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APPLICATION OF LINEAR PROGRAMMING FOR PROFIT MAXIMIZATION IN VEGAS RESTAURANT AND BAKERY USING SIMPLEX METHOD

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ABSTRACT

This paper aims to apply linear programming for profit maximization in a Vegas Restaurant and Bakery using the Simplex method. This study was carried out to seek the optimal solution for the bakery that will maximize its monthly profit. Data was collected as extracts from the records on different types of bread packages adopted in the company. Survey data is analyzed to determine the decision-making style and define the problem. A linear programming model is developed for profit maximization and the model equations are solved using the Atozmath.com solver. Results revealed that the company makes a monthly profit of N979,030 and that Milk bread, Sardine bread, wheat bread, and Banana bread contributed objectively to the maximum profit. Hence, the firm should produce more of these types of bread and sold to maximize the company's profit. Sensitivity analysis was also performed.

1. Introduction

Linear Programming (LP) is a problem-solving approach developed to help managers make decisions. In Statistics and Mathematics, Linear Programming (LP) is a technique for optimizing linear objective functions, subject to linear equality and linear inequality constraint(s). Informally, linear programming determines how to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model and provides some list of requirements as linear equations.

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Numerous linear programming applications can be found in today's competitive business environment. Such applications of LP can be found in transportation problems, military applications, Distribution centers, manufacturing plants, production, and manpower management. The term linear programming was first used by G.B. Dantzig in 1947 during World War II, while working with the US Air Force, he developed the LP model primarily for solving military logistics problems. But now, it is extensively being used in all functional areas of management, airlines, agriculture, military operations, oil refining, education, energy planning, health care system, etc. Refer to specific problems of optimization that assume that both constraints and objective functions are linear. LP modeling solves particular mathematical problems by forming specific rules that deal with the allocation of limited resources under strict technological or practical restrictions when a certain course of action has to be chosen [1]. In 1992, Taha applied linear programming in poultry farms to determine the accurate proportion of calcium, fiber, and protein in food.

Linear programming model to optimize the water resource in irrigation was used and an optimal way of water resource usage was obtained [2]. They considered the numerous constraints by developing a separate mathematical model to achieve their goal. They noted that mathematical programming quantifies an optimal way of combining scarce resources to satisfy the proposed goals; that is to analyze the cases where the available resources must be combined in a way to maximize the profit or minimize cost.

The authors in [3] used a linear programming approach in the Profit Planning of an NGO-run Enterprise. In [4], the authors evaluated and improved the effectiveness of the rolling mill in the production of medium steel sections in the selected company from the metallurgical industry. Authors in [5], considered the Profit Optimization using a Linear Programming Model in their study of the Ethiopian Chemical Company. In 2018, a group of researchers in [6], worked on Profit Maximization in a Product Mix Company Using Linear Programming. [7] looked at the Impact of Linear Programming on Profit Maximization in Production Firms. Also, [8] considered maximizing the logistic model for customer churn prediction using genetic algorithms. In [9], the authors apply a linear programming, investigating on profit maximization of Bagels and Donuts for Sale in Economics.

Also, he applied a linear programming model to aggregate production planning of coated peanut products. International Conference on Green Agro-industry and Bio-economy.

In this paper, we applied the Linear P rogramming model for profit maximization of Vegas Restaurant, and Bakery Abakaliki, Ebonyi state using the Simplex Method.

In Nigeria, most decisions have been taken by government or non-governmental organizations, whether it is profitable or not for companies, manufacturing, or service industries are based on trial and error. Qualitative decisions, like intuition, and judgmental approaches are more dominant and the application of model-based decisions like linear programming models have little or no application. Hence, it is initiated by this work to conduct an assessment of the application of linear programming in this particular company as a case study.

The work aims to use linear programming (LP) as a mathematical model to maximize the profit of manufacturing industries; a case study of Vegas Restaurant Abakaliki while the objectives to formulate a linear programming model that would suggest a viable product mix to ensure profit for the company, use the simplex method and perform the sensitivity analysis.

II. METHODOLOGY

The method used in this work is the Simplex method to solve (LPP) models which contain two or more decision variables. The Simplex method is an approach for determining the optimal value of a linear program by hand. The method produces an optimal solution to satisfy the given constraints and produce a maximum zeta value. To use the Simplex method, a given linear programming model needs to be in standard form, where slack variables can then be introduced. Using the tableau and pivot variables, an optimal solution can be reached.

The General Model of A Linear Programming Problem (Lpp**)**

$$\underset{\substack{subject\ to\\ AY\leq C}}{\mathrm{M}ax/\mathrm{Min}\mathrm{Z}}$$

The simplest form of linear programming for the profit maximization

$$\underset{\substack{subject\ to\\ AY\leq C}}{\mathrm{M}ax}\ (Z) = B^T Y$$

Where Z denotes the vector of variables, B and A are coefficient vectors of matrices that are known. The expression, "$\mathrm{Max(Z)} = B^T Y$", is the objective function that is to be maximized and the expression $AY \leq C$ is the vector of constraints to be met while optimizing the objective function.

The current study demonstrates the application of linear programming for profit maximization using the Simplex method about market demand and other related constraints.It is a case of Vegas Restaurants and Bakeries, Abakaliki, Nigeria.

The data are taken from Vegas Restaurants and Bakeries, Abakaliki where they intend to manufacture bread consisting of nine different types. The challenge faced by the company is selecting the type of bread among these nine types of bread which give maximum profit when combined. The profit maximization approach is used to address the challenge. The output of the profit maximization model of the current study is expressed in the form of a linear programming statement proclaiming the objective and the constraint functions of the optimization, including its optimal solution.

The analysis of the obtained optimization solutions and their sensitivities are performed to identify the key factors influencing profit maximization and use as the basis to decide the most profitable bread(s). In formulating the model for profit maximization, the current study employs the linear programming method (i.e. the simplex method).

There are five stages of linear programming procedures applied, namely; determining decision variables, (2) formulating the objective function, (3) determining and modeling the constraint functions, (4) formulating the model of the linear programming statement subject to the given constraints, (5) calculating and analyzing the optimization solution of the linear programming statement, and (6) conducting sensitivity analysis to identify the key factors influencing the optimization solution.

The main reason for choosing a Vegas restaurant for this study is because it produces nine different types of Bread which determines the quantity combinations of the products produced (product mix) an important and major management decision. The overall quantity combination of the nine products produced by Vegas Bakery and Restaurant was investigated during the research period and the allocation of resources to the various products. This has been made possible by the records kept by the Production Line Manager and the Sales Department relating to the different brands of products produced by the firm, the technical coefficients, the raw materials available, and their relative prices. Linear programming was used to determine a new quantity combination. The total contribution to profit of each of the products for the month using the new quantity will now be compared with the total profit contribution made by the former product mix determined by the trial-and-error Method. Linear programming needs firstly to be presented in a general standard form to display all properties required of a linear programming problem. This consists of a linear objective function $f(y)$ such that, if in general $b_1, b_1, \dots, b_n$ are real numbers, then the function, f of real variables $y_1, y_2, \dots, y_n$ can be defined as:

$$F(y) = b_1 y_1 + b_2 y_2 +, \dots, b_n y_{n=} \sum_{i}^{n} b_i y_i$$

Other properties include a linear constraint (which is either a linear equation or linear inequality) and a non-negativity constraint. These can be written in mathematical notations as Linear constrained

$$\sum_{j=1}^{n} a_{ij} \leq c_i \forall\ i \in \{1, .., m\}$$

$$y_j \geq 0, \forall j \in \{1, \dots, n\} \text{ (non-negative constraint)}$$

Generally, linear programming is always given as

$$Z = \mathrm{M}ax \sum_{i=1}^{n} b_i y_i$$

$$\boldsymbol{subject}\ to$$

$$\sum_{j=1}^{n} a_{ij} \leq c_i\ , \forall i \in \{1, 2, \dots, m\},$$

$$y_{j \geq 0}\ \forall j \in \{1, \dots, n\}$$

In the below tables; Table 1 shows the nine different types of bread produced by the Vegas Restaurant and Bakery, their production cost, selling price, and profit. Table 2 shows The Raw Materials mix for the production of bread for the Restaurants. The combinations of the quantities of these eight basic raw materials (raw material mix) for bread production per loaf (in grams), and the maximum quantity of each raw material in stock for monthly production are shown in the table. This information is used to determine the production cost (in terms of raw materials) per loaf of bread produced by the firm. The data collected from Vegas Restaurant and Bakery, Abakaliki on her main product record unit were analyzed to determine the exact type of bread that would yield maximum profit to the company.

All the information provided in Tables 1 to 2 was used to form the linear programming model of the maximization type for the data as stated above.

Table 1: **Nine different types of bread, production cost, selling price, and profit**

Names of bread	Production cost per loaf(])	Selling price per loaf(])	Profits
Jumbo Bread(y_1)	270	450	180
Fruit Bread(y_2)	290	500	210
Milk Bread(y_3)	290	500	210
Coconut Bread(y_4)	290	500	210

Sardine Bread(y_5)	290	500	210
Wheat Bread(y_6)	200	350	150
Fantasia Bread(y_7)	200	500	300
Special Bread(y_8)	200	350	150
Banana Bread(y_9)	190	500	310

Table 2: **The Raw Materials mix for the production of bread**

Raw Materials	Types of bread and their raw material mixture									Total quantity per month
	$y1$	$y2$	$y3$	$y4$	$y5$	$y6$	$y7$	$y8$	$y9$	
Flour	556	556	556	556	556	667	714	625	581	70,000,000
Yeast	3	3	3	3	3	3	6	3	5	7,000,000
Milk	2	2	2	2	2	-	6	6	2	24,400
Egg	-	13	13	19	19	-	8	9	-	69540
Water	833	833	278	444	444	667	514	313	872	2,100,000
Flavor		1	1	1	-	-	1	1	0.3	1,400
Butter	11	13	13	11	11	1	14	5	2	840,000
Sugar	4	50	50	4	4	-	7	6	4	11,200,000

2. 1 LINEAR PROGRAMMING MODEL FUNCTION

From Tables 1 and 2, we formulate the Linear Programming model of the problem of Vegas Restaurant. Let F represent the profit function that we seek to maximize, we have the LP model for the problem as:

$$MaxF(y) = 180y_1 + 210y_2 + 210y_3 + 210y_4 + 210y_5 + 150y_6 + 300y_7 + 150y_8 + 310y_9 .$$

Subject to:

$$+556y_2 + 556y_3 + 556y_4 + 556y_5 + 667y_6 + 714y_7 + 625y_8 + 581y_9 \leq 70000000$$
$$3y_1 + 3y_2 + 3y_3 + 3y_4 + 3y_5 + 3y_6 + 6y_7 + 3y_8 + 5y_9 \leq 7000000$$

$$2y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + 3y_6 + 6y_7 + 3y_8 + 2y_9 \leq 24400$$
(P_1)

$$13y_2 + 13y_3 + 19y_4 + 19y_5 + 8y_7 + 9y_8 \leq 69540$$

$$833y_1 + 833y_2 + 278y_3 + 444y_4 + 444y_5 + 667y_6 + 514y_7 + 313y_8 + 872y_9 \leq 2100000$$

$$y_1 + y_2 + y_3 + y_4 + y_7 + y_8 + 0.3y_9 \leq 1400$$

$$11y_1 + 13y_2 + 13y_3 + 11y_4 + 11y_5 + y_6 + 14y_7 + 5y_8 + 2y_9 \leq 840000$$
$$4y_1 + 50y_2 + 50y_3 + 4y_4 + 4y_5 + 7y_7 + 6y_8 + 4y_9 \leq 11200000$$

$$y_1, y_2, y_3, y_4 + y_5, y_6, y_7, y_8, y_9 \geq 0$$

To be able to solve for the optimality of this problem, we first introduce eight slack variables, $S_i (i = 1,2, \ldots ,8)$

This will change the inequalities signs in the constraint aspect of the model to equality signs. A slack variable will account for the unused quantity of raw material (if any) it represents at the end of the production. As a result, the above LP model becomes:

$$F(y) = 180y_1 + 210y_2 + 210y_3 + 210y_4 + 210y_5 + 150y_6 + 300y_7 + 150y_8 + 310y_9 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + 0s_7 + 0s_8$$

Subject to:

$$556y_1 + 556y_2 + 556y_3 + 556y_4 + 556y_5 + 667y_6 + 714y_7 + 625y_8 + 581y_9 + s_1 = 70000000$$

$$3y_1 + 3y_2 + 3y_3 + 3y_4 + 3y_5 + 3y_6 + 6y_7 + 3y_8 + 5y_9 + s_2 = 7000000$$

$$2y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + 3y_6 + 6y_7 + 3y_8 + 2y_9 + s_3 = 24400$$
(P_2)

$$13y_2 + 13y_3 + 19y_4 + 19y_5 + 8y_7 + 9y_8 + s_4 = 69540$$

$$833y_1 + 833y_2 + 278y_3 + 444y_4 + 444y_5 + 667y_6 + 514y_7 + 313y_8 + 872y_9 + s_5 = 2100000$$

$$y_1 + y_2 + y_3 + y_4 + y_7 + y_8 + 0.3y_9 + s_6 = 1400$$

$$11y_1 + 13y_2 + 13y_3 + 11y_4 + 11y_5 + y_6 + 14y_7 + 5y_8 + 2y_9 + s_7 = 840000$$

$$4y_1 + 50y_2 + 50y_3 + 4y_4 + 4y_5 + 7y_7 + 6y_8 + 4y_9 + s_8 = 1120000$$

$$y_1, y_2, y_3, y_4 + y_5, y_6, y_7, y_8, y_9, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \geq 0$$

III. ANALYSIS AND RESULTS

Applying the Simplex Method to the Vegas Restaurant Model (P_2) generates the Excel

report.

Table 3: Excel report

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Total profit
Quantity of product	0	0	1391.634558	0	2707.828987	729.4362384	0	0	27.88480708	
Unit profit	180	210	210	210	210	150	300	150	310	978,947.07

Constraints	Types of bread and quantities of raw materials used.									Used	Available
Flour	556	556	556	556	556	667	714	625	581	2,782,036.77	70,000,000.00
Yeast	3	3	3	3	3	3	6	3	5	14,626.12	7,000,000.00
Milk	2	2	2	2	2	0	6	6	2	24,400.00	24,400.00
Egg	0	13	13	19	19	0	8	9	0	69,540.00	69,540.00
Water	833	833	278	444	444	667	514	313	872	2,100,000.00	2,100,000.00
Flavor	1	1	1	1	0		1	1	0.3	1,400.00	1,400.00
Butter	11	13	13	11	11	1	14	5	2	48,662.57	840,000.00
Sugar	4	50	50	4	4	0	7	6	4	80,524.58	11,200,000.00

Table 4: Objective Cell (Max) and Variable Cells

Objective Cell (Max)

Name	**Initial Value**	**Final Value**
Total profit	832996.1554	978947.0703

Variable Cells

Name	**Initial Value**	**Final Value**
Qty of Jumbo bread(Y1)	0	0
Qty of Fruit bread(Y2)	0	0
Qty of Milk bread(Y3)	1376.844732	1391.634558
Qty of Coconut bread(Y4)	0	0
Qty of Sardine bread(Y5)	2556.724673	2707.828987
Qty of Wheat bread(Y6)	0	729.4362384
Qty of Fantasia bread(Y7)	23.15526777	0
Qty of Special bread(Y8)	0	0
	0	27.88480708

From the model, the Excel solver was implemented to get the result as shown in table 4 above. The result on the table shows that Vegas Restaurant makes an optimal profit of approximately, N979030 with an approximate production of 1392, 2708, 729, and 28 loaves of Milk, Sardine, Wheat, and Banana types of bread respectively. This means that the Restaurant is advised to produce more loaves of Milk, Sardine, and Wheat bread because of high demand.

IV SENSITIVITY ANALYSIS

Here, we try to identify the key factors influencing the profit maximization of the Restaurants.

Table 5: Sensitivity Analysis

Variable Cells

	Final	Reduced	Objective	Allowable	Allowable
Name	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
Qty of product Y1	0	-79.66423307	180	79.66423307	1E+30
Qty of product Y2	0	-124.8125937	210	124.8125937	1E+30
Qty of product Y3	1391.634558	0	210	307.5446487	71.99170774
Qty of product Y4	0	-72.01519308	210	72.01519308	1E+30
Qty of product Y5	2707.828987	0	210	105.2186498	109.9099055
Qty of product Y6	729.4362384	0	150	43.40771342	62.02288267
Qty of product Y7	0	-47.27346115	300	47.27346115	1E+30
Qty of product Y8	0	-143.7138038	150	143.7138038	1E+30
Qty of product Y9	27.88480708	0	310	31895.85457	38.53461022

Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
Flour Used	2782036.775	0	70000000	1E+30	67217963.23
Yeast Used	14626.12338	0	7000000	1E+30	6985373.87
Milk Used	24400	0.158852828	24400	321214.2976	16195.78947

Egg Used	69540	5.780643125	69540	20963.17585	51485.14128
Water Used	2100000	0.224887556	2100000	67217963.23	486533.971
Flavor Used	1400	72.01519308	1400	3956.305574	1391.180723
Butter Used	48662.57396	0	840000	1E+30	791337.426
Sugar Used	80524.58307	0	11200000	1E+30	11119475.42

DISCUSSION OF THE RESULTS

The result obtained from Excel as seen in Table 3, revealed that Vegas restaurant can make a maximum monthly profit of N979,030 with an approximate production of 1392, 2708, 729, and 28 loaves of Milk bread, Sardine bread, Wheat bread, and Banana bread respectively. On the Analysis result, it is seen in the objective function that this Maximum profit will remain unchanged even with an increase in the quantity of Milk bread, Sardine bread, Wheat bread, and Banana bread up to 306, 105, and 31896 loaves respectively. The maximum profit also remains the same with a reduction of the quantity of Milk bread, Sardine bread, Wheat bread, and Banana bread down to 78, 110, 62, and 39 loaves respectively. It is also noted that Jumbo bread, Fruit bread, coconut bread, Fantasia, and Special bread contributions didn't attract any profit to the company to warrant its inclusion and as seen from the result, any unit profit increase of these bread will lead to a reduction of the maximum profit by N80, N125, N72, N47, and N144 respectively.

The Constraints Analysis shows that the quantities of materials like Flour, Yeast, Butter and Sugar used in the product mix to get the maximum profit is less than the original available quantities. This implies that Vegas restaurant can actually reduce its cost of purchasing these raw materials and save money too. From the shadow price of the Constraints, it also revealed that a unit increase in the quantity of Milk, Egg, Water, and Flavor will increase the monthly maximum profit of Vegas Restaurant by N0.16, N6, N0.22, and N72 respectively. The analysis also shows that Milk, Egg, Water, and Flavour are binding Constraints LHS constraint value is equal to the RHS constraint value with its shadow prices as non-zero while Flour, yeast, Butter, and Sugar are non-binding constraints.

CONCLUSION AND RECOMMENADTIONS

The results of the LP model fitted to the data collected from Vegas Restaurant and Bakery are only based on the cost of raw materials and oven capacity for bread production. Therefore, it is to note that if the information on other elements of cost of production such as labor and time of the process, and other costs is available and incorporated into the LP model formulation and analysis, the results reported here might be remarkably different. Nonetheless, findings from this work could still serve as useful guides to the management of Vegas Restaurant and Bakery in the formulation of production and marketing strategies for their product.

From the Excel result, we therefore recommend that Vegas Restaurant and Bakery should be producing approximately 1392 loaves of Milk bread, 2708 loaves of Sardine bread, 729 loaves of Wheat bread, and 28 loaves of Banana bread to maximize more Profit. It is also suggested that they produce others in less quantity or ignore them since they are not yielding any profit. The paper encourages similar companies to adopt the model to maximize their profits.

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