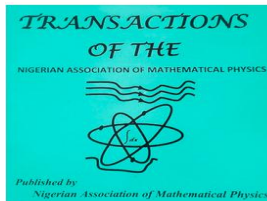


**Journal of  
The Nigerian Association of  
Mathematical Physics**  
Journal homepage: <https://nampjournals.org.ng>



## DERIVATION OF MASS-ENERGY RELATION FOR COMPUTATION OF ENERGIES OF ATOMIC AND NUCLEAR PARTICLES

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### ARTICLE INFO

#### Article history:

Received xxxxx

Revised xxxxx

Accepted xxxxx

Available online xxxxx

#### Keywords:

Speed of light, radiation law, blackbody, Planck, Wein, mass-energy relation, Bahjat, Einstein, transcendental equation

### ABSTRACT

*In this study, a new mass-energy concept (mvc) has been derived by employing blackbody relation and Einstein's relativistic relation. Einstein's relation,  $mc^2$ , would apply perfectly to particles with the speed of light that is photons. The method employed semi-classical Newtonian concept of relativistic mass-energy theory, Bahjat mass-energy concept (mbc), Blackbody radiation and Plank's radiation law. A solution of a transcendental equation was obtained using graphical means from which the conversion factor in the relativistic energy relation,  $mc^2$ , is found to be  $vc$ , where  $v = 1.8 \times 10^8 \text{ms}^{-1}$  and  $c$  remains the speed of light. Results in the calculation of atomic mass units shows that Einstein's mass-energy relation over estimates the energies of nuclear matter while that of Bahjat underestimates the same. Therefore, the new mass-energy concept,  $E = mc^2$ , becomes  $E = mvc$ , where  $v$  is the speed for particles with mass and  $v = c$  for a photon. Examples can be seen clearly in the unified atomic mass units and the calculation of binding energy.*

### INTRODUCTION

The concepts of Einstein's mass-energy relation for centaury, developed from the theoretical view, pointed out dual realities: matter and field. Field represents energy, matter represents mass. The greatest part of energy is concentrated in matter; but the field surrounding the particle represents energy, though in incomparable smaller quantity. Mass energy equivalence implies that we cannot differentiate between matter and the field[1]. The term relativistic mass, questioned by sciencescholars leads to the agreement that increase in energy or mass of a particle with velocity results from some change in the internal structure of the particle [2]. In addition, Einstein did not seem to be consistent when he discussed the need for relativistic mass, how mass changes with speed[1]. This have led to some argument on the interpretations of Einstein's statement.

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<https://doi.org/10.60787/jnamp-v66-317>

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<https://doi.org/10.60787/jnamp-v66-317>

Some physicists prefer rest mass of photon to be zero, whereas some others prefer relativistic mass of photon to be ratio of energy to the square of velocity of light[1]. The Einstein’s relativistic mass-energy relation overestimates the nuclear energy. Therefore, the relativistic formula was suggested with a new conversion factor, other than  $c^2$  to help in the calculation of nuclear energy [3].

**THEORY**

**The Einstein relativistic mass-energy theory**

Einstein’s mass-energy equivalence  $E = mc^2$  was derived using Newtonian mechanics [4]. Mass consists of matter associated with volume. Light (photon) has no mass and volume, but has speed more than the speed of nuclear matter,  $3 \times 10^8 ms^{-1}$  [5].

The Einstein mass-energy equivalence denotes relativistic energy. In the theory of special relativity[6,7,8]

The relativistic momentum is concerned with the motion of a particle whose velocity approaches the speed of light [9].

**Theories of Einstein mass-energy( $E = m_0c^2$ ) from Newtonian mechanics**

Newton’s second law of motion states that the force (F) acting on a particle is equal to the rate of change of its momentum (p). Below are the highlights of the derivation of the relativistic mass, momentum, and energy.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt} \tag{1}$$

Since the kinetic energy is the work done by a particle, the following equation is obtained:

$$dK = dW = Fds \tag{2}$$

Substituting equation (1) for F into equation (2) the following equation was obtained

$$dK = Fds = \left( m \frac{dv}{dt} + v \frac{dm}{dt} \right) ds \tag{3}$$

The derivative of equation (3) is taken with respect to time, t as

$$dK = Fds = m \frac{ds}{dt} dv + v \frac{ds}{dt} dm \tag{4}$$

where,  $\frac{ds}{dt} = v$

Note that the term  $c^2 dm$  allows the hypothesis of variable mass as it actually occurs at high speed. Also,  $c^2 dm$  is equal to the kinetic energy.

$$dK = mv dv + v^2 dm \tag{5}$$

where,  $dK =$  Kinetic energy

$$c^2 dm = mv dv + v^2 dm \tag{6}$$

Making  $\frac{dm}{m}$ , the subject of relation in equation (6), the following equation results:

$$\frac{dm}{m} = \frac{v}{c^2 - v^2} dv \tag{7}$$

Integrating equation (7) we obtain:

$$\int_{m_0}^m \frac{dm}{m} = \int_0^v \frac{v}{c^2 - v^2} dv \tag{8}$$

$$[\ln(m)]_{m_0}^m = -\frac{1}{2} [(c^2 - v^2)]_0^v \tag{9}$$

$$\ln m - \ln m_0 = -\frac{1}{2} (c^2 - v^2) + \frac{1}{2} \ln c^2 \tag{10}$$

$$\ln \frac{m}{m_0} = \frac{1}{2} \ln \frac{c^2}{c^2 - v^2} \tag{11}$$

$$\frac{m}{m_0} = \sqrt{\frac{c^2}{c^2 - v^2}} \tag{12}$$

$$m = m_o \left( \frac{1}{1-\frac{v^2}{c^2}} \right)^{-1/2} \tag{13}$$

where  $m$  is the relativistic mass of the particle,  $m_o$  is the rest mass of the particle,  $v$  is the velocity of the particle and  $c$  is the speed of light(Annamalai, 2023g).

**The relativistic momentum**

$$P = mv \tag{14}$$

Equation (13) were substituted into equation (14)

$$P = m_o v \left( \frac{1}{1-\frac{v^2}{c^2}} \right)^{-1/2} \tag{15}$$

Since the relativistic energy was given as ( $E = mc^2$ )

Equation (15) becomes:

$$E = m_o c^2 \left( \frac{1}{1-\frac{v^2}{c^2}} \right)^{-1/2} \tag{16}$$

The relationship between the relativistic energy and the relativistic momentum becomes:

$$E^2 = m_o^2 c^4 \left( \frac{1}{1-\frac{v^2}{c^2}} \right)^{-1} \tag{17}$$

$$E^2 = \frac{m_o^2 c^2 (v^2 - v^2 + c^2)}{1-\frac{v^2}{c^2}} \tag{18}$$

$$E^2 = \frac{m_o^2 c^2 v^2 - m_o^2 c^2 v^2 + m_o^2 c^4}{1-\frac{v^2}{c^2}} \tag{19}$$

$$E^2 = \left( m_o v \left( \frac{1}{1-\frac{v^2}{c^2}} \right)^{-1/2} \right)^2 c^2 + \frac{m_o^2 c^2 (c^2 - v^2)}{c^2} \tag{20}$$

From the above expression of equation (20), the energy-momentum relation is obtained

$$E^2 = P^2 c^2 + m_o^2 c^4 \tag{21}$$

If particle is at rest, then  $P = 0$  thus, the rest energy is becomes  $E = m_o c^2$ .

The Relativistic Mass-energy equivalence( $E = mc^2$ ) was derived by employing equation (13) above by squaring both sides to obtain

$$m^2 c^2 - m^2 v^2 = m_o^2 c^2 \tag{22}$$

Where,  $m_o^2 c^2$  is the rest mass energy of the particle

By differentiating the equation with respect to time we obtain

$$2mc^2 \frac{dm}{dt} - 2mv \frac{d(mv)}{dt} = 0 \tag{23}$$

From equation (23), the following result is obtained:

$$c^2 \frac{dm}{dt} = v \frac{d(mv)}{dt} \tag{24}$$

$$\frac{dE}{dt} = Fv = v \frac{d(mv)}{dt} = c^2 \frac{dm}{dt} \tag{25}$$

$$dE = c^2 dm \tag{26}$$

The kinetic energy of the particle  $K$  is given as:

$$\int_0^K dE = \int_{m_o}^m c^2 dm \tag{27}$$

$$K = c^2(m - m_o) \tag{28}$$

The total energy of the particle is the sum of its kinetic energy and the rest mass-energy  $m_o c^2$

Total Energy ( $E$ ) = Kinetic Energy ( $K$ ) + Rest Mass-Energy  $m_o c^2$

$$E = c^2(m - m_0) + m_0c^2 \text{ (Annamalai, 2023h)} \tag{29}$$

$$\text{Hence, } E = mc^2 \tag{30}$$

**Bahjat Mass-Energy concept(mbc):**

Bahjat reported that, the Einstein’s relativistic mass–energy theory  $E = mc^2$  overestimates the nuclear energy. Therefore, the relativistic formula was suggested with a new conversion factor (b) other than  $c^2$  to help perfect the calculation of nuclear energy (Bahjat, 2008)

The new energy converting factor by Bahjat was expressed as *bc instead of  $c^2$*  and the Einstein relativistic mass-energy  $E = mc^2$  theory was transformed to  $E = mbc$ . Where b is obtained as  $0.6 \times 10^8 m/s$  (Bahjat, 2008).

**METHODOLOGY**

The method employed the aforementioned literatures from Newtonian concept of relativistic mass-energy theory, Bahjat mass-energy concept (mbc) and Blackbody radiation comprising of Planck’s radiation law and Wien’s displacement law.

**The New mass-energy concept (mvc) involved the following steps:**

From Plank’s energy and frequency relation, the following equation is obtained

$$E = hf \tag{31}$$

From the frequency and wavelength relation, the following is obtained

$$f = \frac{c}{\lambda} \tag{32}$$

Substituting (32) into (31)

$$E = \frac{hc}{\lambda} \tag{33}$$

From the momentum and wavelength relation, the following is obtained

$$\lambda = \frac{h}{p} \tag{34}$$

Where, momentum was expressed as  $P = mc$

Substituting (34) into (33)

$$E = mc^2 \tag{35}$$

The average Energy of a Blackbody is related with temperature as follow

$$E = kT_{blackbody} \tag{36}$$

Using Plank’s blackbody radiation law to obtain Wein’s displacement relation  $\left(T = \frac{b}{\lambda_{max}}\right)$

Plank’s Blackbody Radiation Law

$$U(\lambda) = \frac{8\pi hc}{\lambda^5} \left[ e^{\frac{hc}{\lambda k_B T}} - 1 \right]^{-1} \tag{37}$$

At Maximum wavelength the following condition holds

$$\left(\frac{dU}{d\lambda}\right)_{\lambda_{Max}} = 0 \tag{38}$$

Using Quotient rule to differentiate (38) with respect to wavelength the following were obtained

$$\frac{8\pi hc \left[ \left( e^{\frac{hc}{\lambda k_B T}} - 1 \right)^{-2} \times -5\lambda^{-6} \right] - \left( \lambda^{-5} \times e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{k_B T} \times \frac{1}{\lambda^2} \right)}{\left( e^{\frac{hc}{\lambda k_B T}} - 1 \right)^2} = 0 \tag{39}$$

Multiplying both side of (38) with  $\frac{\left( e^{\frac{hc}{\lambda k_B T}} - 1 \right)^2}{8\pi hc}$

$$\left[ \left( e^{\frac{hc}{\lambda k_B T}} - 1 \times -5\lambda^{-6} \right) - \left( \lambda^{-5} \times e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{k_B T} - \frac{1}{\lambda^2} \right) \right] = 0 \tag{40}$$

$$\left[ e^{\frac{hc}{\lambda k_B T}} - 1 \times -5\lambda^{-6} + \lambda^{-6} e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{\lambda k_B T} \right] = 0 \tag{41}$$

$$\left[ e^{\frac{hc}{\lambda k_B T}} - 1 \times -5 + e^{\frac{hc}{\lambda k_B T}} \times \frac{hc}{\lambda k_B T} \right] = 0 \tag{42}$$

For simplification, (42) exponential power is equate to x as follow

$$\frac{hc}{\lambda k_B T} = x \tag{43}$$

By substituting (43) into (42) we obtained the following expression

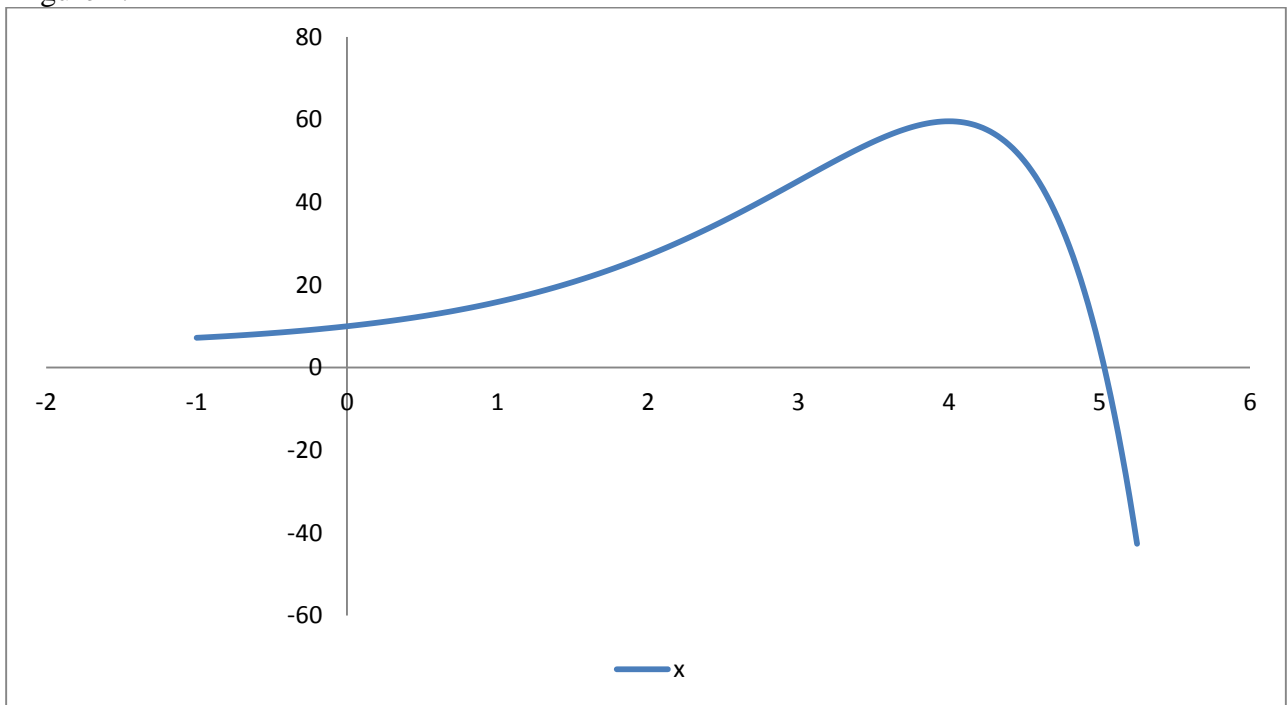
$$(e^x - 1) - 5 + e^x \cdot x = 0 \tag{44}$$

$$-5e^x + 5 + e^x x = 0 \tag{45}$$

$$e^x(x - 5) = -5 \tag{46}$$

### RESULTS AND DISCUSSION

Equation (46) is a transcendental equation, where  $x$  signifies the trivial function of wavelength at maximum temperature and the solution can only be obtained graphically for the numerical value of  $x$ . The graphical representation of the transcendental function,  $f(x) = e^x(x - 5) + 5$  is shown in Figure 1.



**Figure 1: Graphical plot of the transcendental function  $f(x) = e^x(x - 5) + 5$**

The graphical plot of transcendental function was obtained, by employing and taking the differential of Plank radiation law at maximum wavelength as given in equation (38). The trivial function  $x$ , was obtained graphically on the x-axis with a numerical value given as 5. The trivial function  $x$ , is of significance in determining the magnitude of Wein’s displacement constant as given in equation (47). The new mass-energy concept (mvc) is established by relating the blackbody energy as derived in equation (50) with Einstein mass-energy theory given in equation (35). Hence the energy

converting factor  $v$  was obtained numerically as  $1.8 \times 10^8 m/s$  which holds for speed of particle with mass. The mathematical steps are as follow:

The Weins Temperature- Wavelength relation can be observed from equation (43) as in the following

$$\lambda_{Max} = \frac{hc}{xK_B T} = \frac{b}{T} \tag{47}$$

Where  $b = \frac{hc}{xK_B} = \text{Wien's displacement constant} = 2.879546 \times 10^{-3} mK$

$$T = \frac{b}{\lambda_{Max}} \tag{48}$$

Substituting (48) for Temperature value at maximum wavelength into (36)

$$E = K_B \frac{b}{\lambda_{Max}} \tag{49}$$

Substituting (34) for value of maximum wavelength into (49)

$$E = \frac{K_B b}{h} mc \tag{50}$$

By adding and taking the average of equation (35) and (50)

$$E = \frac{\frac{K_B b}{h} mc + mc^2}{2} \tag{51}$$

$$E = \frac{(\frac{K_B b}{h} + c)mc}{2} \tag{52}$$

Where the New mass-energy converting factor  $v$  is deduced from equation (52) as follow

$$\frac{\frac{K_B b}{h} + c}{2} = v \tag{53}$$

Since the following constant have their appropriate numerical values as follow:

$$\begin{aligned} K_B &= 1.3806488 \times 10^{-23} J/K \\ b &= 2.879546 \times 10^{-3} mK \\ h &= 6.62606957 \times 10^{-34} Js \text{ and} \\ c &= 3 \times 10^8 m/s \end{aligned}$$

When substituting the corresponding constants value into equation (53) we have the following:

$$v = 1.8 \times 10^8 m/s \tag{54}$$

Hence, the new mass-energy concept is as follow

$$E = mvc \tag{55}$$

**Binding Energy  $E = \Delta mc^2$ :**

The Binding Energy calculated using Einstein’s mass-energy theory, Bahjat mass-energy relation and our new mass-energy relation are given as follow:

$$BE = [(M_p + M_n) - M_{A,Z}]c^2 \tag{56}$$

$$BE = [(M_p + M_n) - M_{A,Z}]bc \tag{57}$$

$$BE = [(M_p + M_n) - M_{A,Z}]vc \tag{58}$$

Table 1 shows the results in the calculation of atomic mass units (u) in energy (MeV), which clearly indicates that Einstein’s mass-energy relation over estimates the energies of nuclear matter while that of Bahjat underestimates the same.

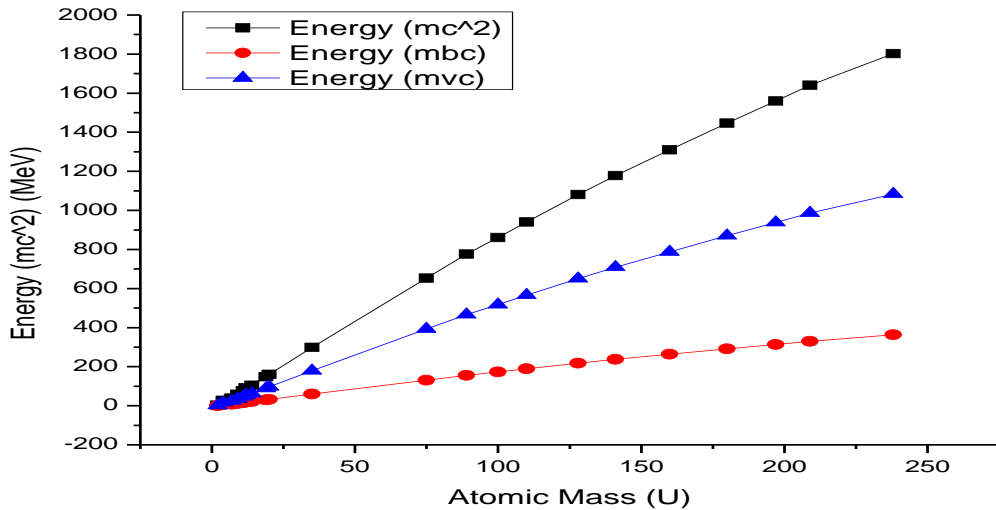
**Table 1: The values of unified atomic mass unit (u) using Einstein, Bahjat and the new mass-energy relation**

nified atomic mass unit (u)	Einstein $E = mc^2 (MeV)$	Bahjat $E = mbc (MeV)$	New Mass-energy concept $E = mvc (MeV)$
1u	931.5	187.607	560.43

**Table 2: Binding Energy values using Einstein, Bahjat and New mass-energy concept**

Element	Atomic Mass (u)	Energy ( $mc^2$ ) (MeV)	Energy (mbc) (MeV)	Energy (mvc) (MeV)
Lithium	7	39.24503	7.904071	23.61148
Beryllium	9	58.16565	11.71474	34.99493
Boron	11	76.20602	15.34813	45.84878
Carbon	12	92.16261	18.56184	55.44894
Nitrogen	14	104.6596	21.07877	62.96767
Oxygen	16	127.92464	25.49664	76.81280
Fluorine	19	147.803	29.76798	88.92455
Neon	20	160.6465	32.3547	96.65176
Sodium	23	180.9189	37.23701	111.52093
Magnesium	24	199.92479	40.50595	120.60720
Aluminum	27	224.94095	46.35297	139.59307
Silicon	28	235.94388	48.72832	146.42560
Sulfur	32	271.76272	56.6088	171.4172
Chlorine	35	298.2123	60.06088	179.4172
Argon	36	303.41856	61.73208	183.2108
Potassium	39	339.2374	68.59517	205.6132
Calcium	40	349.2104	71.4298	212.6143
Scandium	45	395.16565	81.2348	246.7132
Titanium	48	423.65225	87.3119	261.7269
Vanadium	51	450.652	92.4502	276.9362
Chromium	52	469.52	95.2348	284.7132
Manganese	55	500.652	101.4298	302.6143
Iron	56	510.652	103.2348	308.7132
Cobalt	59	546.7318	111.2348	332.7132
Nickel	58	530.652	108.2348	324.7132
Copper	63	589.2348	121.2348	358.7132
Zinc	65	615.2348	127.2348	373.7132
Gallium	69	666.2348	138.2348	407.7132
Germanium	72	693.2348	144.2348	422.7132
Arsenic	75	729.2348	152.2348	447.7132
Selenium	74	713.2348	149.2348	440.7132
Bromine	79	772.2348	162.2348	474.7132
Krypton	84	831.2348	175.2348	508.7132
Rubidium	85	846.2348	177.2348	514.7132
Strontium	88	882.2348	185.2348	539.7132
Yttrium	89	897.2348	187.2348	545.7132
Zirconium	90	912.2348	189.2348	551.7132
Niobium	93	948.2348	197.2348	576.7132
Molybdenum	96	984.2348	205.2348	601.7132
Rhodium	101	1043.2348	218.2348	635.7132
Palladium	102	1058.2348	220.2348	641.7132
Silver	108	1117.2348	233.2348	675.7132
Cadmium	112	1153.2348	241.2348	699.7132
Indium	115	1189.2348	249.2348	723.7132
Tin	119	1225.2348	257.2348	747.7132
Antimony	122	1261.2348	265.2348	771.7132
Tellurium	128	1320.2348	278.2348	805.7132
Iodine	127	1305.2348	276.2348	800.7132
Xenon	136	1394.2348	291.2348	844.7132
Cesium	133	1358.2348	283.2348	829.7132
Barium	137	1409.2348	291.2348	844.7132
Lanthanum	139	1435.2348	297.2348	851.7132
Cerium	140	1445.2348	299.2348	855.7132
Praseodymium	141	1455.2348	301.2348	859.7132
Neodymium	142	1465.2348	303.2348	863.7132
Europium	152	1574.2348	318.2348	897.7132
Gadolinium	157	1633.2348	331.2348	931.7132
Terbium	159	1659.2348	337.2348	946.7132
Dysprosium	163	1700.2348	345.2348	961.7132
Ytterbium	173	1809.2348	360.2348	995.7132
Lutetium	175	1835.2348	366.2348	1010.7132
Hafnium	178	1871.2348	374.2348	1025.7132
Tantalum	181	1907.2348	382.2348	1040.7132
Tungsten	184	1943.2348	390.2348	1055.7132
Rhenium	187	1979.2348	398.2348	1070.7132
Osmium	192	2038.2348	411.2348	1104.7132
Iridium	193	2048.2348	413.2348	1109.7132
Platinum	195	2074.2348	419.2348	1116.7132
Gold	197	2100.2348	425.2348	1123.7132
Mercury	201	2141.2348	433.2348	1138.7132
Thallium	205	2182.2348	441.2348	1153.7132
Lead	208	2218.2348	449.2348	1168.7132
Bismuth	209	2228.2348	451.2348	1173.7132
Polonium	210	2238.2348	453.2348	1178.7132
Astatine	211	2248.2348	455.2348	1183.7132
Radium	226	2466.2348	499.2348	1277.7132
Actinium	227	2476.2348	501.2348	1282.7132
Thorium	232	2535.2348	514.2348	1316.7132
Protactinium	231	2525.2348	512.2348	1312.7132
Uranium	238	2644.2348	539.2348	1380.7132
Neptunium	237	2634.2348	537.2348	1376.7132
Plutonium	244	2753.2348	564.2348	1444.7132
Americium	243	2743.2348	562.2348	1440.7132
Curium	247	2784.2348	570.2348	1455.7132
Berkelium	247	2784.2348	570.2348	1455.7132
Californium	251	2825.2348	578.2348	1470.7132
Einsteinium	252	2835.2348	580.2348	1475.7132
Fermium	257	2894.2348	593.2348	1509.7132
Mendelevium	258	2904.2348	595.2348	1514.7132
Nobelium	262	2945.2348	603.2348	1529.7132
Lawrencium	261	2935.2348	601.2348	1525.7132
Rutherfordium	263	2961.2348	607.2348	1532.7132
Dubnium	263	2961.2348	607.2348	1532.7132
Seaborgium	266	3002.2348	615.2348	1547.7132
Bhassium	267	3012.2348	617.2348	1552.7132
Hassium	270	3053.2348	625.2348	1567.7132
Mt	270	3053.2348	625.2348	1567.7132
Darmstadtium	271	3063.2348	627.2348	1572.7132
Roentgenium	272	3073.2348	629.2348	1577.7132
Copernicium	277	3132.2348	642.2348	1611.7132
Nihonium	284	3251.2348	669.2348	1679.7132
Oganesson	284	3251.2348	669.2348	1679.7132

The binding energy in Table 2, using Einstein’s mass-energy theory, Bahjat mass-energy relation and New mass-energy concept, are calculated using equations (56),(57) and(58) and Table 1.



**Fig 2: The graphical Plot of Energy as a function of Atomic Mass (u)**

The graphical Plot of Energy as a function of Atomic Mass (u) is observed in figure 2 which employed table 2, shows that, the Einstein’s mass-energy relation over estimates the energies of

nuclear matter while that of Bahjat underestimates the same. Therefore, the new mass-energy concept,  $E = mc^2$ , becomes  $E = mvc$ , where  $v$  is the speed for particles with mass and  $v = c$  for a photon.

## CONCLUSION

Einstein's mass-energy relation,  $E = mc^2$ , overestimates the energies of particles that have mass because it is only photons, or massless energy mediators, that have the speed  $3 \times 10^8 \text{ms}^{-1}$ . Therefore, it is crucial to employ the new mass-energy relation,  $E = mvc$ , in the computation of nuclear and particle energies. Hence, the parameter  $v = 1.8 \times 10^8 \text{ms}^{-1}$  can be used to replace  $c$ . Examples can be seen clearly in the unified atomic mass units and the calculation binding energies.

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