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TEMPERATURE DEPENDENT NON-REACTING VISCOUS FLOW ON POROUS PLATE IN A PRESENCE OF CONSTANT MAGNETIC FIELD.

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ABSTRACT

In the present study, an analysis is Carried out to study viscous dissipation effects on free convective temperature non-reacting viscous flows on a porous plate in a presence of constant magnetic field. The coupled non-linear partial differential equations are simplified with the help of asymptotic expansion the simplified equations are solved numerically by using finite difference method. The effects of different parameters on the dimensionless velocity and temperature profiles are shown graphically. It is observed that increasing the suction velocity and Eckert number cause an increase in both order zero velocity and order zero temperature profiles respectively.

1. Introduction

Fluid convection at vertical plates resulting from buoyancy forces find application in several industrial and technological field such as nuclear reactors, heat exchanges, electronic cooling equipment and aeronautics among others.

[1] studied the unsteady free convection flow near a moving infinite flat plate in a rotating mixture of an incompressible fluid. Both soret (thermal diffusion) Dufour (diffusion-thermo) and radiation effects were considered when there was no chemical reaction. They imposed a time dependent perturbation on the constant plate temperature and concentration and assuming a differential approximation for the radioactive flux, the coupled non-linear problem was solved for the temperature and the concentration.

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Further, a critical value for the soret and radiation was determined as 0.1 and the effect of Dufour soret and radiation was shown while both Dufour and soret have no effect on the temperature field. They both affect the concentration field with Dufour causing an overwhelming increase and soret just a slight decrease further, while radiation decreases both the temperature and concentration fields. [2] discussed the effects of a transverse magnetic field on transient natural convection in a vertical channel due to asymmetric heating of channel walls and analytically investigated them. The solutions of the linear system of equations were derived by Laplace transform technique Results were presented for the velocity and temperature profiles and skin friction at hot and cold walls channel. He observed that damping force exerted by the Hartman number played a similar role as the flow resistance offered by the presence of solid matrix in the porous medium. [3] Presented analytical solution of free convective and mass transfer flow in a vertical channel by two vertical parallel plates. A fully developed laminar flow was considered with uniform temperature and concentration on the plates. The diffusion –thermo effect was taken into consideration. It was shown that Dufour effect on the flow results in an anomalous phenomenon in temperature and velocity distribution when D>>Pr. [4] considered the effect of viscous dissipation on the natural convection flow of incompressible fluid along a uniformly heated sphere with heat generation. And their investigation revealed that increase in heat generation parameter and skin friction coefficient, decreases the rate of heat transfer in terms of nusselt number for any specific value of Prandtl number but an increase value of Pr affects negatively the skin friction coefficient and positively affects the rate of heat transfer in terms of nusselt number for any specific value of heat generation.

Furthermore, [5] presented unsteady free convection flow through a porous vertical flat plate immersed in a porous medium in the presence of magnetic field with radiation, introducing a time dependent suction to the plate .A similarity procedure is adopted by taking a time dependent similarity parameter .In their analysis they considered Darcy-Forchemier model and the corresponding momentum and energy equations were solved numerically for cooling and heating of the plate by employing Nacchyshein-Swigert iteration technique along with the sixty order Runge-Kutta integration Scheme .Non –dimensional velocity and temperature profiles were presented graphically for different values of the parameter entering into the problem. [6] studied the effects of stratified viscous fluid on MHD free convection flow with heat and mass transfer in the presence of radiation and heat source. They showed that velocity increases with the increase in stratification parameter as well as the skin friction. [7] discussed unsteady natural convection flow of a viscous dissipative fluid along a semi-infinite vertical plate subjected to periodic surface temperature oscillation. An electrical network model based on the network simulation method was developed to solve the governing equations. The accuracy and effectiveness of the method was demonstrated. The increasing of the viscous dissipation and the decreasing in the prandtl number lead to a decrease in Nusselt number and an increase in the local skin friction [8] investigated both impacts of electrical as well as magnetic strength with viscous dissipation for homogeneous chemical processing with higher order over a stretching porous sheet in the governing flow. The numerical results have been reported for the geometrical model under the physical significance of dimensionless numbers Hartmann, porous parameter, electric parameter, mixed convection, thermophoresis factor, random motion of Brownian, Prandtl, Schmidt, Eckert number, also chemical reaction parameter. The physical significance of these parameters are given numerically by using R-K-F method and presented graphically. [9] carried out an investigation to exhibit the duality in the solution for the MHD hybrid nano fluid flow due to the porous shrinking sheet with thermal radiation and viscous dissipation. The physical flow problem was modelled into system of partial differential equations. These governing equations were converted into nonlinear ODE and the dual solutions were obtained using the hypergeometric function. Recently, [10] investigated analytical study of steady incompressible viscous two-dimension, couple stress boundary layer flow for hybrid nanofluids with the influence of viscous dissipation and thermal radiation. The three types of nanoparticles were considered and engine oil was taken as base fluid. Proper transformations were used to change a set of PDEs into nonlinear ODEs. The authors solved this set of equations using the homotopy analysis approach (HAM). The results were planned with the aid of graphs involving the magnetic field, nanoparticle volume concentration, couple stress parameter, Eckert number, thermal radiation parameter, and Prandtl number. The influence of various temperature and velocity parameters was intended. The structures of flow features, such as temperature and velocity profiles, were simulated and evaluated using a physical description in response to changes in developing factors.

Therefore, captivated and motivated by the usages of these researches above, the present study will be devoted to study Viscous dissipation effects on free convective temperature dependent non-reacting viscous flows on porous plate in a presence of constant magnetic field. Also, to best of my knowledge no one has carried out a study like this. Hence, there is need to undergo this study. The novelty of this work lying on the use of asymptotic technique in decoupling the non-linear PDEs before solving them numerically. Generally, in the literature when solving problem like this that involves infinite boundary condition an arbitrary number is always assigned for infinity values but in this work a transformation is used to transform infinite domain to a finite domain before proceeding to solve the problem.

2. Mathematical Formulation.

Considered an unsteady two – dimensional free convection flow, the coordinate origin at an arbitrary point on an infinite. Porous limiting vertical plate or wall. The x-axis is along the plate in the upward direction and the y-axis normal towards it. The fluid is viscous and incompressible. The flow is induced either by the motion of the plate or by heating it or by both. The plate initially at rest and with a constant temperature T_w is suddenly moved with the velocity $u_0 f(t^{'})$ in its own plane along the x'-axis and its temperature is instantaneously increased (or decreased) by the quantity $\left(T_w^{'} - T_\infty^{'}\right)$ g(t') for $t^{'}$ >0;with u_0 along a constant velocity $T_w (\neq T_\infty)$ a constant temperature for the plate, f(t') and g(t') two arbitrary functions of non-dimensional time t. The flow geometry is shown in fig. 1 below

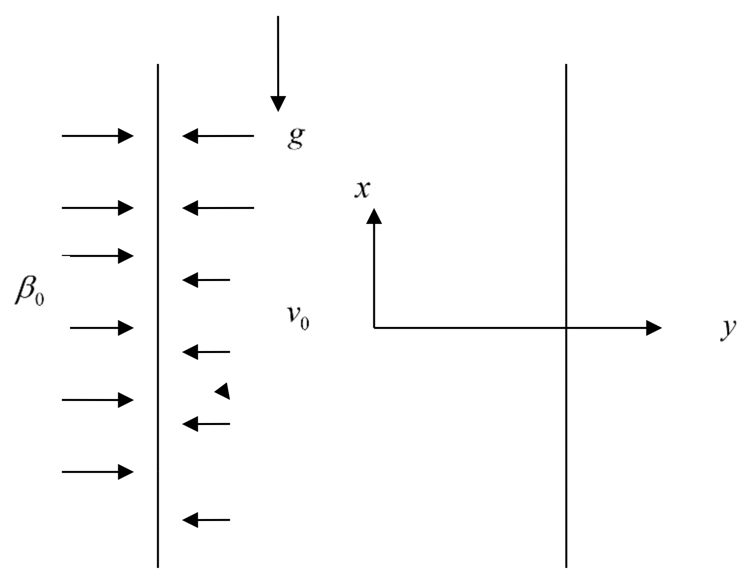

Fig.1: Configuration of the problem

Also, an external magnetic field β_0 Is applied in the positive y'-direction . By assuming a very small magnetic Reynolds number the induced magnetic field is neglected and assumed that the fluid is non-reacting one and there is heat generation due to viscous dissipation under this assumption the governing equations are:

The equation of continuity on integrating becomes

$$v' = cons\tan t = v_0' \qquad (2.1)$$

Where v_0' is the normal velocity of suction or injection at the wall according as $v_0'<0$ or $v_0'>0$ respectively $v_0'=0$ represents the case of non-permeable wall .The remaining basic equations of motion and energy for these problems are :

$$\frac{\partial u'}{\partial t'} + v_0'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta'\left(T - T_\infty\right) - \frac{\sigma\beta_0'}{\rho}.u' \qquad (2.2)$$

$$\frac{\partial T'}{\partial t'} + v_0'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_P}\frac{\partial^2 T'}{\partial y'^2} + \frac{\sigma\beta_0' u'^2}{\rho c_P} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \qquad (2.3)$$

Assuming that no slipping occurs between the plate and the fluid the initial and boundary conditions for equations (2.1) -(2.3) are

$$u'(y',t') = 0 \quad and \quad T'(y',t') = T_w' \qquad y' \geq 0 \ and \ t' \leq 0 \qquad (2.4)$$

$$u'(0,t') = \left[u_0 f(t'),0,0\right] \ and \ T'(0,t') = T'_\infty + (T_w' - T_\infty')g(t') \ \ for, \ t' > 0 \qquad (2.5)$$

$$u'(\infty,t') \to 0 \ \ and \ T'(\infty,t') \to T_\infty \ \ for, t' \geq 0 \qquad (2.6)$$

We now introduce the following non-dimensional quantities into equations (2.1) - (2.3)

$$\bar{y} = \frac{y' u_0}{\nu}, \bar{t} = \frac{t' u_0^2}{\nu}, \bar{u} = \frac{u'}{u_0}, \bar{v} = \frac{v_0'}{u_0}, \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'} \qquad (2.7)$$

Now, substituting (2.7) into (2.2) - (2.6) simplify and neglecting the bar symbol for clarity, the dimensionless equations become

$$\frac{\partial u}{\partial t} + v_0\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G\theta - H_a^2 u \qquad (2.8)$$

$$\frac{\partial \theta}{\partial t} + v_0\frac{\partial \theta}{\partial y} = \frac{1}{P}\frac{\partial^2 \theta}{\partial y^2} + H_a^2\, Ecu^2 + Ec\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.9)$$

The corresponding initial and boundary conditions are

$u(y,0)=0 \qquad \theta(y,0)= 0 \qquad$ for $y \geq 0$

$u(0,t) = f(t) \qquad \theta(0,t)= g(t) \qquad$ for $t > 0 \qquad (2.10)$

$u(\infty,t) \to 0 \qquad \theta(\infty,t) \to 0 \qquad \text{for } t>0$

where the dimensionless parameters are defined in the nomenclature.

We now consider the asymptotic expansions of temperature(θ) and velocity (u) in Ha as

$$\begin{aligned} u &= u_0 + H_a u_1 + H_a^2 u_2 + \\ \theta &= \theta_0 + \theta_1 H_a + H_a^2 \theta_2 + \end{aligned} \cdots \qquad (2.11)$$

Substituting (2.11) into equations (2.8) - (2.10) and simplify to have

H_a^0

$$\frac{\partial u_0}{\partial t} + v_2 \frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} + G\theta_0 \qquad (2.12)$$

u_0(y,0)=0 u_0(0,t)=f(t) u_0(∞,t)=0

H_a^1

$$\frac{\partial u_1}{\partial t} + v_0 \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + G\theta_1 \qquad (2.13)$$

u_1(y,0)=0 u_1(0,t)=0 u_1(∞,t)=0

H_a^2

$$\frac{\partial u_2}{\partial t} + v_0 \frac{\partial u_2}{\partial y} = \frac{\partial^2 u_2}{\partial y^2} + G\theta_2 - u_0 \qquad (2.14)$$

$$u_2(y,0)=0, \quad u_2(0,t)=0, \quad u_2(\infty,t)=0 \qquad (2.15)$$

H_a^0

$$P\frac{\partial \theta_0}{\partial t} + v_0 P\frac{\partial \theta_0}{\partial y} = \frac{\partial^2 \theta_0}{\partial y^2} + PEc\left(\frac{\partial u_0}{\partial y}\right)^2 \qquad (2.16)$$

$$\theta_0(y,0)=0, \quad \theta_0(0,t)=g(t), \quad \theta_0(\infty,t)=0$$

H_a^1

$$P\frac{\partial \theta_1}{\partial t} + v_0 P\frac{\partial \theta_1}{\partial y} = \frac{\partial^2 \theta_1}{\partial y} + PEc\left(\frac{\partial u_1}{\partial y}\right)^2 \qquad (2.17)$$

$$\theta_1(y,0)=0, \quad \theta_1(0,t)=0, \quad \theta_1(\infty,t)=0$$

H_a^2

$$p\frac{\partial\theta_2}{\partial t}+v_0 p\frac{\partial\theta_2}{\partial y}=\frac{\partial^2\theta_2}{\partial y^2}+pEcu_0{}^2+PEc\left(\frac{\partial u_2}{\partial y}\right)^2 \quad (2.18)$$

$\theta_2(y,0)=0, \quad \theta_2(0,t)=0, \quad \theta_2(\infty,t)=0$

We carry out one more transformation from the infinite domain to finite domain using

$$x=e^{-y}; \quad (2.19)$$

$$\frac{\partial u_0}{\partial t}=x^2\frac{\partial^2 u_0}{\partial x^2}+(v_0+1)x\frac{\partial u_0}{\partial x}+G\theta_0 \quad (2.20)$$

$u_0(x,0)=0, \quad u_0(0,t)=0\ , \quad u_0(1,t)=f(t)$

$$\frac{\partial u_1}{\partial t}=x^2\frac{\partial^2 u_1}{\partial x^2}+(v_0+1)x\frac{\partial u_1}{\partial x}+G\theta_1 \quad (2.21)$$

$u_1(x,0)=0, \quad u_1(0,t)=0, \quad u_1(1,t)=0$

$$\frac{\partial u_2}{\partial t}=x^2\frac{\partial^2 u_2}{\partial x^2}+(v_0+1)x\frac{\partial u_2}{\partial x}+G\theta_2-u_0 \quad (2.22)$$

$u_2(x,0),=0, \quad u_2(0,t)=0, \quad u_2(1,t)=0$

$$\frac{\partial\theta_0}{\partial t}=\frac{x^2}{p}\frac{\partial^2\theta_0}{\partial x^2}+\frac{(v_0 p+1)}{p}x\frac{\partial\theta_0}{\partial x} \quad (2.23)$$

θ_0 (x,0)=0 $\quad \theta_0$ (0,t)=0 $\quad \theta_0$ (1,t)=g(t)

$$\frac{\partial\theta_1}{\partial t}=\frac{x^2}{p}\frac{\partial^2\theta_1}{\partial x^2}+\frac{(v_0 p+1)}{p}x\frac{\partial\theta_1}{\partial x} \quad (2.24)$$

θ_1 (x,0)=0 $\quad \theta_1$ (0,t)=0 $\quad \theta_1$ (1,t)=0

$$\frac{\partial\theta_2}{\partial t}=\frac{x^2}{p}\frac{\partial^2\theta_2}{\partial x^2}+\frac{(v_0 p+1)}{p}x\frac{\partial\theta_2}{\partial x}+Ecu_0{}^2 \quad (2.25)$$

θ_2 (x,0)=0 $\quad \theta_2$ (0,t)=0 $\quad \theta_2$ (1,t)=0

Numerical solution

Getting a closed form solution for equations (2.20) - (2.25) with the boundary conditions are not possible. Consequently, we resolved the problems numerically by finite difference method. It is this method because it is very easy to apply to problem like this without any complication. The discretization of this method is shown in equation (2.26)

Assume g(t) = f(t) = 1.

We define
$$\left.\begin{aligned}
&\frac{\partial\theta_0}{\partial t} = \frac{\theta_{0i,j+1}-\theta_{0ij}}{k}, \frac{\partial\theta_0}{\partial x} = \frac{\theta_{0i+1,j}-\theta_{0ij}}{2h}, \frac{\partial^2\theta_0}{\partial x^2} = \frac{\theta_{0i+1,j}-2\theta_{0ij}+\theta_{0i-1,j}}{h^2}\\
&\frac{\partial u_0}{\partial t} = \frac{u_{0i,j+1}-u_{0ij}}{k}, \frac{\partial u_0}{\partial x} = \frac{u_{0i+1,j}-u_{0ij}}{2h}, \frac{\partial^2 u_0}{\partial x^2} = \frac{u_{0i+1,j}-2u_{0ij}+u_{0i-1,j}}{h^2},\\
&\frac{\partial\theta_2}{\partial t} = \frac{\theta_{2i,j+1}-\theta_{2ij}}{k}, \frac{\partial\theta_2}{\partial x} = \frac{\theta_{2i+1,j}-\theta_{2ij}}{2h}, \frac{\partial^2\theta_2}{\partial x^2} = \frac{\theta_{2i+1,j}-2\theta_{2ij}+\theta_{2i-1,j}}{h^2},\\
&\frac{\partial u_2}{\partial t} = \frac{u_{2i,j+1}-u_{2ij}}{k}, \frac{\partial u_2}{\partial x} = \frac{u_{2i+1,j}-u_{2ij}}{2h}, \frac{\partial^2 u_2}{\partial x^2} = \frac{u_{2i+1,j}-2u_{2ij}+u_{2i-1,j}}{h^2}
\end{aligned}\right\} \tag{2.26}$$

Substituting equation (2.26) into equations (2.20) - (2.25) and simplify, we have

$$u_{0i,j+1} = u_{0ij} + \frac{(ih)^2 k}{h^2}(u_{0i+1,j} - 2u_{0ij} + u_{0i-1,j}) + \frac{(v_0+1)k(ih)}{2h}(u_{0i+1,j} - u_{0ij}) + kG\theta_{0ij} \tag{2.27}$$

$$u_0(ih,0) = 0 \qquad u_0(0,jk) = 0 \qquad u_0(1,jk) = 1$$

$$u_{2i,j+1} = u_{2ij} + \frac{(ih)^2 k}{h^2}(u_{2i+1,j} - 2u_{2ij} + u_{2i-1,j}) + \frac{(v_0+1)k(ih)}{2h}(u_{2i+1,j} - u_{2ij}) + kG\theta_{2ij} - k\,u_0 \tag{2.28}$$

$$\theta_{0i,j+1} = \theta_{0ij} + \frac{(ih)^2 k}{ph^2}(\theta_{0i+1,j} - 2\theta_{0ij} + \theta_{0i-1,j}) + \frac{(v_0 p+1)k(ih)}{2hp}(\theta_{0i+1,j} - \theta_{0ij}) + \frac{kEc(ih)^2}{4h^2}(u_{0i+1,j} - u_{0ij})^2 \tag{2.29}$$

$$\theta_0(ih,0) = 0 \qquad \theta_0(0,jk) = 0 \qquad \theta_0(1,jk) = 1$$

$$\theta_{2i,j+1} = \theta_{2ij} + \frac{(ih)^2 k}{ph^2}(\theta_{2i+1,j} - 2\theta_{2ij} + \theta_{2i-1,j}) + \frac{(v_0 p+1)k(ih)}{2hp}(\theta_{2i+1,j} - \theta_{2ij}) + \frac{kEc(ih)^2}{4h^2}(u_{2i+1,j} - u_{2ij})^2 + kEcu_{0ij}^2 \tag{2.30}$$

$$\theta_2(ih,0) = 0 \qquad \theta_2(0,jk) = 0 \qquad \theta_2(1,jk) = 0$$

Equations (2.27) -(2.30) are implemented using Pascal Programming Language the results are presented in figures (2) - (8).

Results and Discussion

Numerical solutions are obtained for the problem of viscous dissipation effects on free convective temperature dependent non-reacting viscous flow on a porous plate in a constant magnetic field. Five basic parameters governed the flow namely, the Prandtl number (P), Suction/Injection (v_0) ,Grashof number and Eckert number. A numerical computation is carried out for various values of the parameters that describe the flow characteristics and the results are display in graphs. Figure 1 gives a vivid description of the flow configuration. Figures 2 – 4 show the order zero velocity profiles for different suction velocity. It is observed that increasing the suction velocity cause an increase in order zero velocity profiles. An increase in Eckert number resulted in escalating the order zero temperature profiles shown in figure 5. In Figure 6, we observed that, the order zero velocity increases as Grashof number increases. Figures 7 and 8 illustrate the effect of Eckert number and Prandtl number on second order temperature profiles. It is observed that the temperature increases as both Eckert number and Prandtl number increase and maximum temperature exists within the fluid.

Conclusion

The main purpose of the present study is to show the nature and importance of free convection temperature dependent non reacting viscous flows on a porous plate in presence of constant magnetic field. This was accomplished by first formulating the general problem under reasonable assumptions. Finally, it is observed that increasing the suction velocity and Eckert number cause an increase in both order zero velocity and order zero temperature profiles respectively.

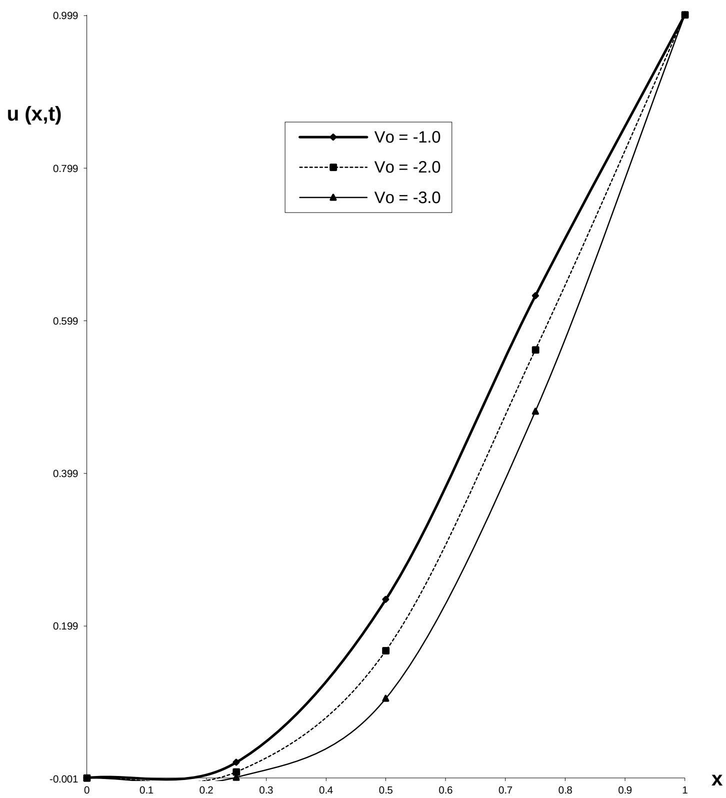

Figure 2: The graph of unsteady velocity distribution u (x,t) against x of a non-reacting flow with viscous dissipation for equation 2.27 when G = 2.0,P = 0.71,Ec = 1.0,t = 0.2 for various Vo

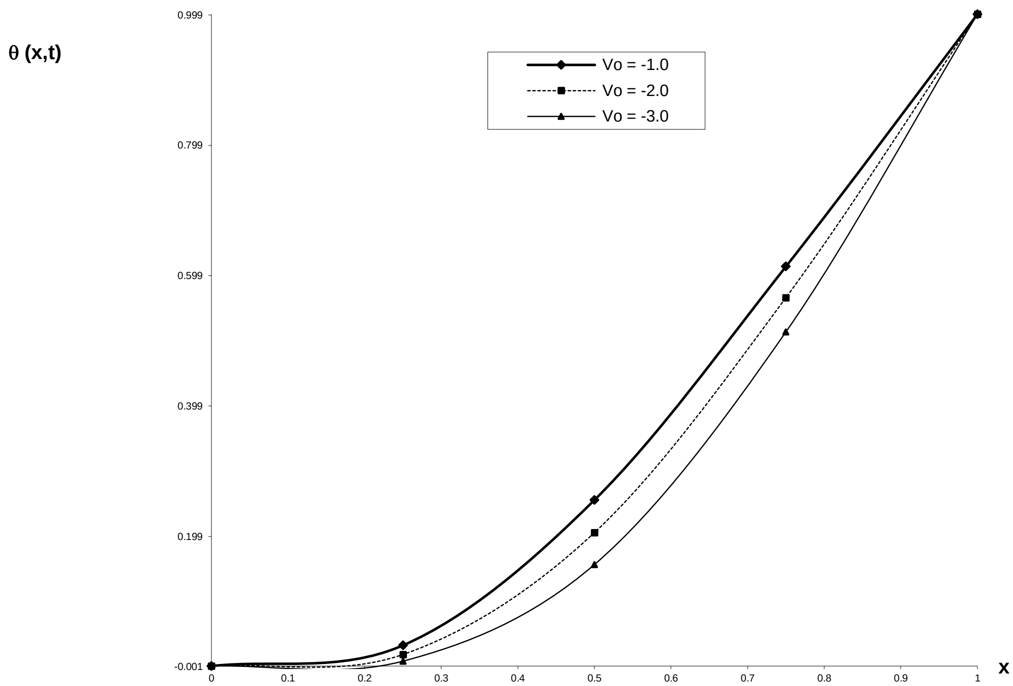

Figure 3: The graph of unsteady temperature distribution q (x,t) against x of a non-reacting flow with viscous dissipation for equation 2.29 when P = 0.71,G = 2.0,Ec = 1.0,t = 0.2 for various Vo

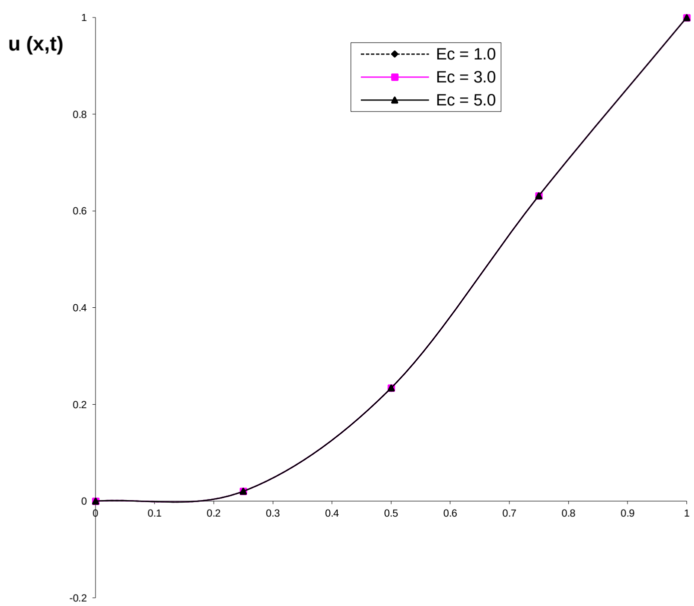

Figure 4: The graph of unsteady velocity distribution u (x,t) against x of a non-reacting flow with viscous dissipation for equation 2.26 when P = 0.71,G = 2.0,Vo = -1.0,t = 0.2 for various Ec

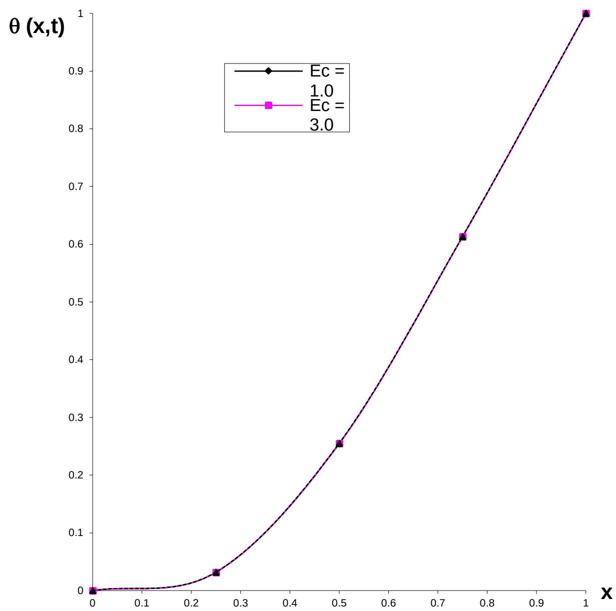

Figure 5: The graph of unsteay temperature distribution q (x,t) against x of a non-reacting flow with viscous dissipation for equation 2.29 when P = 0.71,Vo = -1.0,G = 2.0,t = 0.2 for various Ec

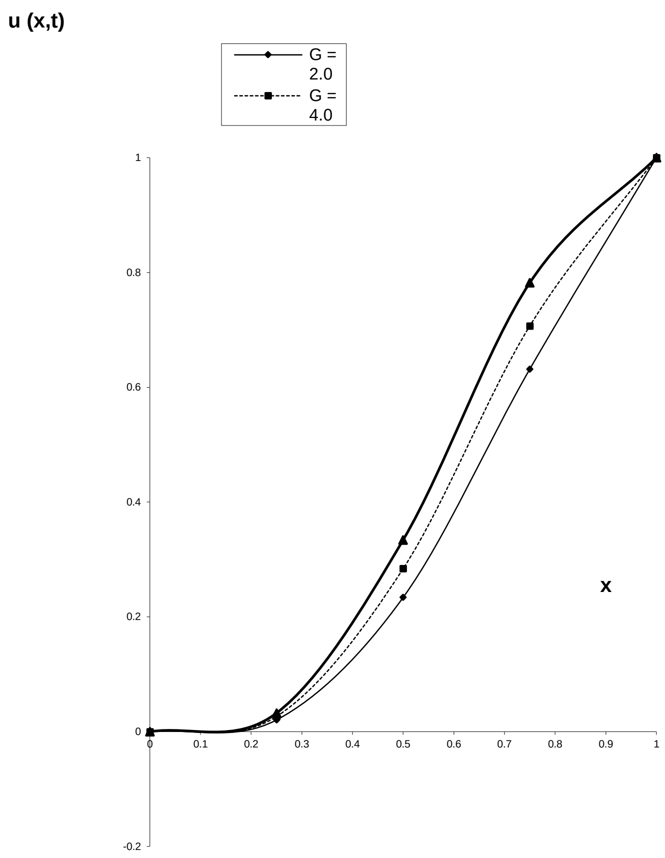

Figure 6:The graph of unsteady velocity distribution u (x,t) against x of a non-reacting flow with viscous dissipation for equation 2.28 when p = 0.71, Vo = -1.0,Ec = 1.0,t = 0.2 for various G

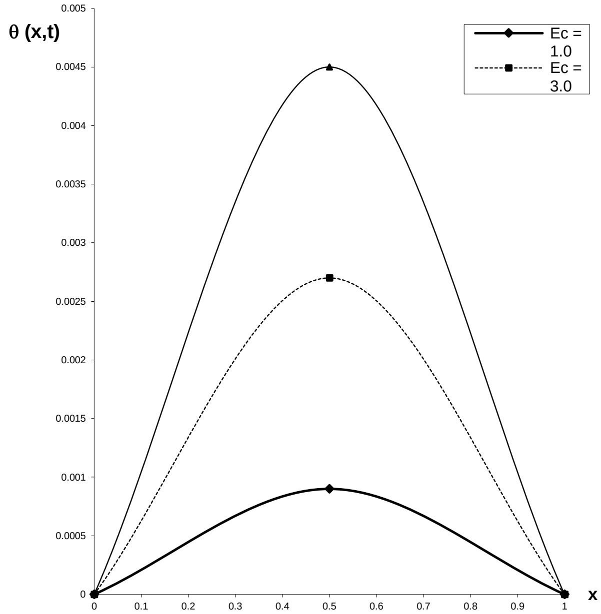

Figure 7: The graph of unsteady temperature distribution q (x,t) against x of a non-reacting viscous flow with viscous dissipation for equation 2. 30 when P = 1.0,G = 1.0,Vo = -1.0,t = 0.06 for various Ec

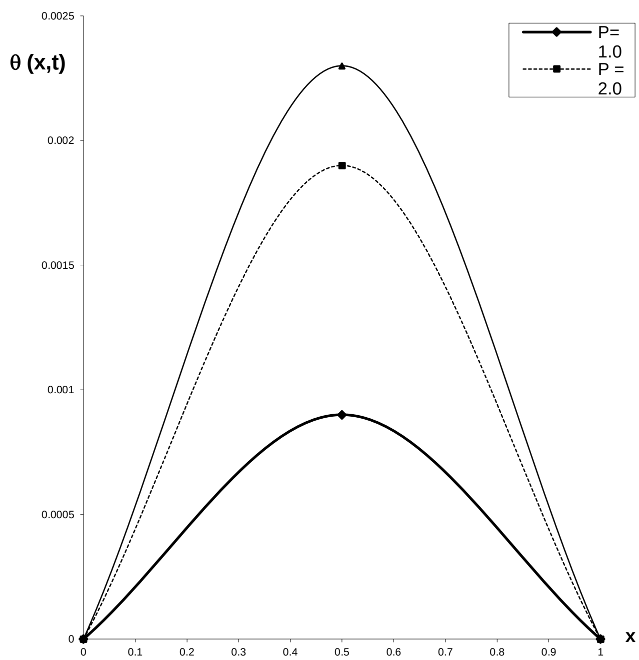

Figure 8: The graph of unsteady temperature distribution q (x,t) against x of a non-reacting flow with viscous dissipation for equation 2.30when Vo = -1.0,G = 1.0,Ec = 1.0,t = 0.06 for various P

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