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TOTAL REVENUE FUNCTION FOR NON REGULAR FIXED LIFETIME INVENTORY SYSTEM: A CASE STUDY OF AIRPLANE SEATS.

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ABSTRACT

The seats in an airplane are considered as non-regular fixed lifetime inventory. An inventory system where items outdate in one period but become useful in the next period. The revenue function per trip was derived for the airplane. The outdates and shortages resulting from low demand and excess demand for seats on the airplane were also derived. The revenue function was simulated in a numerical example.

1. Introduction

The fundamental principle of the fixed lifetime inventory system states that items outdate, when not used to meet demand during their useful lifetimes and discarded from the inventory [1], [2], [3]and [4]. This is because expired items are dangerous and can cause harm to the consumers. However, the non-regular fixed lifetime inventory system as defined by [5], is one where items can outdate in one period but become useful in the next period. This kind of inventory systems exist in real life and their total cost functions is of great importance to researchers. This work derives the cost function for one of such inventory systems, seats on an airplane.

This class of inventory include; rooms in a hotel, airtime in a television or radio station, spaces on daily newspaper, spaces on monthly or quarterly magazines, seats on a car or a bus taking a trip from point A to point B and seats in an airplane flying from point A to point B, [5], [6].

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The seats in an airplane is the main focus of this work. Generally, airline seats are subdivided into economy class, premium economy class (not available in a Nigeria), business class and first class. We give a description of these subclasses.

Economy Class: The economy class is the most basic in any airplane. The seats are also known as a coaches or standard seats. The seats are the narrowest ranging from 16 inches to just over 19 inches wide, the distance from one seat to the seat in front or behind (usually referred to as legroom) ranges from 30 inches to 34 inches. The economy class offer little or no in-flight entertainment and the ticket price are the lowest. The economy seats are more in number on any airplane and most people book them for their travels.

Premium Economy Class: The premium economy class is a travel class offered by some airlines (not in Nigeria), its usually positioned between the standard economy class and business class in terms of price, comfort, service, and available amenities. The premium class emerged as a respond to government and multinational company's requiring a travelling class slightly higher than the regular economy class for their staff. This class is not operated by airlines in Nigeria.

Business Class: The class lying between the economy class and the first class is the business class. The seats are generally 21 inches in width and have more legroom of about 57 inches. This means that there is extra inches of comfort, at most twice the legroom in economy class. The extra legroom is designed for people who need to be on their laptops, talk on their phones, attend online meetings or work on reports. Free food and drinks are always available for customers flying business class. Most airlines go all out to treat their business class passengers like royals in order for them to come back.

First Class: The topmost luxurious seating arrangements in an airplane are the first class seats. They at located at the front of the plane. The seats are typically 2 inches wider than those in business class and have legroom that is approximately 10 to 30 inches lager when compared to business class seats. Some airlines offer personal flight attendants for each passengers travelling first class. Apart from that, the first class passengers are served delicious, freshly cooked food, drinks, slippers, robes, luxe toiletry kits and any other thing the passenger can ask for. The passengers can easily stretch out for a nap and the seats can be converted into a beds. Some airlines have separate washroom for first class passengers. The first class ticket are the most expensive and only a few privileged individuals can try first class.

Model Description and Notation.

The model considers the seats in an airplane as non-regular fixed lifetime inventory [9]. The seats are subdivided into economy class, business class and first class. The total number of seats in the plane is the sum of all the seats in the three classes. The revenue per flight is the sum of revenue from all three subclasses [7], [8]. Seats not used to meet demand on a particular flight are considered outdated, but become useful by the next flight. Demands that cannot be satisfied during a flight are lost.

Notation for the model

$\lambda = N + M + P$ is total number of seats on the plane

N = total number of first class seats

q = number of first class seats used in a flight

M = total number of business class seats

l = number of business class seats used in a flight

P = total number of economy class seats

k = number of economy class seats used in a flight

x = price per first class seat

y = price per business class seat

z = price per economy class seats

$\theta =$ outdate cost

$\beta =$ shortage cost

R = fixed set up cost for the airline

B = fixed maintenance cost for the airline

Assumptions of the Model

1. All the seats are available to meet demand for any flight.
2. Any seat not used in a particular flight, outdates.
3. When there is excess demand, shortage occur and the excess demand is lost.
4. There is a fixed set up cost for the airline.
5. There is a fixed maintenance cost for the airline.

Derivation of revenue function

Revenue per flight is the sum of revenues from the three subclasses [8]. We consider two different cases, first when all the seats are used to meet demand and second, when only a part of the seats are used to meet demand.

Revenue Generated from First Class.

Case 1: when all the seats in first class are used during a trip.

Revenue from first class $= Nx$ (1)

Case 2: when only part of the seats in first class are used during a trip.

Revenue from first class $= qx$ (2)

Where q is the number of first class seats used during the trip and $N-q$ is the number of seats not used. Since seats not used during a trip outdate, the number of outdated seats is $N-q$ and with an outdate cost of θ per seat ,our outdate cost is

Outdate cost $= \theta \int_{0}^{N-q} (N-q-t)dt$ (3)

When the demand is more than the number of seats that is $t > N$, shortage occur and excess demand is lost. The cost per shortage is β and shortage cost is

$$\text{Shortage cost } = \beta\int_0^\infty (t-N)dt \qquad (4)$$

Revenue Generated by Business Class.

Case 1: when all the seats in business class are used to meet demand, we have

$$\text{Revenue from business class } = M\ y \qquad (5)$$

Case 2: when only part of business class seats are used to meet demand, we have

$$\text{Revenue from business class } = l\ y \qquad (6)$$

l is the number of business class seats used to meet demand and $M-l$ is the number of outdates (unused seats). The cost per outdate is θ and the outdate cost is

$$\text{Outdate cost } = \theta\int_0^{M-l} (M-l-t)dt \qquad (7)$$

The shortage cost for the business class is

$$\text{Shortage cost } = \beta\int_0^\infty (t-M)dt \qquad (8)$$

Revenue Generated by Economy Class.

Case 1. When all the seats in economy class is used to meet demand

$$\text{Revenue from economy class } = Pz \qquad (9)$$

Case 2. When only part of the seats in economy class is used to meet demand

$$\text{Revenue from economy class } = k\ z \qquad (10)$$

Where k is the number of used seats in the economy class and $P-k$ is the number of unused seats

$$\text{Outdate cost } = \theta\int_0^{P-k} (P-k-t)dt \qquad (11)$$

$$\text{And Shortage cost } = \beta\int_0^\infty (t-P)dt \qquad (12)$$

Therefore, the total revenue for the case where all seats are used to meet demand is sum of revenue from each class, that is

$$\text{Total Revenue } Nx + My + Pz \qquad (13)$$

and total revenue when only part of the seats were used to meet demand is

Total Revenue $qx + ly + kz$ (14)

The total cost function is the sum of revenues generate, shortage cost, outdate cost, set up cost and maintenance cost. When all seats are used to meet demand, outdate and shortage costs are equal zero and the cost function is given by equation (15)

$$TCF = Nx + My + Pz + R + B \qquad (15)$$

When only a part of the seats are used to meet demand, the cost function is given by equation (16)

$$TRF = qx + ly + kz + \theta\left(\int_0^{N-q}(N-q-t)dt + \int_0^{M-l}(M-l-t)dt + \int_0^{P-k}(P-k-t)dt\right)$$
$$+\beta\left(\int_0^{\infty}(t-N)dt + \int_0^{\infty}(t-M)dt + \int_0^{\infty}(t-P)dt\right) + R + B \qquad (16)$$

Next, we simulate the model for three different flights for an airplane with 150 seats made up of 20 first class seats, 40 business class seats and 90 economy class seats. The results are shown on Tables 1 to 3.

Table 1. The case where all seats are used to meet demand during a flight.

Category	No. of available seats	No. of seats used	Cost per seat	Outdate	Shortage	Revenue
First Class	20	20	950,000	-	-	19,000,000
Business Class	40	40	800,000	-	-	32,000,000
Economy Class	90	90	350,000	-	-	31,500,000
Total	150	150				82, 500,000

In Table 1, all the seats in three categories were used to meet demand and the shortage and outdate quantities are zero.

Table 2. Only part of the seats were used to meet demand during a flight.

Category	No. of available seats	No. of seats used	No. of seats not used	Cost per seat	Outdate Cost per seat	Total Outdate Cost	Revenue
First Class	20	15	5	950,000	50,000	250,000	14,250,000
Business Class	40	33	7	800,000	40,000	280,000	26,400,000
Economy Class	90	80	10	350,000	30,000	300,000	28,000,000
Total	150	128	22			830,000	68,650,000

In Table 2, only part of the seats in each category were used to meet demand. The others outdate at a cost against the airline company. There was no shortage.

Table 3. When both outdate and shortage occurred in a flight.

Category	No. of available seats	No. of seats used in flight	No. of outdates	No. of shortages	Shortage cost	Outdate cost	Revenue
First Class	20	13	7	-	-	350,000	12,350,000
Business Class	40	40	-	6	240,000	-	32,000,000
Economy Class	90	84	6	-	-	180,000	29,400,000
Total	150	137	13	6	240,000	530,000	73,750,000

In Table 3, there are 7 outdates from first class and 6 outdates from economy class, while the number of shortages from the business class is 6. The airline will be charged for outdates and shortages.

Conclusion.

The seats on an airplane is considered as a non-regular fixed life time inventory. The case were all the seats on the airplane were used to meet demand and the case were only a part of the seats were used to meet demand was considered and the total cost function derived for both cases. A simulation was carried out on the model and the result reported on Tables 1-3.

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