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DERIVATION OF MASS-ENERGY RELATION FOR COMPUTATION OF ENERGIES OF ATOMIC AND NUCLEAR PARTICLES

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ABSTRACT

In this study, a new mass-energy concept (mvc) has been derived by employing blackbody relation and Einstein's relativistic relation. Einstein's relation, mc^2, would apply perfectly to particles with the speed of light that is photons.The method employed semi-classical Newtonian concept of relativistic mass-energy theory, Bahjat mass-energy concept (mbc), Blackbody radiation and Plank's radiation law. A solution of a transcendental equation was obtained using graphical means from which the conversion factor in the relativistic energy relation, mc^2, is found to be vc, where $v = 1.8 \times 10^8 ms^{-1}$ and c remains the speed of light. Results in the calculation of atomic mass units shows that Einstein's mass-energy relation over estimates the energies of nuclear matter while that of Bahjat underestimates the same. Therefore, thenew mass-energy concept, $E = mc^2$, becomes $E = mvc$, where v is the speed for particles with mass and $v = c$ for a photon. Examples can be seen clearly in the unified atomic mass units and the calculation of binding energy.

INTRODUCTION

The concepts of Einstein's mass-energy relation for centaury, developed from the theoretical view, pointed out dual realities: matter and field. Field represents energy, matter represents mass. The greatest part of energy is concentrated in matter; but the field surrounding the particle represents energy, though in incomparable smaller quantity. Mass energy equivalence implies that we cannot differentiate between matter and the field[1]. The term relativistic mass, questioned by sciencescholars leads to the agreement that increase in energy or mass of a particle with velocity results from some change in the internal structure of the particle [2]. In addition, Einstein did not seem to be consistent when he discussed the need for relativistic mass, how mass changes with speed[1].This have led to some argument on the interpretations of Einstein's statement.

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Some physicists prefer rest mass of photon to be zero, whereas some others prefer relativistic mass of photon to be ratio of energy to the square of velocity of light[1]. The Einstein's relativistic mass-energy relation overestimates the nuclear energy. Therefore, the relativistic formula was suggested with a new conversion factor, other than $c^2$to help in the calculation of nuclear energy [3].

THEORY

The Einstein relativistic mass-energy theory

Einstein's mass-energy equivalence $E = mc^2$ was derived using Newtonian mechanics [4]. Mass consists of matter associated with volume. Light (photon) has no mass and volume, but has speed more than the speed of nuclear matter, $3 \times 10^8 ms^{-1}$ [5].

The Einstein mass-energy equivalence denotes relativistic energy. In the theory of special relativity[6,7,8]

The relativistic momentum is concerned with the motion of a particle whose velocity approaches the speed of light [9].

Theories of Einstein mass-energy$\left(E = m_0c^2\right)$ from Newtonian mechanics

Newton's second law of motion states that the force (F) acting on a particle is equal to the rate of change of its momentum (p). Below are the highlights of the derivation of the relativistic mass, momentum, and energy.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dv}{dt} \quad (1)$$

Since the kinetic energy is the work done by a particle, the following equation is obtained:

$$dK = dW = Fds \quad (2)$$

Substituting equation (1) for F into equation (2) the following equation was obtained

$$dK = Fds = \left(m\frac{dv}{dt} + v\frac{dv}{dt}\right)ds \quad (3)$$

The derivative of equation (3) is taken with respect to time, t as

$$dK = Fds = m\frac{ds}{dt}dv + v\frac{ds}{dt}dv \quad (4)$$

$$where, \frac{ds}{dt} = v$$

Note that the term c^2dm allows the hypothesis of variable mass as it actually occurs at high speed. Also, c^2dm is equal to the kinetic energy.

$$dK = mvdv + v^2dm \quad (5)$$

where, $dK =$ Kinetic energy

$$c^2dm = mvdv + v^2dm \quad (6)$$

Making $\frac{dm}{m}$, the subject of relation in equation (6), the following equation results:

$$\frac{dm}{m} = \frac{v}{c^2 - v^2}dv \quad (7)$$

Integrating equation (7) we obtain:

$$\int_{m_0}^{m}\frac{dm}{m} = \int_0^v \frac{v}{c^2 - v^2}dv \quad (8)$$

$$[ln(m)]_{m_0}^{m} = -\frac{1}{2}[(c^2 - v^2)]_0^v \quad (9)$$

$$ln\, m - ln\, m_0 = -\frac{1}{2}(c^2 - v^2) + \frac{1}{2}lnc^2 \quad (10)$$

$$ln\frac{m}{m_0} = \frac{1}{2}ln\frac{c^2}{c^2 - v^2} \quad (11)$$

$$\frac{m}{m_0} = \sqrt{\frac{c^2}{c^2 - v^2}} \quad (12)$$

$$m = m_o\left(\frac{1}{1-\frac{v^2}{c^2}}\right)^{-1/2} \tag{13}$$

where m is the relativistic mass of the particle, m_o is the rest mass of the particle, v is the velocity of the particle and c is the speed of light(Annamalai, 2023g).

The relativistic momentum

$$P = mv \tag{14}$$

Equation (13) were substituted into equation (14)

$$P = m_o v\left(\frac{1}{1-\frac{v^2}{c^2}}\right)^{-1/2} \tag{15}$$

Since the relativistic energy was given as $(E = mc^2)$

Equation (15) becomes:

$$E = m_0 c^2\left(\frac{1}{1-\frac{v^2}{c^2}}\right)^{-1/2} \tag{16}$$

The relationship between the relativistic energy and the relativistic momentum becomes:

$$E^2 = m_0^2 c^4\left(\frac{1}{1-\frac{v^2}{c^2}}\right)^{-1} \tag{17}$$

$$E^2 = \frac{m_0^2 c^2(v^2 - v^2 + c^2)}{1-\frac{v^2}{c^2}} \tag{18}$$

$$E^2 = \frac{m_0^2 c^2 v^2 - m_0^2 c^2 v^2 + m_0^2 c^4}{1-\frac{v^2}{c^2}} \tag{19}$$

$$E^2 = \left(m_o v\left(\frac{1}{1-\frac{v^2}{c^2}}\right)^{-1/2}\right)^2 c^2 + \frac{m_0^2 c^2(c^2 - v^2)}{\frac{c^2 - v^2}{c^2}} \tag{20}$$

From the above expression of equation (20), the energy-momentum relation is obtained

$$E^2 = P^2 c^2 + m_0^2 c^4 \tag{21}$$

If particle is at rest, then $P = 0$ thus, the rest energy is becomes$E = m_o c^2$.

The Relativistic Mass-energy equivalence$(E = mc^2)$ was derived by employing equation (13) above by squaring both sides to obtain

$$m^2 c^2 - m^2 v^2 = m_0^2 c^2 \tag{22}$$

Where, $m_0^2 c^2$ is the rest mass energy of the particle

By differentiating the equation with respect to time we obtain

$$2mc^2\frac{dm}{dt} - 2mv\frac{d(mv)}{dt} = 0 \tag{23}$$

From equation (23), the following result is obtained:

$$c^2\frac{dm}{dt} = v\frac{d(mv)}{dt} \tag{24}$$

$$\frac{dE}{dt} = Fv = v\frac{d(mv)}{dt} = c^2\frac{dm}{dt} \tag{25}$$

$$dE = c^2 dm \tag{26}$$

The kinetic energy of the particle K is given as:

$$\int_0^K dE = \int_{m_0}^m c^2 dm \tag{27}$$

$$K = c^2(m - m_0) \tag{28}$$

The total energy of the particle is the sum of its kinetic energy and the rest mass-energy $m_0 c^2$

Total Energy (E) = Kinetic Energy (K) + Rest Mass-Energy $m_0 c^2$

$E = c^2(m - m_0) + m_0c^2$(Annamalai, 2023h) (29)

Hence, $E = mc^2$ (30)

Bahjat Mass-Energy concept(mbc):

Bahjat reported that, the Einstein's relativistic mass–energy theory $E = mc^2$ overestimates the nuclear energy. Therefore, the relativistic formula was suggested with a new conversion factor (b)other than c^2 to help perfect the calculation of nuclear energy (Bahjat, 2008)

The new energy converting factor by Bahjat was expressed as $bc\ instead\ of\ c^2$ and the Einstein relativistic mass-energy $E = mc^2$ theory was transformed to

$E = mbc$. Where b is obtained as $0.6 \times 10^8 m/s$(Bahjat, 2008).

METHODOLOGY

The method employed the aforementioned literatures from Newtonian concept of relativistic mass-energy theory, Bahjat mass-energy concept (mbc) and Blackbody radiation comprising of Planck's radiation law and Wien's displacement law.

The New mass-energy concept (mvc) involved the following steps:

From Plank's energy and frequency relation, the following equation is obtained

$$E = hf \quad (31)$$

From the frequency and wavelength relation, the following is obtained

$$f = \frac{c}{\lambda} \quad (32)$$

Substituting (32) into (31)

$$E = \frac{hc}{\lambda} \quad (33)$$

From the momentum and wavelength relation, the following is obtained

$$\lambda = \frac{h}{p} \quad (34)$$

Where, momentum was expressed as $P = mc$

Substituting (34) into (33)

$$E = mc^2 \quad (35)$$

The average Energy of a Blackbody is related with temperature as follow

$$E = kT_{blackbody} \quad (36)$$

Using Plank's blackbody radiation law to obtain Wein's displacement relation$\left(T = \frac{b}{\lambda_{max}}\right)$

Plank's Blackbody Radiation Law

$$U(\lambda) = \frac{8\pi hc}{\lambda^5}\left[e^{\frac{hc}{\lambda k_B T}} - 1\right]^{-1} \quad (37)$$

At Maximum wavelength the following condition holds

$$\left(\frac{dU}{d\lambda}\right)_{\lambda_{Max}} = 0 \quad (38)$$

Using Quotient rule to differentiate (38) with respect to wavelength the following were obtained

$$\frac{8\pi hc\left[\left(e^{\frac{hc}{\lambda k_B T}} - 1 \times -5\lambda^{-6}\right) - \left(\lambda^{-5} \times e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{k_B T} - \frac{1}{\lambda^2}\right)\right]}{\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)^2} = 0 \quad (39)$$

Multiplying both side of (38) with $\frac{\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)^2}{8\pi hc}$

$$\left[\left(e^{\frac{hc}{\lambda k_B T}} - 1 \times -5\lambda^{-6}\right) - \left(\lambda^{-5} \times e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{k_B T} - \frac{1}{\lambda^2}\right)\right] = 0 \qquad (40)$$

$$\left[e^{\frac{hc}{\lambda k_B T}} - 1 \times -5\lambda^{-6} + \lambda^{-6} e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{\lambda k_B T}\right] = 0 \qquad (41)$$

$$\left[e^{\frac{hc}{\lambda k_B T}} - 1 \times -5 + e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{\lambda k_B T}\right] = 0 \qquad (42)$$

For simplification, (42) exponential power is equate to x as follow

$$\frac{hc}{\lambda k_B T} = x \qquad (43)$$

By substituting (43) into (42) we obtained the following expression

$$(e^x - 1) - 5 + e^x . x = 0 \qquad (44)$$

$$-5e^x + 5 + e^x x = 0 \qquad (45)$$

$$e^x(x - 5) = -5 \qquad (46)$$

RESULTS AND DISCUSSION

Equation (46) is a transcendental equation, where x signifies the trivial function of wavelength at maximum temperature and the solution can only be obtained graphically for the numerical value ofx.The graphical representation of the transcendental function,$f(x) = e^x(x - 5) + 5$ is shown in Figure 1.

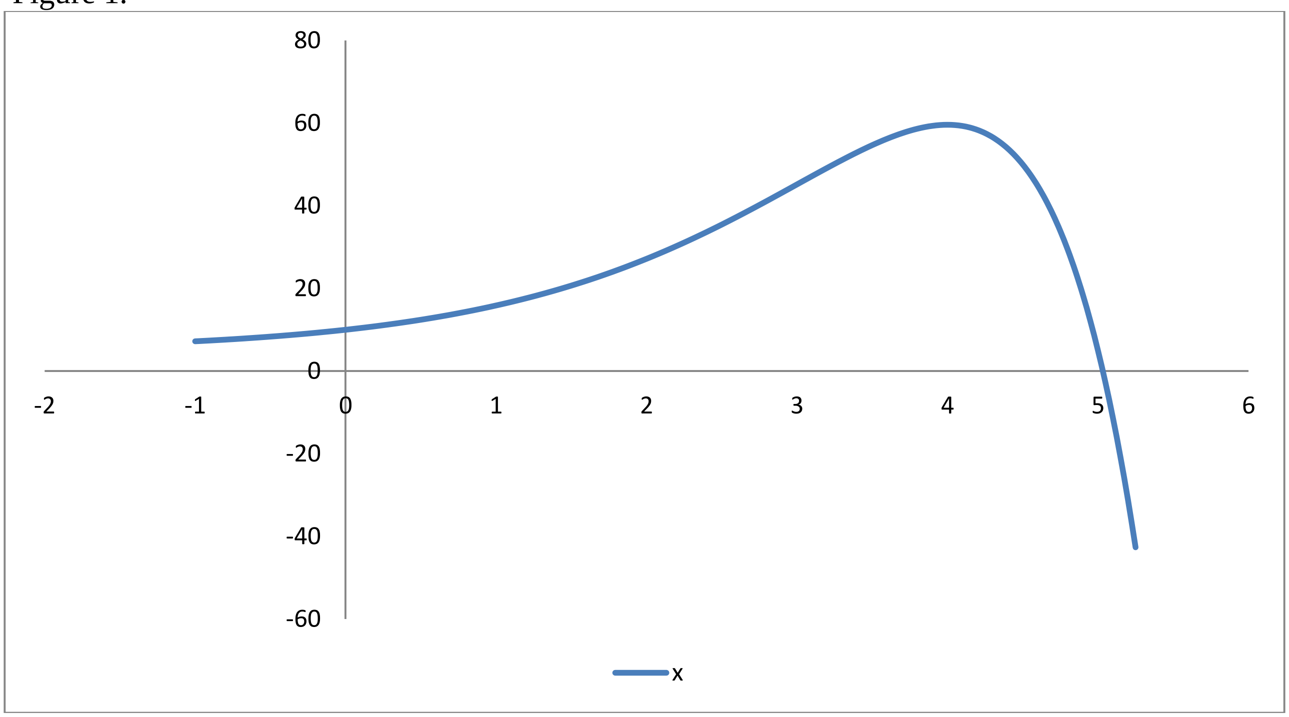

Figure 1: Graphical plot of the transcendental function $f(x) = e^x(x - 5) + 5$

The graphicalplot of transcendental function was obtained, by employing and taking the differential of Plank radiation law at maximum wavelength as given in equation (38). The trivial function x, was obtained graphically on the x-axis with a numerical value given as 5.The trivial function x, is of significance in determining the magnitude of Wein's displacement constant as given in equation (47). The new mass-energy concept (mvc) is established by relating the blackbody energy as derived in equation (50) with Einstein mass-energy theory given in equation (35). Hence the energy converting factor v was obtained numerically as $1.8\times 10^{8} m/s$which holds for speed of particle with mass. The mathematical steps are as follow:

The Weins Temperature- Wavelength relation can be observed from equation (43) as in the following

$$\lambda_{Max} = \frac{hc}{xK_B T} = \frac{b}{T} \quad (47)$$

Where $b = \frac{hc}{xK_B} = Wien's\ displacement\ constant = 2.879546 \times 10^{-3} mK$

$$T = \frac{b}{\lambda_{Max}} \quad (48)$$

Substituting (48) for Temperaturevalue at maximum wavelength into (36)

$$E = K_B \frac{b}{\lambda_{Max}} \quad (49)$$

Substituting (34) for value of maximum wavelength into (49)

$$E = \frac{K_B b}{h} mc \quad (50)$$

By adding and taking the average of equation (35) and (50)

$$E = \frac{\frac{K_B b}{h} mc + mc^2}{2} \quad (51)$$

$$E = \frac{\left(\frac{K_B b}{h} + c\right) mc}{2} \quad (52)$$

Where the New mass-energy converting factor v is deduced from equation (52) as follow

$$\frac{\frac{K_B b}{h} + c}{2} = v \quad (53)$$

Since the following constant have their appropriate numerical values as follow:

$K_B = 1.3806488 \times 10^{-23} J/K$

$b = 2.879546 \times 10^{-3} mK$

$h = 6.62606957 \times 10^{-34} Js$ and

$c = 3 \times 10^{8} m/s$

When substituting the corresponding constants value into equation (53) we have the following:

$$v = 1.8 \times 10^{8} m/s \quad (54)$$

Hence, the new mass-energy concept is as follow

$$E = mvc \quad (55)$$

Binding Energy $E = \Delta mc^2$:

The Binding Energy calculated using Einstein's mass-energy theory, Bahjat mass-energy relation and our new mass-energy relation are given as follow:

$$BE = \left[\left(M_p + M_n\right) - M_{A,Z}\right] c^2 \quad (56)$$

$$BE = \left[\left(M_p + M_n\right) - M_{A,Z}\right] bc \quad (57)$$

$$BE = \left[\left(M_p + M_n\right) - M_{A,Z}\right] vc \quad (58)$$

Table 1 shows the results in the calculation of atomic mass units (u) in energy (MeV), which clearly indicates that Einstein's mass-energy relation over estimates the energies of nuclear matter while that of Bahjat underestimates the same.

Table 1: The values of unified atomic mass unit (u) using Einstein, Bahjat and the new mass-energy relation

nified atomic mass unit (u)	Einstein $E = mc^2\ (MeV)$	Bahjat $E = mbc\ (MeV)$	New Mass-energy concept $E = mvc\ (MeV)$
1u	931.5	187.607	560.43

Table 2: Binding Energy values using Einstein, Bahjat and New mass-energy concept

$Element$	$Atomic\ Mass$ (u)	ergy$Energy\ (mc^2)$ (MeV)	$Energy\ (mbc)$ (MeV)	$Energy\ (mvc)$ (MeV)
uterium	2	2.224422	0.448006	1.338307
lium	4	28.10801	5.661041	16.91098
hium	7	39.24503	7.904071	23.61148
ryllium	9	58.16565	11.71474	34.99493
ron	11	76.20602	15.34813	45.84878
rogen	14	104.6596	21.07877	62.96767
orine	19	147.803	29.76798	88.92455
rbon	12	92.16261	18.56184	55.44894
on	20	160.6465	32.3547	96.65176
lorine	35	298.2123	60.06088	179.4172
senic	75	652.5707	131.4298	392.6143
rium	89	775.7318	156.2348	466.7132
olybdenum	100	860.5225	173.3119	517.7269
dmium	110	940.652	189.4502	565.9362
lurium	128	1081.452	217.8078	650.6475
aseodymium	141	1177.935	237.2397	708.6957
sprosium	160	1309.471	263.7315	787.8334
fnium	180	1446.313	291.2919	870.1634
ld	197	1559.416	314.0712	938.2108
muth	209	1640.262	330.3538	986.8511
anium	238	1801.714	362.8708	1083.988

The binding energy in Table 2, using Einstein's mass-energy theory, Bahjat mass-energy relation and New mass-energy concept, are calculated using equations (56),(57) and(58) and Table 1.

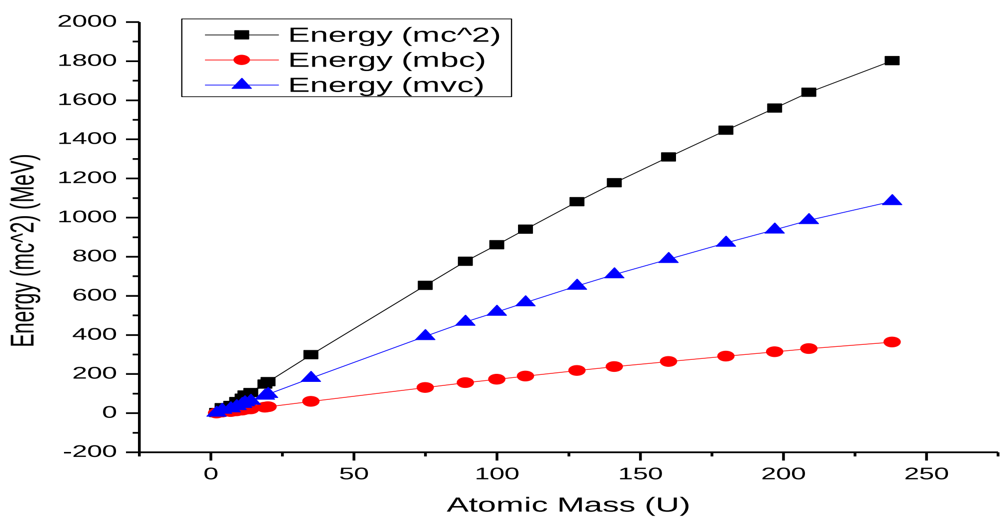

Fig 2: The graphical Plot of Energy as a function of Atomic Mass (u)

The graphical Plot of Energy as a function of Atomic Mass (u) is observed in figure 2 which employed table 2, shows that, the Einstein's mass-energy relation over estimates the energies of nuclear matter while that of Bahjat underestimates the same. Therefore, thenew mass-energy concept, E = mc^2, becomes E = mvc, where v is the speed for particles with mass and v = c for a photon.

CONCLUSION

Einstein's mass-energy relation,$E = mc^2$, overestimates the energies of particles that have mass because it is only photons, or massless energy mediators, that havethe speed 3 x $10^8 ms^{-1}$. Therefore, it is crucial to employ the new mass-energy relation, E = mvc, in the computation of nuclear and particle energies. Hence, the parameter v = 1.8 x $10^8 ms^{-1}$ can be used to replace c. Examples can be seen clearly in the unified atomic mass units and the calculation binding energies.

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