because we only expanded the last $(a+b)(a+b)$ and not the whole of $(a+b)(a+b)(a+b)$. To expand all, we expand the product $(a+b)(a^2 + 2ab + b^2)$ by multiplying each of a and b in the first bracket with each of a^2, $2ab$ and b^2 in the second bracket and adding all together as follows: $aa^2 + a2ab + ab^2 + ba^2 + b2ab + bb^2$ which gives $a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$. This again should give $a^3 + 3a^2b + 3ab^2 + b^3$. Thus. the binomial expansion for $(a+b)^3$ is $a^3 + 3a^2b + 3ab^2 + b^3$ and we write $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

The expression $(a+b)^3$ is called a binomial expression. It is 'binomial' because there are two terms a and b which are added and raised to some power (the power is 3 in this case). Other binomial expressions include $(a-b)^2$, $(2a+3b)^3$, $(3a-b)^5$, $(\frac{2}{3}a+\frac{2}{5}b)^2$, $(4a-2b)^6$, $(a+7b)^{14}$, $(2a-\frac{1}{3}b)^{99}$ and infinitely many more. Thus, it is more convenient to denote a general form of a binomial expression as $(ax+by)^n$ where a and b are coefficients, x and y are variables and n is a positive integer (there also exist cases where n is any real number, but we shall not treat such here). The task in binomial expansion is to express each of these as sums of products of the variables in the brackets with their corresponding coefficients. One may have observed how laborious the above task would pose as n gets larger and as a and b becomes more complex, hence the need for easier and faster methods.

4 METHODS OF EXPANDING A BINOMIAL EXPRESSION

In this section, we present, on one hand, two conventional methods of expanding a binomial expression and, on the other hand, the tabular method (alternative method). The task will be to expand the binomial expression $(2x–y)^6$ by the three methods.

4.1 Pascal's Triangle Method

Let us expand the binomial expression $(2x-y)^6$ by employing the Pascal's triangle method. First, sketch the expansion leaving out the coefficients from Pascal's triangle. Decrease the powers of the first term $2x$ of the expression from 6 to 0 and increase the powers of the second term $(-y)$ of the expression from 0 to 6 at each term of the expansion.

$$(2x+(-y))^6 = [\ \ ](2x)^6(-y)^0 + [\ \ ](2x)^5(-y)^1 + [\ \ ](2x)^4(-y)^2 + [\ \ ](2x)^3(-y)^3 + [\ \ ](2x)^2(-y)^4 + [\ \ ](2x)^1(-y)^5 + [\ \ ](2x)^0(-y)^6$$

Next, construct a Pascal's triangle up to the sixth level and fill in the square brackets with the numbers at the sixth level of the triangle as indicated below.

$n=0$: 1

$n=1$: 1 1

$n=2$: 1 2 1

$n=3$: 1 3 3 1

$n=4$: 1 4 6 4 1

$n=5$: 1 5 10 10 5 1

$n=6$: 1 6 15 20 15 6 1

Figure 1.1: Pascal's triangle for the expansion of $(2x–y)^6$

Evaluate the powers of the brackets.

$$[1]64x^6 - [6]32x^5y + [15]16x^4y^2 - [20]8x^3y^3 + [15]4x^2y^4 - [6]2xy^5 + [1]y^6$$

Multiply out the values of the Pascal's triangle (in the square brackets) with remaining adjacent expression.

$$(2x-y)^6 = 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$$

4.2 Factorial Method

This section contains an expansion of binomial expression using the factorial method. Again the task will be to expand $(2x - y)^6$. First, we have to look at some elementary terminologies.

4.2.1 Factorial

The factorial of a number is the product of all the consecutive natural numbers between the number and 1 inclusive. In other words, let n represent a positive integer. The factorial of n is expressed as $n \times (n - 1) \times (n - 2) \times (n - 3) \times ... \times 1$. For example, the factorial of the number 7 is $7 \times 6 \times 5 \times 4\ 3 \times 2 \times 1 = 5040$. Also, $1! = 1$ and, intriguingly, $0! = 1$.

4.2.2 Combination

The term, "combination" is defined thus:

Let n and r be non-negative integers such that $n \geq r$. Then ${}^{n}\mathrm{C}_{r}$ (pronounced n combination r) is defined as $\frac{n!}{r!(n-r)!}$.

For example,

$$^{8}\mathrm{C}_{5} = \frac{8!}{5!\,(8-5)!} = \frac{8!}{5!\,3!} = \frac{8 \times 7 \times 6 \times 5 \times 4\ 3 \times 2 \times 1}{(5 \times 4\ 3 \times 2 \times 1) \times (3 \times 2 \times 1)} = \frac{40320}{720} = 56$$

.

4.2.3 The binomial expansion

By this method, the general formula for the expansion of the expression $(ax + by)^n$ is given by

$$\sum_{r=0}^{n} {}^{n}\mathrm{C}_{r}\,(ax)^{n-r}(by)^{r}$$

Thus, the expansion of the expression $(2x - y)^6$ is obtained using

$$\sum_{r=0}^{6} {}^{6}\mathrm{C}_{r}(2x)^{6-r}(-y)^{r}$$

That is,

$$\begin{aligned}
&{}^{6}\mathrm{C}_{0}(2x)^{6-0}(-y)^{0} + {}^{6}\mathrm{C}_{1}(2x)^{6-1}(-y)^{1} + {}^{6}\mathrm{C}_{2}(2x)^{6-2}(-y)^{2} + {}^{6}\mathrm{C}_{3}(2x)^{6-3}(-y)^{3} + {}^{6}\mathrm{C}_{4}(2x)^{6-4}(-y)^{4} \\
&\qquad + {}^{6}\mathrm{C}_{5}(2x)^{6-5}(-y)^{5} + {}^{6}\mathrm{C}_{6}(2x)^{6-6}(-y)^{6}
\end{aligned}$$

$$\begin{aligned}
&= \frac{6!}{0!\,(6-0)!}(2x)^{6}(-y)^{0} + \frac{6!}{1!\,(6-1)!}(2x)^{5}(-y)^{1} + \frac{6!}{2!\,(6-2)!}(2x)^{4}(-y)^{2} \\
&\qquad + \frac{6!}{3!\,(6-3)!}(2x)^{3}(-y)^{3} + \frac{6!}{4!\,(6-4)!}(2x)^{2}(-y)^{4} + \frac{6!}{5!\,(6-5)!}(2x)^{1}(-y)^{5} \\
&\qquad + \frac{6!}{6!\,(6-6)!}(2x)^{0}(-y)^{6}
\end{aligned}$$

$$\begin{aligned}
&= \frac{720}{720} \cdot 64x^{6} + 6 \cdot 32x^{5}(-y) + \frac{720}{48} \cdot 16x^{4}y^{2} + \frac{720}{36} \cdot 8x^{3}(-y^{3}) + \frac{720}{48} \cdot 4x^{2}y^{4} + \frac{720}{120} \cdot 2x(-y^{5}) \\
&\qquad + \frac{720}{720}y^{6}
\end{aligned}$$

$$= 64x^6 + 6 \cdot 32x^5(-y) + 15 \cdot 16x^4y^2 + 20 \cdot 8x^3(-y^3) + 15 \cdot 4x^2y^4 + 6 \cdot 2x(-y^5) + y^6$$

$$= 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$$

4.3 Tabular Method

To expand $(2x–y)^6$ using the table method, we draw a table in which the first row contains the ordinals (denoted by *Term*) of the terms. The second row contains the *initial contributions* to the coefficients (denoted by *I-Coef*) in the expanded binomial expression, which is always 1 for the first term. The third row contains the *left contributions* to the coefficients (denoted by *L-coef*) in the expanded binomial expression and it is contributed by the a in $(ax - by)^n$. The fourth row contains the powers of x (denoted by *L-var*) in descending order. The fifth row contains the *right contributions* to the coefficients (denoted by *R-coef*) in the expanded binomial expression which is contributed by the b in $(ax - by)^n$. The sixth row contains the powers of y (denoted by *R-var*) in ascending order. The seventh row contains the product of rows 2 to 6.
The following is the initial table of the expansion of $(2x–y)^6$ and it contains the basic information necessary for the computation of the required terms. In the table, the *Term* rows are written as they are from 1st term to the $(n+1)^{\text{th}}$ term (the 7th term in this case). Write 1 in the first column of the row of *I-coef*. Decrease the power of *L-coef* (in this case 2) by 1. Also, decrease the power of *R-var* by 1, starting with 6. Increase the power of *R-coef* (in this case, –1) by 1, starting with 0. Also, increase the power of *R-var* by 1, starting with 0.

Table 1.1: Initial table of the binomial expansion of $(2x–y)^6$

Term	1st	2nd	3rd	4th	5th	6th	7th
I-Coef	1						
L-coef	2^6	2^5	2^4	2^3	2^2	2^1	2^0
L-var	x^6	x^5	x^4	x^3	x^2	x^1	x^0
R-coef	-1^0	-1^1	-1^2	-1^3	-1^4	-1^5	-1^6
R-var	y^0	y^1	y^2	y^3	y^4	y^5	y^6
Prod							

The final table is obtained when the *I-Coef* entries and the *Prod* entries have been added to the initial table. The *I-Coef* entries of the 2nd term is obtained by multiplying 1st *I-Coef* entry by the power at the 1st *L-coef* entry and dividing the result by the cardinal of '1st'. Similarly, the *I-Coef* entries of the 3nd term is obtained by multiplying 2nd *I-Coef* entry by the power at the 2nd *L-coef* entry and dividing the result by the cardinal of '2nd'.
In general,
I-coef of next entry = *L-coef* of current entry × *I-coef* of current entry ÷ cardinal of current *Term.*
The *Prod* entries are the results of *I-coef* × *L-coef* × *L-var* × *R-coef* × *R-var* of corresponding entries.

Table 1.2: Final table of the binomial expansion of $(2x–y)^6$

Term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
I-Coef	1	**6**	**15**	**20**	**15**	**6**	**1**
L-coef	2^6	2^5	2^4	2^3	2^2	2^1	2^0
L-var	x^6	x^5	x^4	x^3	x^2	x^1	x^0
R-coef	-1^0	-1^1	-1^2	-1^3	-1^4	-1^5	-1^6
R-var	y^0	y^1	y^2	y^3	y^4	y^5	y^6
Prod	$\mathbf{64x^6}$	$\mathbf{-192x^5y}$	$\mathbf{240x^4y^2}$	$\mathbf{-160x^3y^3}$	$\mathbf{60x^2y^4}$	$\mathbf{-12xy^5}$	$\mathbf{y^6}$

Thus, we have the resultant expansion:

$$= 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$$

5 HYPOTHESIS TESTING OF THE METHODS OF EXPANDING A BINOMIAL EXPRESSION

Some 132 randomly selected 100 level students in a Nigerian University are tested based on the three methods of expanding a binomial expression. The students were grouped into three and a question of 20 marks is administered to them using one method for each group. The first method uses the Pascal's triangle ("Pascal" group) while the second method uses factorial ("Factorial" group) – these are the conventional methods. Their efficiency in students understanding of the concept of binomial expansion is compared with that of a third (alternative or tabular) method ("Tabular" group). There are two problems to analyse, each of which will have two related samples.

We use a single tailed hypothesis testing since we are interested in whether the alternative approach is to be adopted rather than the two conventional approaches. We would only adopt the alternative approach if it improves students' performance relative to each conventional approach by a predetermined margin of marks. Since we are testing one new method over two existing methods, we need three samples. Thus, we would have two null and two alternative hypotheses. The following is the sample table for the scores.

Table 5.3: Student's scores for the three methods of teaching binomial expansion

								…Continues
No.	**Pascal (X1)**	**Factoria (X2)**	**Tabular (X3)**		**No.**	**Pascal (X1)**	**Factorial (X2)**	**Tabular (X3)**
1	10	3	20		**23**	17	9	17
2	13	3	13		**24**	8	2	18
3	13	3	20		**25**	14	7	20
4	11	18	13		**26**	14	18	17
5	13	10	17		**27**	20	3	20
6	18	7	18		**28**	16	7	16
7	7	14	12		**29**	11	8	17
8	13	6	17		**30**	8	8	20
9	14	5	6		**31**	18	1	19
10	15	12	7		**32**	14	10	17
11	7	1	14		**33**	20	2	18
12	13	3	17		**34**	14	3	19

13	13	9	18		**35**	17	4	18
14	20	10	19		**36**	7	9	19
15	6	16	19		**37**	13	15	20
16	7	6	20		**38**	20	20	15
17	14	4	18		**39**	19	20	18
18	12	3	19		**40**	8	20	20
19	16	2	8		**41**	20	10	19
20	13	2	20		**42**	10	15	19
21	20	18	19		**43**	13	10	20
22	15	18	12		**44**	10	3	20
					Mean	$\overline{x}_1 = 13.5$	$\overline{x}_2 = 8.6$	$\overline{x}_3 = 17.1$
					Variance	$s_1{}^2 = 17$	$s_2{}^2 = 36$	$s_3{}^2 = 12.$

5.1 The tabular verses the Pascal's triangle method

In this problem we want to test the claim that the tabular method of teaching binomial expansion is *better* (easier to understand by students) than the Pascal's triangle method by a margin mark of 2. For the null hypothesis (H_0), the mean score of the tabular method (with a sample mean of 17.1) is not 2 marks better than the mean score of the Pascal's triangle method (with a sample mean of 13.5). For the alternative hypothesis (H_1), the mean score of the tabular method is 2 marks better than the mean score of the Pascal's triangle method.

$$H_0: \mu_3 - \mu_1 \leq 2$$

$$H_1: \mu_3 - \mu_1 > 2$$

The sample size of 44 is reasonably large with unknown population standard deviations and has the standard normal distribution. Using a significance level of 0.05, we see that $Z_{0.05} = 1.645$. Thus the test statistic is obtained as follows:

$$Z = \frac{(\overline{x}_3 - \overline{x}_1) - (\mu_3 - \mu_1)}{\sqrt{\frac{s_1{}^2}{n_1} + \frac{s_3{}^2}{n_3}}}$$

$\sigma_1{}^2 = 17.1$ and $\sigma_3{}^2 = 12.5$.

$$Z = \frac{(17.1 - 13.5) - 2}{\sqrt{\frac{17.1}{44} + \frac{12.5}{44}}}$$

$$Z = \frac{3.6 - 2}{\sqrt{\frac{29.6}{44}}}\text{L}$$

$$Z = \frac{1.6}{\sqrt{\frac{37}{55}}}$$

$$Z = \sqrt{\frac{16^2}{10^2} \times \frac{55}{37}}$$

$$Z = \sqrt{\frac{704}{185}}$$

$$Z = 1.9507$$

Since $1.9507 > 1.645$ our decision is to reject the null hypothesis H_0.

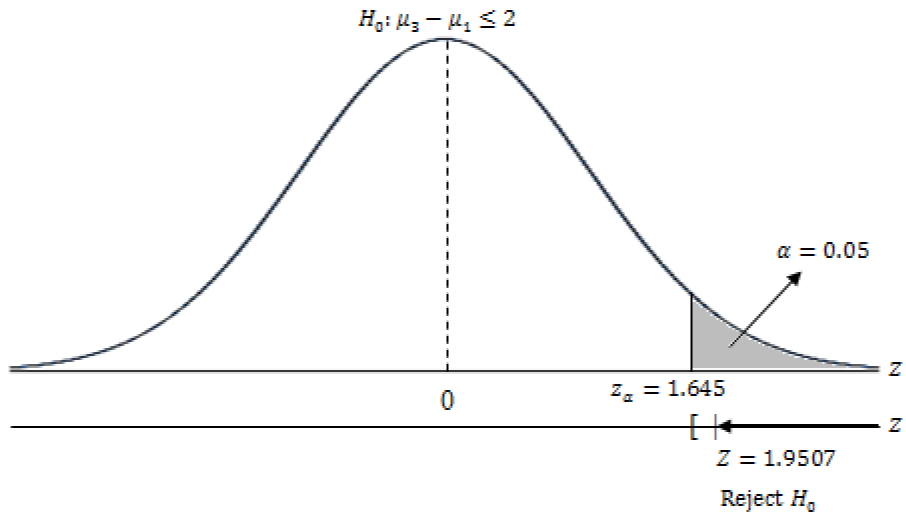


Figure 5.1: Test statistic illustration for tabular versus the Pascal's triangle method

5.2 The factorial method versus the tabular method

In this problem we want to test the claim that the tabular method of teaching binomial expansion is better than the factorial method by a margin mark of 6. For the null hypothesis (H_0'), the mean score of the tabular method (with a sample mean of 17.1) is not 6 marks better than the mean score of the factorial method (with a sample mean of 8.6). For the alternative hypothesis (H_1'), the mean score of the tabular method is 6 marks better than the mean score of the factorial method.

$$H_0': \mu_3 - \mu_2 \leq 6$$

$$H_1': \mu_3 - \mu_2 > 6$$

Using a significance level of 0.05, again $Z_{0.05} = 1.645$.

Since population standard deviation is unknown with a sample size of 44 we have

$$Z = \frac{(\overline{x}_3 - \overline{x}_2) - (\mu_3 - \mu_2)}{\sqrt{\frac{{s_2}^2}{n_2} + \frac{{s_3}^2}{n_3}}}$$

${s_2}^2 = 36.0$ and ${s_3}^2 = 12.5$.

$$Z = \frac{(17.1 - 8.6) - 6}{\sqrt{\frac{36.0}{44} + \frac{12.5}{44}}}$$

$$Z = \frac{2.5}{\sqrt{\frac{97}{88}}}$$

$$Z = \sqrt{\frac{2.5^2}{10^2} \times \frac{88}{97}}$$

$$Z = \sqrt{\frac{550}{97}}$$

$$Z = 2.3812$$

Since $2.3812 > 1.645$ we reject the null hypothesis $H_0{}'$.

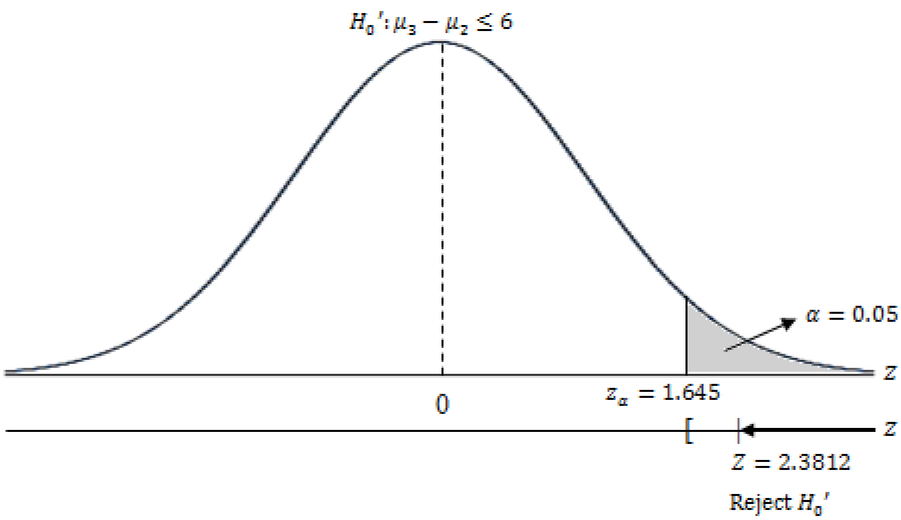


Figure 5.2: Test statistic illustration for tabular versus the factorial method

5.3 Result interpretation of the comparative analysis

The data provide sufficient evidence, at the 5% level of significance, to conclude that the alternative teaching method (the tabular method) is better than the Pascal's triangle method as far as students' understanding of the concept is concerned and students would earn 2 marks more using the alternative method.

The data provide sufficient evidence, at the 5% level of significance, to conclude that the alternative teaching method (the tabular method) is better than the factorial method in students' understanding of the concept and students would earn 6 marks more using the tabular method.

6 TIME COMPLEXITY OF THE TEACHING METHODS

Time complexity is a computational terminology used to describe the amount of time taken by an algorithm to run as a function of the length of the input [1]. The basic difference between the three methods can be reduced to the way in which the initial parts of the coefficients of the variables in the expanded binomial expression are obtained. Hence it is sufficient to compare only the time complexity of obtaining the initial coefficients We analyse the time it takes to obtain these initial coefficients in each method. Here, the highest single mathematical operation is multiplication or division. The remaining task of obtaining the full expanded binomial expression is generally the same.

6.1 Coding the Pascal's triangle method

The following is the algorithm used in obtaining the initial part of the coefficients of the expanded binomial expression using the Pascal's triangle method.

```
void OnInit()
 {
  int P0, P1, P2, P3, P4, P5, P6, P7, P8,P9, P10, P11, P12, P13, P14;
  int I1, I2, I3, I4, I5, I6, I7, I8,I9, I10, I11, I12, I13;
  for (int row = 0; row <= 6; row++)
    {
     for (int col = 1; col <= 13; col++)
       {
        if (row == 0)
          {
           I7 = 1;
          }
        else
          {
           I1 = P0 + P2;
           I2 = P1 + P3;
           I3 = P2 + P4;
           I4 = P3 + P5;
           I5 = P4 + P6;
           I6 = P5 + P7;
           I7 = P6 + P8;
           I8 = P7 + P9;
           I9 = P8 + P10;
           I10 = P9 + P11;
           I11 = P10 + P12;
           I12 = P11 + P13;
           I13 = P12 + P14;
          }
       }
     P1 = I1; P2 = I2; P3 = I3; P4 = I4; P5 = I5; P6 = I6; P7 = I7; P8 = I8; P9 = I9; P10 = I10; P11 = I11; P12 = I12; P13 = I13;
    }
  if(I1 != 0) Print(I1);
  if(I2 != 0) Print(I2);
  if(I3 != 0) Print(I3);
  if(I4 != 0) Print(I4);
  if(I5 != 0) Print(I5);
  if(I6 != 0) Print(I6);
  if(I7 != 0) Print(I7);
  if(I8 != 0) Print(I8);
  if(I9 != 0) Print(I9);
  if(I10 != 0) Print(I10);
  if(I11 != 0) Print(I11);
  if(I12 != 0) Print(I12);
  if(I13 != 0) Print(I13);
 }
```

Figure. 6.1: The MQL4 code for the Pascal's triangle method

6.2 Coding the factorial method

The following is the algorithm used in obtaining the initial part of the coefficients of the expanded binomial expression using the factorial method.

```
void OnInit()
  {
   int I1, I2, I3, I4, I5, I6, I7;
   I1 = (6 * 5 * 4 * 3 * 2 * 1) / ((1) * (6 * 5 * 4 * 3 * 2 * 1));
   I2 = (6 * 5 * 4 * 3 * 2 * 1) / ((1) * (5 * 4 * 3 * 2 * 1));
   I3 = (6 * 5 * 4 * 3 * 2 * 1) / ((2 * 1) * (4 * 3 * 2 * 1));
   I4 = (6 * 5 * 4 * 3 * 2 * 1) / ((3 * 2 * 1) * (3 * 2 * 1));
   I5 = (6 * 5 * 4 * 3 * 2 * 1) / ((4 * 3 * 2 * 1) * (2 * 1));
   I6 = (6 * 5 * 4 * 3 * 2 * 1) / ((5 * 4 * 3 * 2 * 1) * (1));
   I7 = (6 * 5 * 4 * 3 * 2 * 1) / ((6 * 5 * 4 * 3 * 2 * 1) * (1));
   Print(I1);
   Print(I2);
   Print(I3);
   Print(I4);
   Print(I5);
   Print(I6);
   Print(I7);
  }
```

Figure. 6.2: The MQL4 code for the Factorial method

6.3 Coding the tabular method

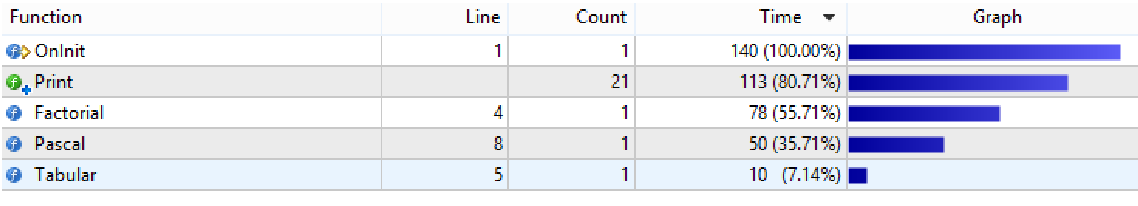
The following is the algorithm used in obtaining the initial part of the coefficients of the expanded binomial expression using the tabular method.

```
void OnInit()
  {
   int I1, I2, I3, I4, I5, I6, I7;
   I1 = 1;
   I2 = (6 * I1) / 1;
   I3 = (5 * I2) / 2;
   I4 = (4 * I3) / 3;
   I5 = (3 * I4) / 4;
   I6 = (2 * I5) / 5;
   I7 = (1 * I6) / 6;
   Print(I1);
   Print(I2);
   Print(I3);
   Print(I4);
   Print(I5);
   Print(I6);
   Print(I7);
  }
```

Figure. 6.3: The MQL4 code for the Tabular method

6.4 Time complexity of the three methods compared

The following table contains the results of the runtimes of the various methods, their percentage runtimes and the bar graph representing the runtimes.



Function	Line	Count	Time	Graph
OnInit	1	1	140 (100.00%)	
Print		21	113 (80.71%)	
Factorial	4	1	78 (55.71%)	
Pascal	8	1	50 (35.71%)	
Tabular	5	1	10 (7.14%)	

Figure 6.4: The runtime of the various codes used in obtaining the initial coefficients of the three methods of expanding a binomial expression.

The entire program ran in 140 milliseconds which represents 100%. The factorial method block of codes ran in 78 milliseconds. This represents 55.71% of the total time it took the entire program to run. The Pascal's triangle method block of codes ran in 50 milliseconds. This represents 35.71% of the total time it took the program to run. while the Tabular method block of codes ran in 10 seconds, representing only 7.14% of the total time it took the entire program to run.

7 Conclusion

The problem presented is real and not hypothetical. It is aimed at determining the better method of teaching binomial expansion based on students' understanding of the topic. The authors are teachers of higher institutions of learning and are concerned about the students' failures rate in the topic and in mathematics in general, hence the need to devise an alternative method. From the statistical analysis of the data obtained, it is obvious that the tabular method, which is otherwise referred as the modern method is the easiest to understand by the students and also the method that proves to be the fastest in solving problems in practice. The tabular method also proves to have an advantage over the other two methods in computational parlance. It is the shortest to code and the fastest to be processed by the computer.

Acknowledgment

This paper though simple is a collective effort, and we extends our appreciation to those who played a role, big or small, in the successful completion of this research. We appreciate the efforts of friends and colleagues, who provided encouragement and feedback during the process of the research. We thank the various reviewers for their constructive criticisms.

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