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Investigation of electromagnetic form factors, Masses and Magnetic Moments of Nucleon using quark-based MIT Bag Model.

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ABSTRACT

Theoretical results from lattice gauge theory indicate that when distance scale is compared with the size of hadron, quarks interact with an effective interaction which goes linearly with spatial distance. Experimentally, no single quark has been isolated. This leads to concept that large-scale behavior of quarks is characterized by confinement inside hadrons. As the gap between quark and antiquark gets larger, more quark-antiquark pairs are produced such that produced quark is connected to antiquark and vice versa. Isolating quark by separating it from its antiquark would be impossible. With quarks confined inside hadron, useful description of quarks in hadrons is provided by Bag Model. Massachusetts Institute of Technology Bag Model has essential characteristics of quark confinement. This model is used in this work to investigate how quarks can be emptied in new phases of quark matter called the Quark Gluon Plasma. In this model, quarks are treated as massless particles inside a bag of finite dimension and infinitely massive outside the bag. Bag radius of nucleon is determined by MIT bag model based on electric and magnetic form factors of nucleon. Masses and magnetic moments of nucleon are calculated using bag radius. Results obtained show good agreement with results in literature.

1. INTRODUCTION

The study of the electric and magnetic form factors of the nucleons is of fundamental importance in understanding their electromagnetic structure. The electromagnetic form factors of the nucleon have been a longstanding subject of interest in nuclear and particle physics, and have been the subject of sustained experimental and theoretical investigations for almost 50 years. The nucleon magnetic moment is the intrinsic dipole moments of proton and neutron of symbol $\mu_p and\ \mu_n$. Its magnetic strength is measured by its magnetic moment [1]. Until the 1960s, nucleons were thought to be elementary particles not made up of smaller parts. Now, they are known to be composite particles, made up of three quarks bound together by strong interaction called nuclear force. Nucleons sit at the boundary where particle physics and nuclear physics overlap.

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Particle physics particularly quantum chromodynamics, provides fundamental equations that describe the properties of quarks and of strong interaction. These equations describe quantitatively how quarks can bind together into protons and neutrons (and all other hadrons). However, when multiple nucleons are assembled into an atomic nucleus (nuclide), these fundamental equations become too difficult to solve directly. Instead, nucleons and their interactions are studied by approximations and models such as nuclear shell models. These models can successfully describe nuclide properties as, for example, whether or not a particular nuclide undergoes radioactive decay. The original MIT bag model was presented about three decades ago [2, 3]. This model is defined by the equation of motion and boundary condition for each field degree of freedom inside the bag and homogeneous boundary condition at the surface of the bag. Hadrons are considered as static extended objects in space. The internal structure of these objects includes quarks and varying gluon field. In MIT bag model, it is supposed that a region of space called "bag" including hadrons fields are fixed. The pressure of hadron constituents in the surface is constant and the vacuum around the bag imposes an external pressure on the surface of the bag. As this external pressure increases more than the internal one, the bag shrinks [4]. Bag is the only parameter of this theory. Hadrons constituent fields in the bag can carry any spin or quantum numbers. In this paper we generally supposed that the fields in the bag are massless, that is, free Lagrangian is considered for the fields without any interaction in Lagrangian. Although the fields in the bag are free in first approximation, at the next level, it is suggested that the fields couple weakly. Weak coupling is considered for the quantum numbers in the hadron. The hadron fields in the bag are colored quarks and gluons [3]. Models such as MIT bag model describe two features of Quantum Chromodynamics (QCD) in the quark model [5], asymptotic in short distances and Confinement in large distances. The goal of this paper is to perform the numerical analysis of masses and magnetic moment of the nucleon with MIT bag model based on the electric and magnetic form factors. Finally, the obtained results are compared with the experimental and previous calculated values.

II. ELECTRIC AND MAGNETIC FORM FACTORS OF THE NUCLEON

Elastic electron scattering of the lightest nuclei, hydrogen and deuterium, yields information about the nuclear building blocks, the proton and the neutron [5].

The nucleon Electric and magnetic form factors $G_E \; and \; G_M$, were determined by first converting the experimental elastic e-p cross section $\sigma = (E, \theta)$, to the reduced cross section σ_R [6], as:

$$\sigma_R(Q^2, \varepsilon) = \varepsilon(1+\tau)\frac{E}{E'}\frac{\sigma(E,\theta)}{\sigma_{Mott}} = \tau G_{M_p}^2(Q^2) + \varepsilon G_{E_p}^2(Q^2) \tag{1}$$

, where $\sigma_{Mott} = \frac{\propto^2 cos^2(\frac{\theta}{2})}{4E^2 sin^4(\frac{\theta}{2})}$ and $\varepsilon = (1 + 2(1+\tau)tan^2(\frac{\theta}{2}))^{-1}$ is the transverse polarization of virtual nucleon. By measuring the reduced cross section σ_R at several ε points for a fixed charge operator Q^2, and by making a linear fit to ε, we obtain $\tau G_{M_p}^2(Q^2)$ from the intercept and $G_{E_p}^2(Q^2)$ from the slope in eq.(1).

Quasielastic e-p spectra at each kinematic point were obtained as function of missing mass squared, $W^2 = M^2 - 2M(E - E') - Q^2$. In this portion, $\varepsilon = \left(1 + 2(1+\tau')tan^2\left(\frac{\theta}{2}\right)\right)^{-1}$ is the longitudinal polarization of the virtual photon, with $\tau' = \frac{V^2}{Q^2}$ and $V = E - E'$

The measured e-p cross section per nucleon,$\sigma(E - E', \theta)$,were converted to reduce cross sections, defined as:

$$\sigma_R = \varepsilon(1+\tau')\frac{\sigma(E,E',\theta)}{\sigma_{Mott}G_D^2} = \frac{R_T}{G_D^2} + \varepsilon\frac{R_L}{G_D^2} \tag{2}$$

To extract the neutron form factors, $R_L \; and \; R_T$ were fitted with the model shapes for both the quasielastic and inelastic contributions. The quasielastic component was modeled with a nonrelativistic plane wave impulse approximation (PWIA) calculation [7] using Paris deuterium wave function [8].

In the PWIA, quasielastic portion of R_L is proportional to $(G_E^P)^2 + (G_E^n)^2$ and is also proportional [9] to $(G_M^P)^2 + (G_M^n)^2$. The neutron form factors were determined by subtracting the proton form factors measured from the coefficients of the quasielastic fits [10].

III. NUCLEON BAG RADIUS

In the other hand, the nucleon form factors can be calculated in other theory models. There are many calculations of the nucleon electromagnetic form factors within different hadronic models. Indeed the understanding of these form factors is extremely important in any effective theory or models of strong interaction [11]. The electromagnetic form factors of proton and neutron are calculated using MIT bag model wave function and parameters [12, 13]. The MIT bag model is a conceptually very simple phenomenological model developed in 1974 at the Massachusetts Institute of Technology in Cambridge (USA) shortly after the formulation of Quantum chromodynamics (QCD) used in theoretical physics for strong interaction between quarks mediated by gluons. Quarks are fundamental particles that make up composite hadrons such as the proton, neutron and pion. QCD is a type of quantum field theory called non-abelian gauge theory (or Yang-Mills theory) with symmetry group SU (3). The analog of electric charge is a property called color. Gluons are the force carriers of the theory, just as photons are for electromagnetic force in quantum electrodynamics. MIT Bag model soon became a major tool for hadrons theorists. According to the model, quarks are forced by a fixed external pressure to move only inside a given spatial region. Within this region (bag) quarks occupy single-particle orbitals similar to nucleons in the nuclear shell model [4]. The corresponding wave function can be obtained by solving the Dirac equation for free fermions, and appropriate boundary conditions at the bag surface guarantee that no quark can leave the bag. In the following, we shall investigate spherical bags; this is for sure the most obvious assumption for hadrons in the ground state, and it has the additional advantage that the solutions can be found analytically. Since the surface of the bag is spherical and all quarks are in the lowest eigen mode, the electric and magnetic form factors for the nucleon can be written as [11]:

$$G_E(Q^2) = \int_0^R 4\pi r^2 dr j_0(Qr)[g^2(r) + f^2(r)] \tag{3}$$

$$G_M(Q^2) = 2m_N \int_0^R 4\pi r^2 dr \frac{j_i(Qr)}{Q}[2g(r)f(r)] \tag{4}$$

Where m_N is the nucleon mass. The function $g(r)$ and $f(r)$ are defined by:

$$g(r) = Nj_0(\frac{\omega r}{R}) \tag{5}$$

$$f(r) = Nj_i(\frac{\omega r}{R}) \tag{6}$$

Where $\omega = 2.04$ in the lowest mode and $N^2 = {}^{\omega}/_{8\pi R^3}\, j_0^2(\omega)(\omega - 1)$. According to the nucleon electric and magnetic form factors calculated, the radius of the bag can be obtained based on Eq.3 and Eq.4 at each Q^2. The results are shown in Tables I and II. It is seen th/at R value is inversely proportional to the increase of charge carrier Q^2. Since increases cause an external pressure, B, imposed on the surface of the bag and makes it contract, hence its radius decreases [12].In the calculation, to obtain the static radius of the bag, the limit value of the nucleons bag radius can be calculated in limit $Q^2 \to 0$ based on the fits of the graphs in Figure 3 and 4 which our results show in Table III .

IV. NUCLEON MASS

In the MIT bag model, nucleons masses are determined based on some terms. We summarize them briefly [14]:

(a) The quantum fluctuations contribute two terms which depend only on the radius of the nucleon. The volume term is:

$$E_Y = \frac{4}{3}\pi B R^3 \tag{7}$$

The remainder of the zero-point energy is:

$$E_0 \equiv -\frac{Z_0}{R} \tag{8}$$

we can expect Z_0, to be positive and of order unity.

(b) The quarks contribute their rest and kinetic energies to the nucleon's mass. If N_0, N_S, m_0 *and* m_S are the respective numbers and masses of the non-strange and strange quarks, and if ω is the frequency defined by: $\omega(m\,, R) = \frac{1}{R}[x^2 + (mR)^2]^{\frac{1}{2}}$ (9)

Where $x = x(mR)$ and obtained by:

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{\frac{1}{2}}} \tag{10}$$

Then this term is:

$$E_Q = N_0\omega(m_0, R) + N_s\omega(m_s, R) \tag{11}$$

(c) The gluon interaction has color magnetic exchange and color electric parts. The color magnetic exchange term will be written in the form:

$$E_M = a_{00}M_{00} + a_{0s}M_{0s} + a_{ss}M_{ss} \tag{12}$$

In Eq.(12), M_{00} is the color magnetic interaction between two non-strange quark, M_{0s} is that between a non-strange and strange quark and M_{ss}, the interaction energy between two strange quarks. The values of M_{00} , M_{0s} *and* M_{ss} can be read off in Fig. 1. The value of a_{00} for nucleon is (-3). the color electric energy is given by: $E_E = b\epsilon$ (13)

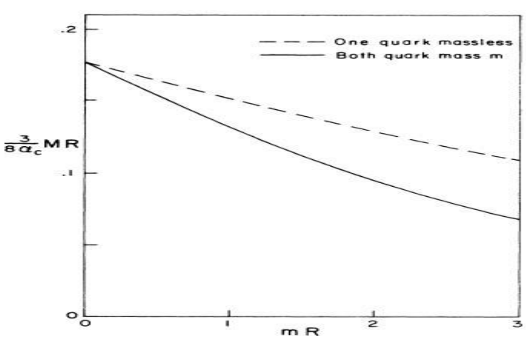

Fig.1 Magnetic gluon exchange energy of two quarks as a function of mR . M is the quantity referred to in Eq.(12). Ref [14]

Where ϵ is the color electric interaction energy of a strange and a non-strange quark including both self-interaction and exchange graphs. The coefficient b is one or zero depending upon whether the quark content of the hadron is mixed or not.

Where for nucleon is zero.

The mass of hadron of radius R is then given by:

$$M(R) = E_V + E_0 + E_Q + E_M + E_E \tag{14}$$

Where the individual terms are given by:

$$E_V = \frac{4}{3}\pi B R^3\,, E_0 = \frac{-Z_0}{R},\; E_Q = N_0\omega(m_0, R) \tag{15}$$

Without considering the mass of quarks in above equation is zero. Then:

$$E_Q = N_0 \frac{1}{R}[x^2 + 0]^{\frac{1}{2}} = \frac{N_0 x}{R} = \frac{3(2.04)}{R} = \frac{6.12}{R} \quad (16)$$

Where due to nucleons do not have strange quark, then Eq. (11) as follow:

$$E_M = a_{00} M_{00} = -3M_{00} \quad (17)$$

The value of M_{00} can be read off of Fig.2. then:

$$\frac{3}{8\propto_C} M_{00} R = 0.175 \ , \propto_C = 0.55 \ , M_{00} R = 0.256 \quad (18)$$

Then, $$M_N = \frac{4}{3}\pi B R^3 - \frac{Z_0}{R} + \frac{6.12}{R} - \frac{0.768}{R} \quad (19)$$

Values of Z_0 and B in Eq.(19) according to Ref.[15] are $Z_O = 1.84$ and $B^{\frac{1}{4}} = 0.145(Gev)$.

Finally, mass of the nucleon is:

$$M_N = \frac{4}{3}\pi(0.145)^4 R^3 - \frac{1.84}{R} + \frac{6.12}{R} - \frac{0.768}{R} \quad (20)$$

According to the calculated static radius, the numerical value of masses of the nucleons can be obtained by others. We have shown our results in Table 3.

V. NUCLEON MAGNETIC MOMENT

The magnetic moment of the proton in MIT bag model is defined as [4]:

$$\mu_P = \int d^3r_1 \int d^3r_2 \int d^3r_3 \, \Psi_P^{\uparrow} \sum_i (\frac{Q_i}{2} \grave{r}_i \times \grave{\propto}_i) \Psi_P \quad (21)$$

here Ψ_P denotes the proton wave function and Q_i is the charge operator. Since the SU (6) wave function of the proton is symmetric under permutations of the indices 1, 2, and 3, we have:

$$\int \Psi_P^{\uparrow} \left(\frac{Q_1}{2} \grave{r}_1 \times \acute{\propto}_2\right) \Psi_P d^3r_1 d^3r_2 d^3r_3 = \int \Psi_P^{\uparrow} \left(\frac{Q_2}{2} \grave{r}_2 \times \acute{\propto}_2\right) \Psi_P d^3r_1 d^3r_2 d^3r_3$$

$$= \int \Psi_P^{\uparrow} (\frac{Q_3}{2} \grave{r}_3 \times \acute{\propto}_3) \Psi_P d^3r_1 d^3r_2 d^3r_3 \quad (22)$$

We insert Ψ_P in Eq. (21) .then:

$$\mu_P = \frac{1}{6}\int d^3r_1 \times [10u^{\uparrow}(1)^{\uparrow}\left(\frac{Q_1}{2}\acute{r}_1 \times \acute{\alpha}_1\right)u^{\uparrow}(1) + 2u^{\uparrow}(1)^{\uparrow}\left(\frac{Q_1}{2}\acute{r}_1 \times \acute{\alpha}_1\right)u^{\uparrow}(1) + 4d^{\uparrow}(1)^{\uparrow}\left(\frac{Q_1}{2}\acute{r}_1 \times \acute{\alpha}_1\right)d^{\uparrow}(1) + 2d^{\uparrow}(1)^{\uparrow}\left(\frac{Q_1}{2}\acute{r}_1 \times \acute{\alpha}_1\right)d^{\uparrow}(1)] \quad (23)$$

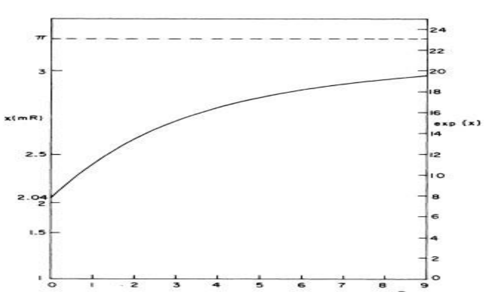

Fig. 2 Eigen frequency x(mR) of the lowest quark Mode with mass in a spherical cavity of radius in Ref [14]

Next we insert the quark charges and wave function of quarks to obtain:

$$\mu_P = \frac{e}{2}N^2 \int_0^R drr^2 = \frac{e}{2}N^2 \int_0^R drr^2 \int d\Omega \, \mathrm{i} j_0(Er) j_1(Er) x_{-1}^{\frac{1}{2}\uparrow}, [(r \times \acute{\propto})\delta_r - \delta_r(r \times \acute{\propto})] x_{-1}^{\frac{1}{2}\uparrow}, \quad (24)$$

Using commutation relations of Pauli matrices, σ, therefore :

$$\mu_P = \frac{e}{2}N^2 \int_0^R drr^2 \int d\Omega \, \mathrm{i} j_0(Er) j_1(Er) \, \frac{1}{4\pi}\begin{pmatrix}1 & \acute{0}\end{pmatrix}[2ir\sigma_r - 2ir\sigma]\binom{1}{0}$$

$$=\frac{eN^2}{4\pi}\int_0^R drr^3\, j_0(Er)j_1(Er)\int d\Omega\,[e_3-\cos\theta\begin{pmatrix}\sin\theta & \cos\varphi\\ \sin\theta & \sin\varphi\\ & \cos\theta\end{pmatrix}] \quad (25)$$

Now we can easily perform the φ , θ and r integrations:

$$\mu_P = e_3\frac{Re}{ER(ER-1)}\frac{4ER-3}{12} \quad (26)$$

According to the calculated static radius, the numerical value of magnetic moment of the nucleon can be obtained by others with similar calculation obtained the magnetic moment of neutron. Results in Table III.

TABLE I

COMPARISON OF CALCULATED MAGNETIC AND ELECTRIC FORM FACTORS (M_f and E_f) OF QUARKS WITH REF[15] . ADDITIONALLY, THE RADIUS OF THE BAG IS SHOWN

Our calc. Results for M_f	Exp. Results for M_f	Our calc. Results for E_f	Exp. Results for E_f	Results for M_f in Ref.[15]	Results for E_f in Ref.[15]	Bag Radius, R_B
0.6503	0.6721	0.4688	0.2713	0.686	0.2651	9.941
0.9106	0.9223	0.5713	0.9971	0.998	0.9922	8.873
2.2073	2.3345	1.2761	2.0627	2.136	2.2553	8.671
3.6744	2.7634	2.2361	3.0442	2.707	3.0350	7 .619
4.6813	3.7941	4.2410	4.0262	3.731	4.0234	6.546
5.2587	4.2712	5.0202	5.0210	4.286	5.0298	5.518
6.2523	5.2821	6.0155	6.0142	5.19	6.0165	4.472

TABLE II

RESULTS FOR ELECTRIC AND MAGNETIC FORM FACTORS (G_E AND G_M) OF NEUTRON AND THE RADIUS OF THE BAG.

Calc. Results of G_E (GeV)	Calc. Results of G_M (GeV)	Exp. Results of G_E (GeV)	Exp. Results of G_M (GeV)	Calc. Results of Bag Radius ,R_B	Exp. Results of Bag Radius , R_B
0.0037	1.7550	0.0025	1.7644	9.0325	8.7629
0.0514	2.5010	0.0844	2.6440	8.0844	7.6713
0.4020	3.2532	0.4450	3.2064	7.0450	6.6131
0.0303	4.2200	0.0480	4.2480	6.1050	5.5629

TABLE III

THE MASSES (M_N) AND MAGNETIC MOMENT (μ_N) OF NEUCLEON AND COMPARISON WITH THOSE OBTAINED IN LITERATURE.

	G_E (GeV)	G_M (GeV)	Bag Radius , R_B	M_f (μ_β)	$E_{f(\mu_\beta)}$	M_N $\times 10^{-27}(kg)$	$\mu_N(\mu_\beta)$
Our calc. Results	0.0037	0.7550	5.0325	0.6503	0.4688	1..6920	0.8902
Ref [4]	0.0321	2.7342	7.3215	2.2073	1.2761	1.6780	0.8821
Ref[15]	0.0425	3.7327	8.4141	3.6744	2.2361	1.6698	0.8582
Ref [17]	0.0542	4.7818	9.5321	4.6813	4.2410	1.6532	0.8442

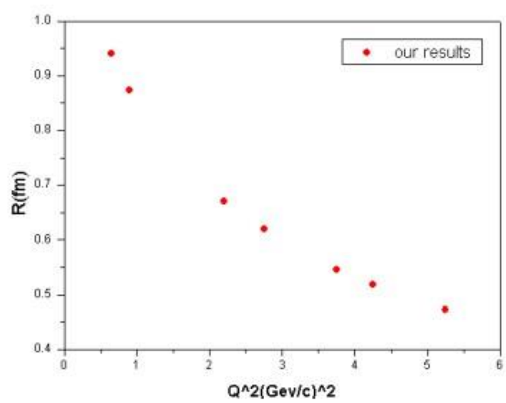

Fig. 3 The bag radius versus Energy of Neutron in Ref [15]

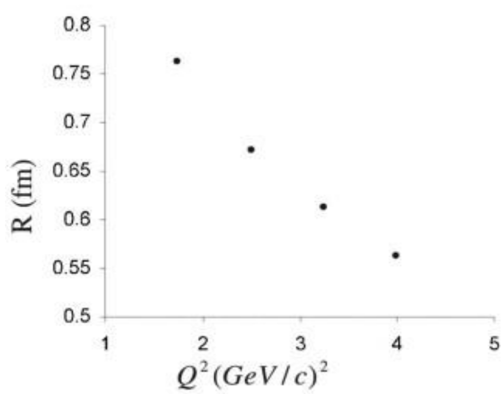

Fig. 4 the bag radius versus Energy of Neutron in ref [17]

CONCLUSION

The electric and magnetic form factors of the nucleon are calculated and through using them in the MIT bag model, the bag radius can be calculated In the limit of 0, the static radius of the bag can be obtained and based on this, masses and magnetic moment of the nucleon can be calculated and compared with the results of others. There should be no difference between the proton and the neutron mass, because the proton and the neutron have the same quark structure. Finally, as we have remarked, the mass and magnetic moment of the nucleon decrease as the MIT bag radius increases. Our calculated mass of nucleon is $1.6780 \times 10^{-27} kg$ coheres with the experimental mass of either proton or neutron $(1.6726 \times 10^{-27} kg)$. The calculated sum of magnetic moment of proton and neutron is $0.879\mu_N$ is in nice agreement with the experimental result of $0.857\mu_N$. This is a consequence of the fact that the magnetic moment of a quark is associated with strong interaction forces and its masslessness inside the bag model. Also, the overlap of the small and large components of Dirac wave function is small. Hence, the small values of mass and magnetic moment of nucleon.

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