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ANALYTICAL INVESTIGATION OF HEAT GENERATING/ABSORBING FLUID ON MHD FULLY DEVELOPED STEADY NATURAL CONVECTION FLOW IN A VERTICAL MICRO-CHANNEL WITH *SYMMETRIC* AND ASYMMETRIC HEATING

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ABSTRACT

This works investigates analytically the influence of heat generating and absorbing fluid behaviours on MHD steady natural convection flow in a vertical parallel plate micro-channel in presence of velocity slip and temperature jump conditions. Flow is assumed to be steady, laminar and fully developed. Exact solution of momentum and energy equations is derived separately for heat generating fluid and heat absorbing fluid. The effects of various flow parameters entering into the problem such as heat generating/absorbing parameter, Hartmann number, Knudsen number, and fluid - wall interaction parameter on the temperature, velocity, rate of heat transfer and skin frictions are provided and discussed with appropriate physical explanations. Results revealed that, increase in Hartmann number reduced the influence of the velocity, and skin friction. The role of heat generating/absorbing fluid parameter, Hartmann number, rarefaction parameter as well as fluid wall interaction parameter on temperature and velocity is significantly pronounced in the case of asymmetric heating. Furthermore, it is observed that that skin friction and rate of heat transfer at cooler wall increases with increase in heat generation parameter while the reverse trend in the case of heat absorption parameter.

1. Introduction

Microflow has been given great importance in recent research activities due to its new application in microfluidic system devices, such as biomedical sample injection, biochemical cell reaction, and microelectric ship cooling. A fundamental understanding of the flow and thermal fields as well as the corresponding characteristics at microscale, which may deviate from those at macroscale, is required for the technological demands. These highly integrated systems are now widely applied to various applications.

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As a basic element in MEMS/NEMS devices, microchannel is often found to be used for integrated cooling or heating in micro-reactor devices. Current applications for such devices include microchannel heatsink, microjet impingement cooling and micro heat pipe [1]. The key quantity in microchannel analysis is Knudsen number (Kn), defined as the ratio of molecular mean free path to the characteristic length $(Kn = \lambda/b)$. Knudsen number is very small for continuum flows. However, for microscale gas flows where the gas mean free path become comparable with the characteristic dimension of the duct, the Knudsen number may be greater than 10^{-3}. Microchannel with characteristic lengths on the order of $100\mu m$ would produce flows inside the slip regime for gas with a typical mean free path of approximately $100nm$ at standard condition. Many researchers have studied the flow and heat transfer characteristics in microchannels. Chen and Weng [2] investigated the fully developed natural convection in a vertical parallel plate micro-channel. Their result reveal that the effects of rarefaction and fluid-wall interaction increase the volume flow rate while it decrease the rate of heat transfer. This result is further extended by taking into account suction/injection on the micro-channel walls by Jha *et al.* [3]. They concluded that skin-friction as well as rate of heat transfer is strongly dependent on suction/injection parameter. The work of Jha *et al.* [4] extends the work of Chen and Weng [2] by considering the influence of externally applied transverse magnetic field on steady natural convection flow of conducting fluid in a vertical microchannel. Some recent works related to the present investigation are found in the literature [4-10].

The study of heat generation/absorption effects in moving fluids is importance in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids. A lot of interests have been built in the study of flow of heat generating/absorbing fluid because as the temperature differences are increased appreciably, the volumetric heat generation/absorption term may employ strong influence on the heat transfer and transitively on the flow [11]. Internal heat generation/absorption plays significant role in various physical phenomena such as convection in earth's mantle [12], application in the field of nuclear energy [13], post-accident heat removal [14], fire and combustion modelling [15], and the development of metal waster from spent nuclear fuel [15]. Chamkha [16] considered non-Darcy fully developed mixed convection flow in a channel embedded in a porous medium in the presence of heat generation/absorption and hydromagnetic effects.

On the other hand, different attributes have been accorded internal heat generation/absorption: for instance, it was assumed to be constant in the study conducted by Inman [17], Ostrach [18], but considered as a function of space by Low [19], Chambre [20] and Toor [21]. In the works of Gee and Lyon [22], Modejski [23] and Toor [24], heat generation is taken to be frictional heating and expansion effects of the working fluid, while Moalem [25] presented heat generation as an inversely proportional value to $(a + bT)$. In addition, Foraboschi and Federico [26] presented the volumetric rate of heat generation/absorption which is directly proportional to $(T - T_0)$ and explained that it is an approximation of the state of some exothermic process with T_0 as the initial temperature.

Despite the fact that there are numerous studies on natural convection flow in channel in the presence of heat generating/absorbing fluids, there are just a few studies regarding natural convection flow in micro-geometry. For instance, Jha *et al.* [27] obtained an exact solution of the steady fully developed mixed convection flow in a vertical micro-concentric-annulus with heat-generating/absorbing fluid, and concluded that the reverse flow formation could be controlled by selecting suitable value of the Knudsen number and heat generation/absorption parameter. Jha and Aina [28] investigated the impact of heat generation/absorption on MHD mixed convection flow in a vertical tube having time periodic boundary condition: steady periodic regime. Fully developed steady natural convection flow of heat generating/absorbing fluid in a vertical annular microchannel was discussed by Jha and Aina [28].

The present study combines the effect of heat generation/absorption, externally applied transverse magnetic field on the steady natural convection flow of conducting fluid in a vertical microchannel with symmetric and asymmetric heating. After an exhaustive literature survey, we found that no prior work has dealt with the problem considered here. The main goal in this paper is to further extend the work of Jha *et al.* [4] by considering heat generation/absorption fluid. The characteristics of various parameters of interest are studied for velocity, temperature, skin friction and rate of heat transfer.

Mathematical Analysis

Convection flow in a vertical micro-channel formed by two infinitely vertical parallel plates separated by a distance b under the effects of heat generating/absorbing fluids and MHD is carried out as illustrated in Figure 1. We choose a Cartesian coordinate system with $x-$axis along the plate in the vertically upward direction while the $y-$axis is orthogonal to the vertical parallel plates as presented in Figure 1. The plates are heated asymmetrically with one plate maintained at a temperature T_1 where the other plate at temperature T_2, where $T_1 > T_2$.

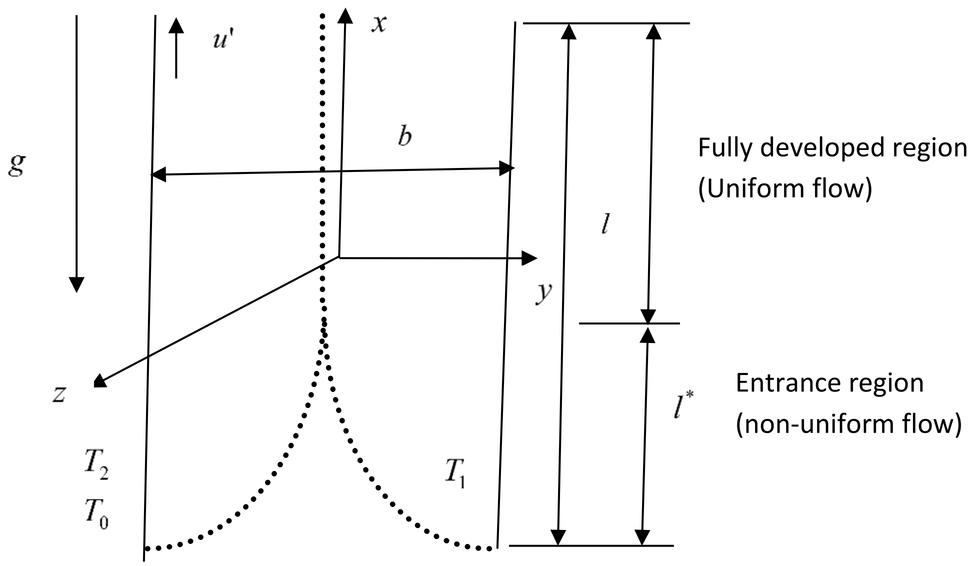

Figure 1: Schematic diagram

A uniform magnetic field B_0 is passed across the micro-channel normal to the plates and a viscous conducting fluid rises in the micro-channel driven by buoyancy forces and retarded by magnetic forces. Following Jha *et al.* [4] and considering the influence heat generating/absorbing fluid, under the usual Boussineq approximation and following non dimensional quantities,

$$y = \frac{y}{b},\ U = \frac{\nu u}{g\beta b^2(T_1 - T_0)},\ \theta = \frac{T' - T_0}{T_1 - T_0},\ \Pr = \frac{\nu}{\alpha},\quad M = B_0 b\sqrt{\frac{\sigma}{\rho\nu}} \tag{1}$$

where B_0 is cconstant strength of applied magnetic field, β is thermal expansion coefficient, b is gap between the walls, T is the temperature, M is Hartmann number, T_0 is the free stream temperature, g is the acceleration due to gravity, ν is the kinematic viscosity of the fluid, μ is the dynamic viscosity and k is thermal conductivity.

The governing equations and boundary conditions can be written in the following dimensionless form:

$$\frac{d^2U}{dY^2} - M^2U + \theta = 0 \tag{2}$$

$$\frac{d^2\theta}{dY^2} \pm S^2\theta = 0 \tag{3}$$

with the boundary conditions in non dimensional form as

$$U(Y) = \beta_v Kn \frac{dU}{dY}, \qquad \theta(Y) = \xi + \beta_v Kn \ln \frac{d\theta}{dY} \qquad \text{at } Y = 0 \tag{4}$$

$$U(Y) = -\beta_v Kn \frac{dU}{dY}, \qquad \theta(Y) = 1 - \beta_v Kn \ln \frac{d\theta}{dY} \qquad \text{at } Y = 1$$

where:

$$\beta_v = \frac{2-\sigma_v}{\sigma_v},\ \beta_t = \frac{2-\sigma_t}{\sigma_t}\frac{2\gamma_s}{\gamma_s + 1}\frac{1}{\Pr},\ Kn = \frac{\lambda}{b},\ \ln = \frac{\beta_t}{\beta_v},\ \xi = \frac{T_2 - T_0}{T_1 - T_0}$$

Referring to the values of σ_v and σ_t given in Eckert and Drake [29] and Goniak and Duffa [30], the value of β_v is near unity, and the value of β_t ranges from near 1 to more than 100 for actual wall surface conditions and is near 1.667 for many engineering applications, corresponding to $\sigma_v = 1$, $\sigma_t = 1$, $\gamma_s = 1.4$ and $\Pr = 0.71$ $(\beta_v = 1, \beta_t = 1.667)$.

The physical quantities used in the above equations are defined in the nomenclature.

METHOD OF SOLUTION

It should be mentioned that the form of the analytical solutions for temperature are different for a heat generating (positive sign in equation (3)) and heat absorbing fluid (negative sign in equation (3)). Closed form solutions are derived for these two cases separately.

CASE 1: HEAT - GENERATING FLUID

For this type of fluid the energy equation (equation (3)) with the positive sign in the second term is a differential equation that has the following closed form solution for temperature:

$$\theta(Y) = A\cos(\delta Y) + B\sin(\delta Y) \tag{5}$$

where A and B are arbitrary constants determined by the boundary condition given in equation (4). Using the boundary conditions, A and B becomes

$$A = \xi + \beta v Kn \ln \delta B \text{ and } B = \frac{1 + \xi\left[\beta v Kn \ln \delta \sin(\delta) - \cos(\delta)\right]}{2\beta v Kn \ln \delta \cos(\delta) - (\beta v Kn \ln)^2 \delta^2 \sin(\delta) + \sin(\delta)} \tag{6}$$

with the solution for temperature already determined, equation (2) can be solved for velocity (U), subjected to the boundary conditions given in equation (4). The exact solution of equation (2) under the appropriate velocity slip condition defined in equation (4) is

$$U(Y) = C\exp(MY) + D\exp(-MY) + \frac{1}{\delta^2 + M^2}\left[A\cos(\delta Y) + B\sin(\delta Y)\right] \tag{7}$$

where:

$C = \frac{F_9 F_{11} - F_8 F_{12}}{F_7 F_{11} - F_8 F_{10}}$ and $D = \frac{F_9 F_{10} - F_7 F_{12}}{F_8 F_{10} - F_7 F_{11}}$

It should be mentioned that in the absence of the fluid heat generation effect, equations (5) –(7) are consistent with those reported by Jha [4].

Skin friction

$$\left.\frac{dU}{dY}\right|_{Y=0} = CM - DM + F_3 \tag{8}$$

$$\left.\frac{dU}{dY}\right|_{Y=1} = CF_4 - DF_5 + F_6 \tag{9}$$

Rate of heat transfer (Nusselt Number)

$$\left.\frac{d\theta}{dY}\right|_{Y=0} = B\delta \tag{10}$$

$$\left.\frac{d\theta}{dY}\right|_{Y=1} = B\delta\cos(\delta) - A\delta\sin(\delta) \tag{11}$$

CASE II: HEAT – ABSORBING FLUID

For this type of fluid, the energy equation (equation (3)) with the negative sign in the second term is a differential equation that has the following closed form solution for temperature:

$$\theta(Y) = E\exp(\delta Y) + F\exp(-\delta Y) \tag{12}$$

where E and F are arbitrary constants determined by the boundary condition given in equation (4). Using the boundary conditions, E and F becomes

$E = \frac{\xi F_{16} - F_{14}}{F_{13} F_{16} - F_{14} F_{15}}$ and $F = \frac{\xi F_{15} - F_{13}}{F_{14} F_{15} - F_{13} F_{16}}$

with the solution for temperature already determined, equation (2) can be solved for velocity (U), subjected to the boundary conditions given in equation (4). The exact solution of equation (2) under the appropriate velocity slip condition defined in equation (4) is

$$U(Y) = G\exp(MY) + H\exp(-MY) - \frac{1}{\delta^2 - M^2}\left[E\exp(\delta Y) + F\exp(-\delta Y)\right] \tag{13}$$

where:

$G = \frac{F_{21} F_{23} - F_8 F_{24}}{F_7 F_{23} - F_8 F_{22}}$ and $H = \frac{F_{21} F_{22} - F_7 F_{24}}{F_8 F_{22} - F_7 F_{23}}$

Skin friction

$$\left.\frac{dU}{dY}\right|_{Y=0} = GM - HM - F_{19} \tag{14}$$

$$\left.\frac{dU}{dY}\right|_{Y=1} = GM\exp(M) - HM\exp(-M) - F_{20} \tag{15}$$

Rate of heat transfer (Nusselt Number)

$$\left.\frac{d\theta}{dY}\right|_{Y=0} = E\delta - F\delta \tag{16}$$

$$\left.\frac{d\theta}{dY}\right|_{Y=1} = E\delta\exp(\delta) - F\delta\exp(-\delta) \tag{17}$$

The dimensionless volume flow rate is:

$$Q = \frac{m}{bU_0} = \int_0^1 U dY$$

Results and Discussion

In this paper, the role of heat generating/absorbing fluids on MHD natural convection flow in a vertical micro-channel with symmetric and asymmetric heating is investigated analytically. The interactive influence of heat generating/absorbing fluids parameter (δ), wall-ambient temperature difference ratio (ξ), Knudsen number $(\beta_v Kn)$, fluid wall interaction parameter $(\ln)$, and Hartmann number (M) on the fluid velocity, temperature, skin friction and rate of heat transfer which is expressed as the Nusselt number are computed. To examine the impact of these controlling parameters, the variation of temperature, velocity, skin friction and rate of heat transfer are presented in Figure 2-14. The present parametric study has been performed in the continuum and slip flow regimes $(Kn \le 0.1)$. The selected reference values of $\beta_v Kn$ and $\ln$ for the present analysis are 0.05 and 1.64 respectively as given in Jha *et al.*[4].

Figure 2 and 3 displays the effects of $\beta_v Kn$ and $\ln$ on temperature profile for different values of wall-ambient temperature difference ratio ($\xi = -1$: one heating and one cooling; $\xi = 0$: one heating and one not heating, $\xi = 1$: both walls are heated), respectively. It can be confirmed that for both heat generating and absorbing fluid, the increase in $\beta_v Kn$ and $\ln$ leads to increase in temperature jump. It is evident that, all the effects of Knudsen number and fluid wall interaction parameter increase with decrease of the wall-ambient temperature difference ratio. Furthermore, except for the case of symmetric heating $(\xi = 1)$, there exist points of inflection inside the micro-channel and the location of point of inflection is strongly depending on wall-ambient temperature difference ratio.

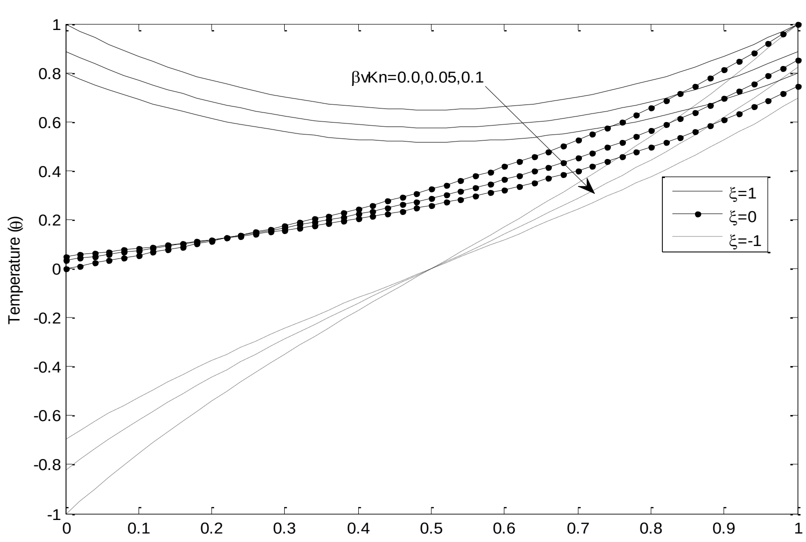

Figure 2: Temperature profile for different values of $\beta_v Kn$ with ln=1.66, δ=2

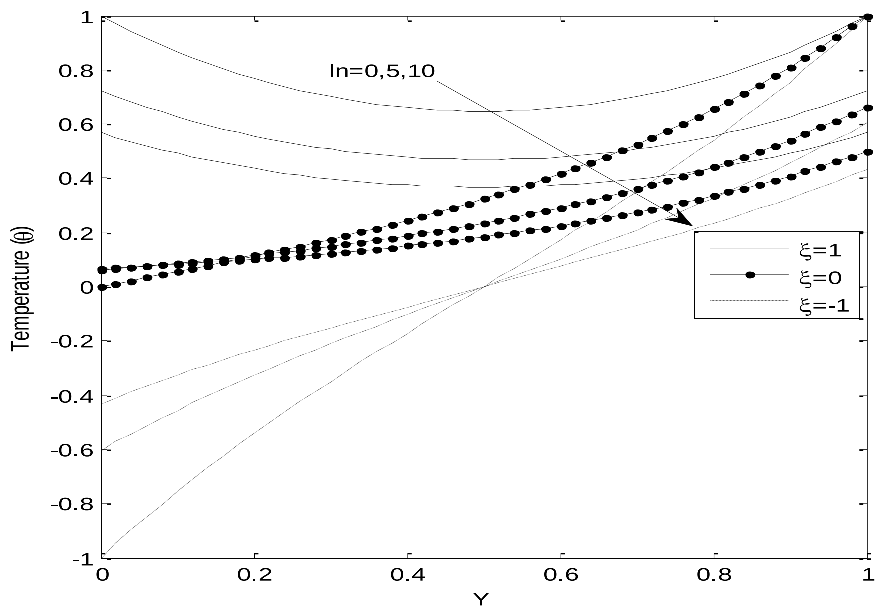

Figure 3: Temperature profile for different value of In with βvKn=0.05, δ=2

Figure 4 and 5 shows the impacts of $\beta_v Kn$ and $\ln$ on velocity profile for different values of wall-ambient temperature difference ratio, respectively. It is exhibited that, increase in $\beta_v Kn$ leads to increase in fluid velocity for both heat generating and absorbing fluid cases. It is found that, the $\ln$ influences the flow for different value of wall-ambient temperature difference ratio. The effect of $\ln$ is observed to horizontally shift the velocity profile to the cooler wall side and also, to reduce the fluid velocity. In addition, as wall-ambient temperature difference ratio increases, the slip induced by the rarefaction effect increase, but the slip induced by fluid wall interaction effect decreases.

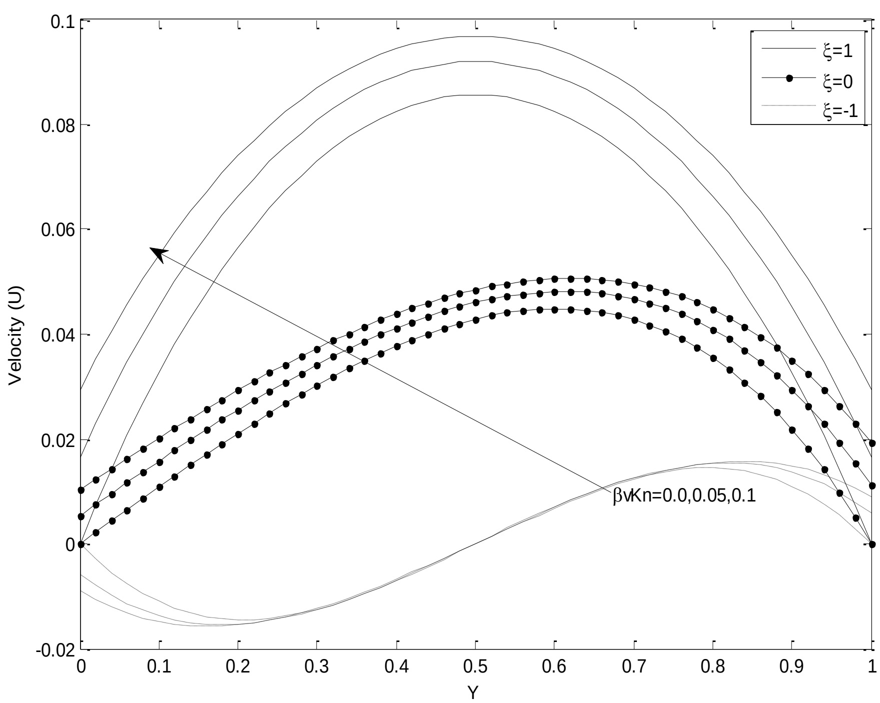

Figure 4: Velocity profile for different value of β_vKn with In=1.667,δ=2, M=0.5

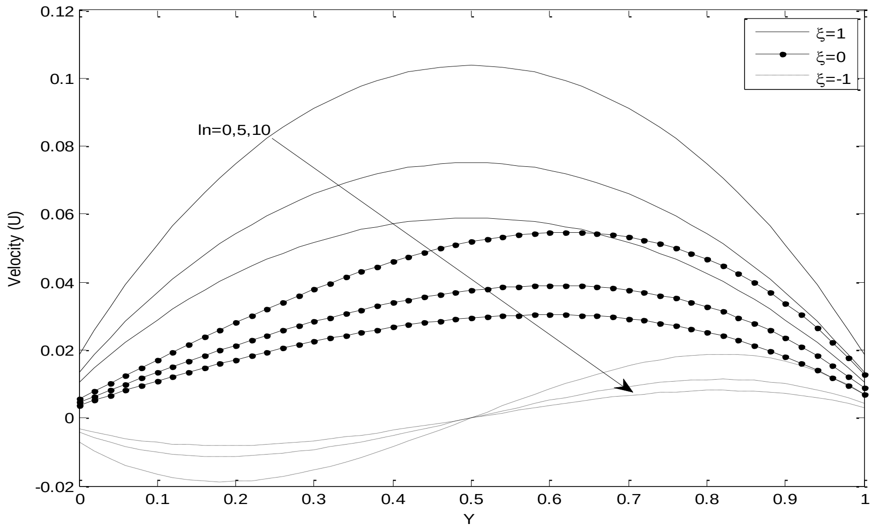

Figure 5: Velocity profile for different vale of In with β_VKn=0.05,δ=2,M=0.5

Figure 6 depict the influence of Hartmann number (M) on velocity profile for different values of wall-ambient temperature difference ratio. As expected, increasing the value of Hartmann number have a tendency to slow down the movement of the fluid in the microchannel for both heat generating and absorbing fluid cases. This is physically truth because, the retarding impact of the magnetic force is strengthened with the increase in Hartmann number. This increase in Hartmann number decreases the thickness of the momentum boundary layer and induces a Lorentz force which opposes the fluid flow.

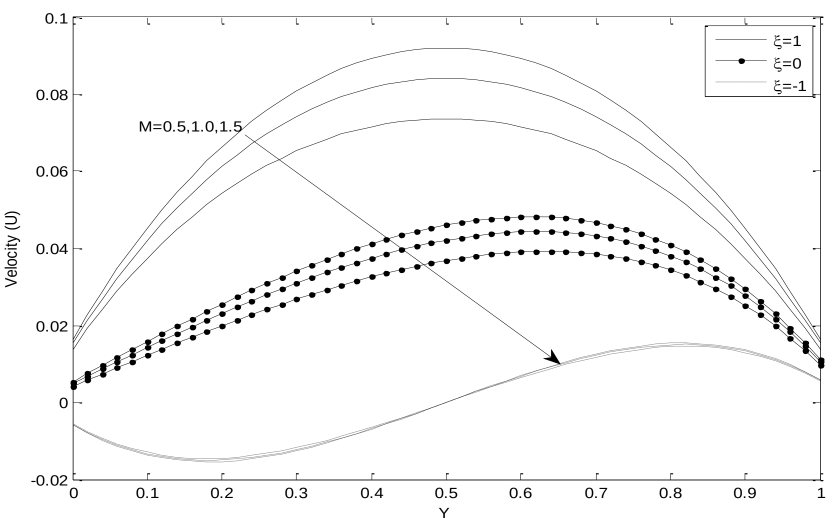

Figure 6: Velocity profile for different value of M with β_VKn=0.05,In=1.667, θ=2

Figures 7(a, b) and 8 (a, b) illustrate the roles of heat generation/absorption parameter (δ) on temperature profile and velocity profile for different values of wall-ambient temperature difference ratio, respectively. It was exhibited that, temperature and velocity is a decreasing function of heat absorption parameter while increasing function of heat generation parameter. The physical reason behind this is that, increasing heat generation/absorption causes the fluid to become warmer and therefore increases its temperature. In addition, the impacts of heat generation/absorption on the temperature and velocity is more important with the increase of wall-ambient temperature difference ratio.

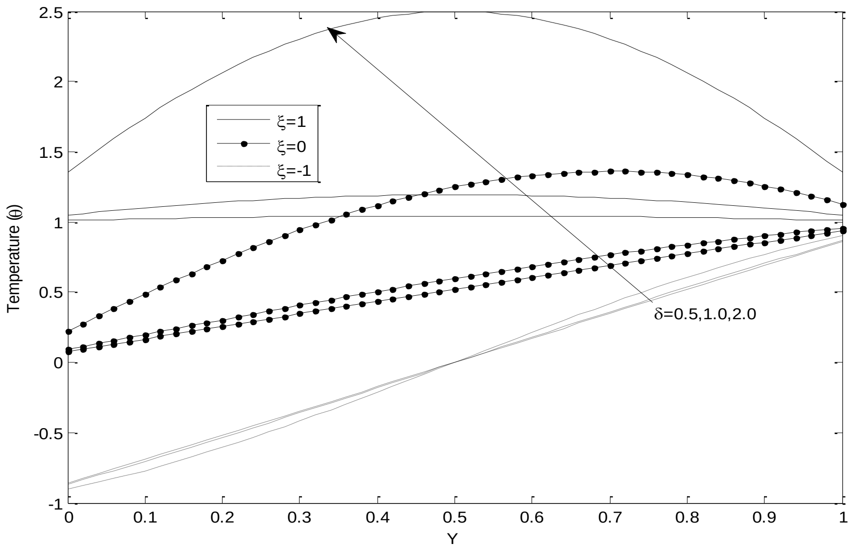

Figure 7a:Temperature profile for different value of δ (Case I) with β_vKn=0.05,ln=1.667

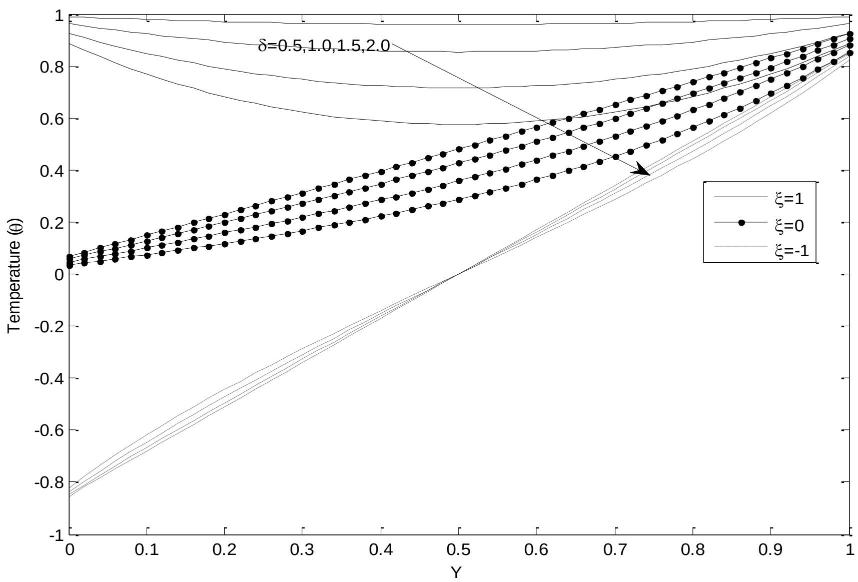

Figure 7b:Temperature profile for different value of δ (Case II) with β_vKn=0.05,ln=1.667

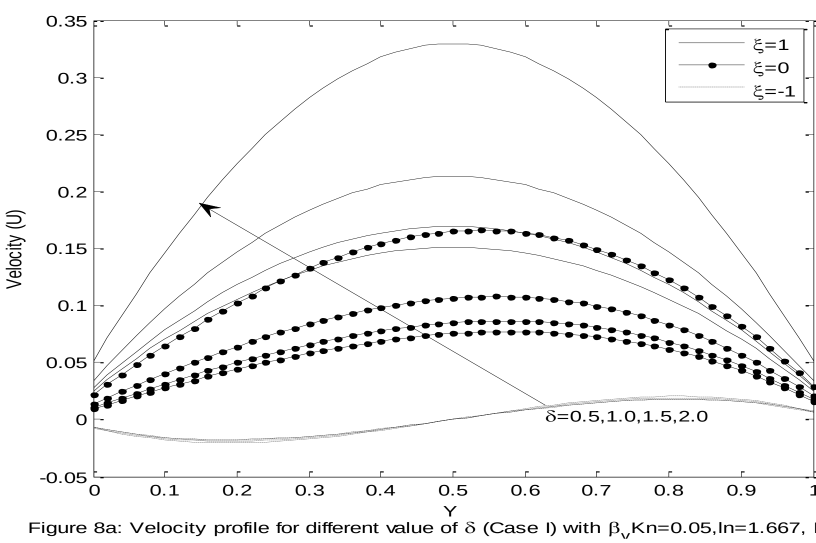

Figure 8a: Velocity profile for different value of δ (Case I) with β_vKn=0.05,ln=1.667, M=0.5

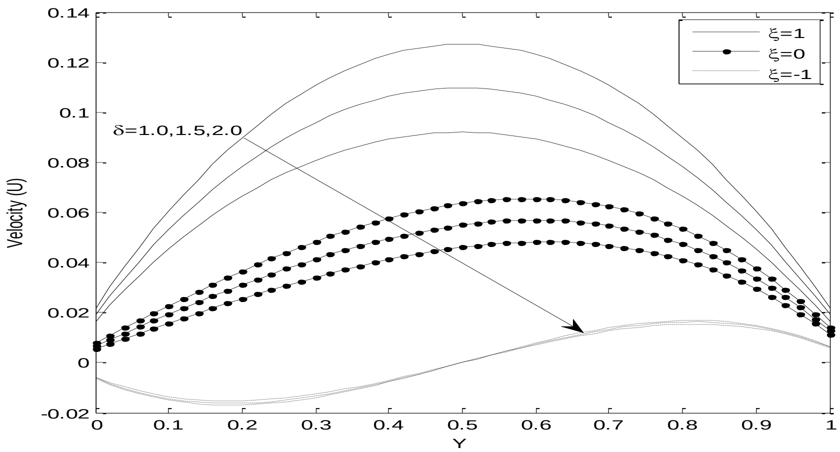

Figure 8b: Velocity profile for different value of δ with β_vKn=0.05,ln=1.667, M=0.5

Figures 9 and 10 show the combined effects of Hartmann number as well as rarefaction on the skin friction at cooler and hotter wall respectively. It is found for different values of wall-ambient temperature difference ratio that, increase in Hartmann number as well as rarefaction leads to decreases in skin friction on both micro-channel walls.

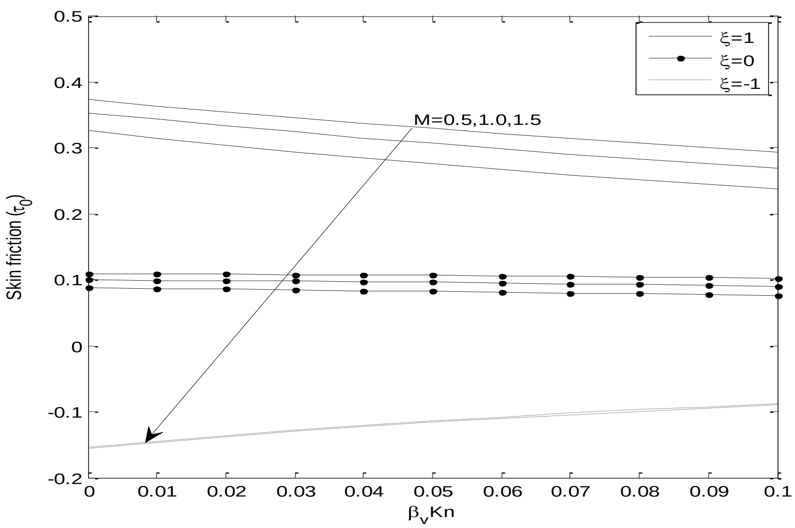

Figure 9: Variation of skin friction for different value of M at Y=0

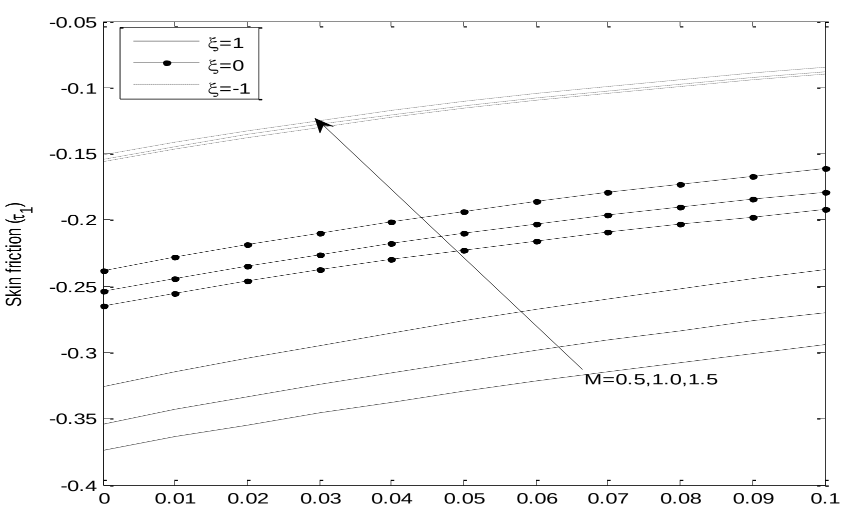

Figure 10: Variation of skin friction for different value of M at Y=1

Figures 11 (a, b) and 12 (a, b) reveal impact of heat generation/absorption parameter (S) and rarefaction parameter $(\beta_v Kn)$ on the skin-friction at microchannel wall $(Y=0)$ and $(Y=1)$, respectively. It is seen that, the skin friction increases with increase in heat generation parameter at microchannel wall $(Y=0)$ and $(Y=1)$ while the reverse trend is seen in the case of heat absorption parameter as shown in 12 (a, b).

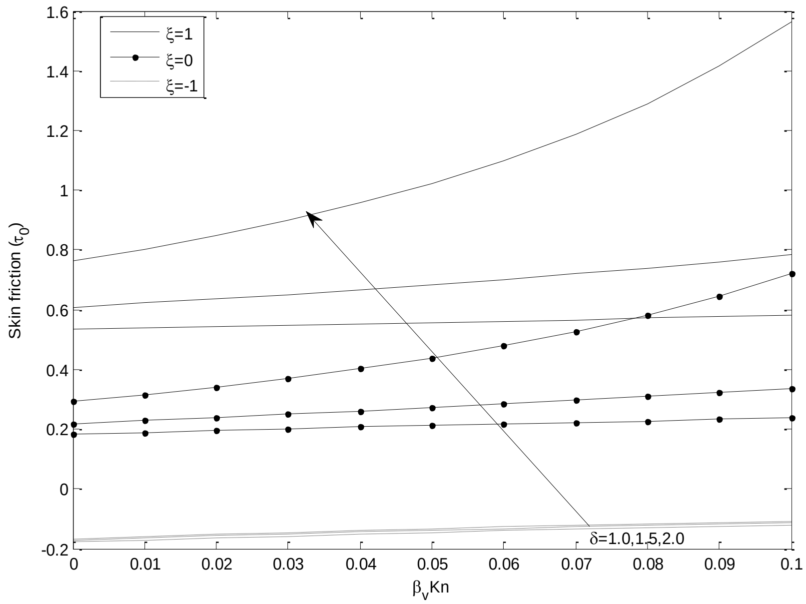

Figure 11a: Variation of skin friction for different value of δ at Y=0 (Case I)

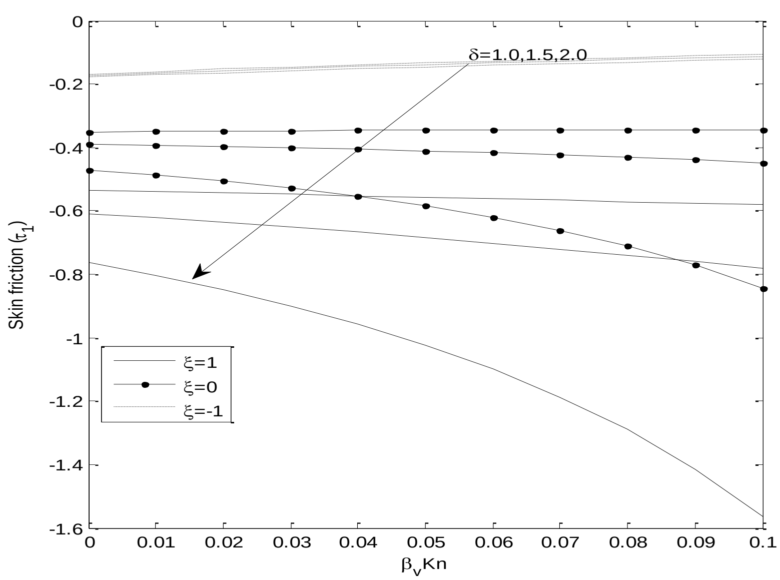

Figure 11b: Variation of skin friction for different value of δ at Y=1 (Case I)

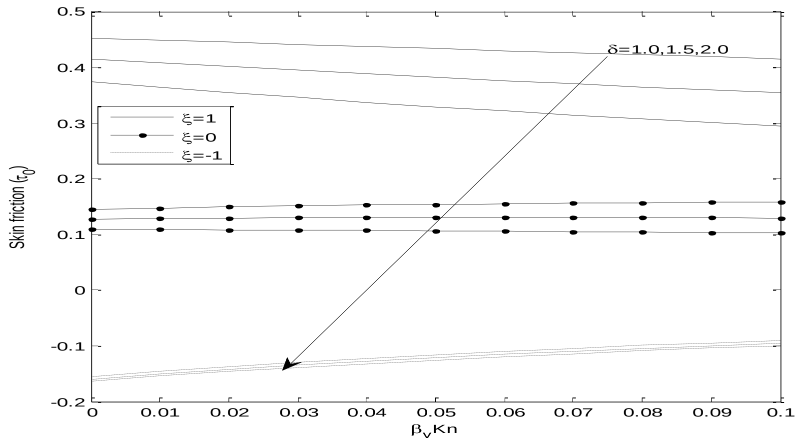

Figure 12a: Variation of skin friction for different value of δ at Y=0 (Case II)

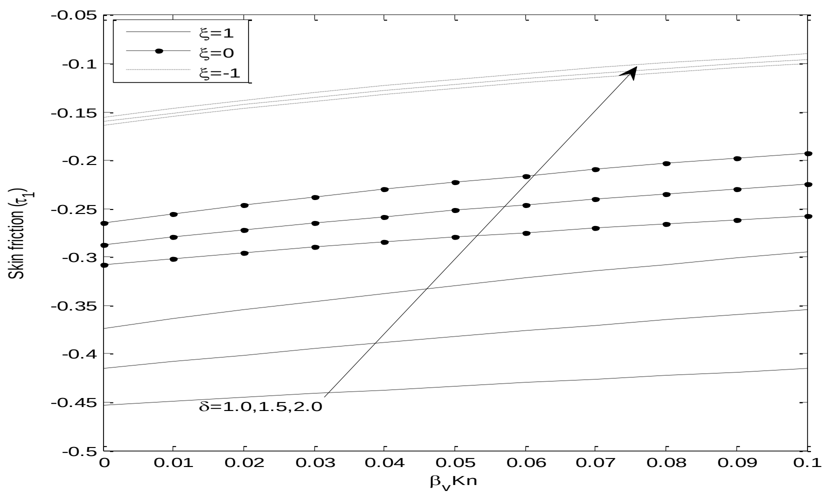

Figure 12b:Variation of skin friction for different value of δ at Y=1 (Case II)

Figures 13 (a, b) and 14 (a, b) shows the impact of heat generation/absorption parameter and rarefaction parameter on the rate of heat transfer at microchannel walls, respectively. It is observed that, the rate of heat transfer increases with increase in heat generation parameter at microchannel walls while the reverse trend is noticed in the case of heat absorption parameter.

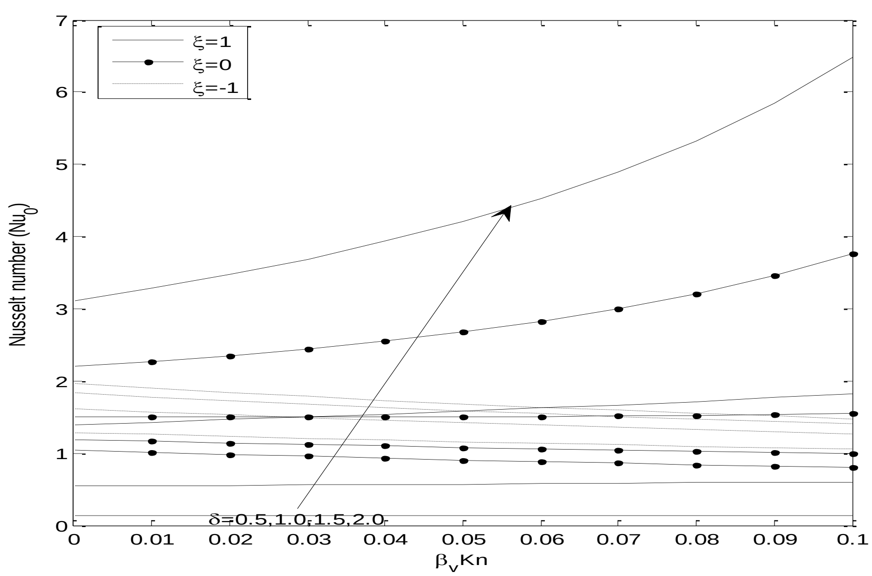

Figure 13a: Variation of rate of heat transfer for different value of δ at Y=0 (Case I)

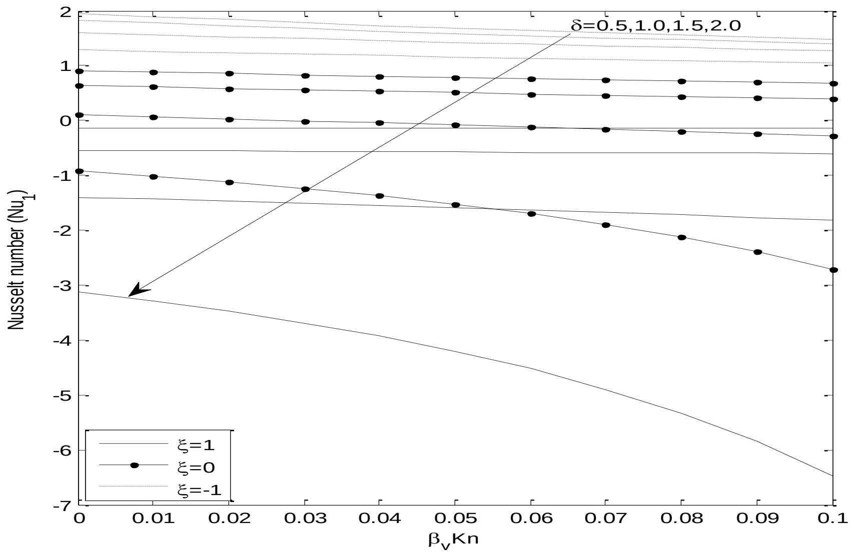

Figure 13b: Variation of rate of heat transfer for different value of δ at Y=1 (Case I)

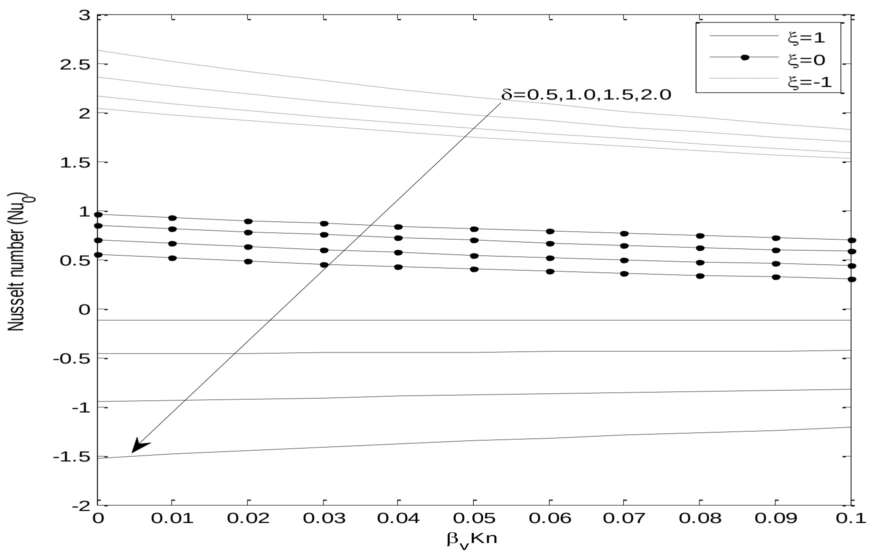

Figure 14a:Variation of rate of heat transfer for different value of δ at Y=0 (Case II)

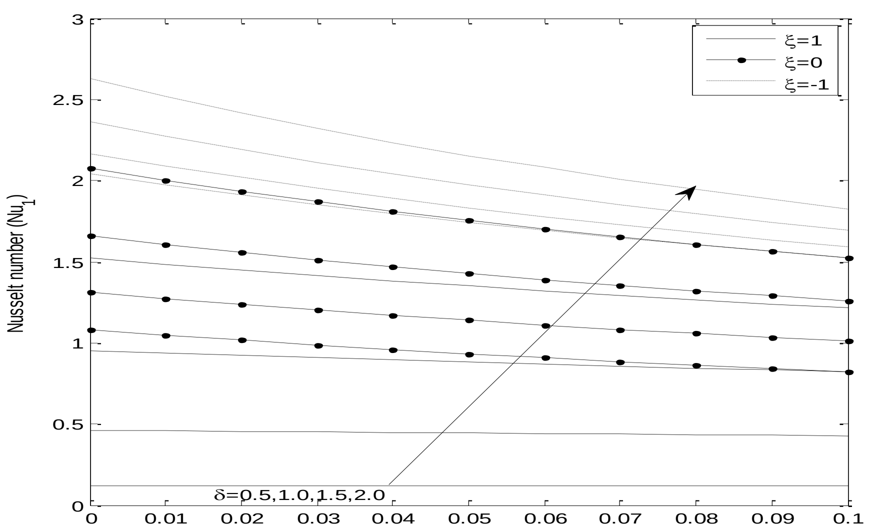

Figure 14b: Variation of rate of heat transfer for different value of δ at Y=1 (Case II)

Conclusion

This paper analytical investigated the role of heat generating and a\absorbing fluid on MHD natural convection flow in a vertical microchannel. The Exact solution of momentum and energy equations is derived separately for heat generating fluid and heat absorbing fluid. The influences of various flow parameters entering into the problem such as heat generating/absorbing fluids parameter, Hartmann number, Knudsen number, and fluid - wall interaction parameter on the velocity, temperature, skin friction and rate of the heat transfer are demonstrated through helpof graphs. The research leads to the following conclusions:

1. The effect of fluid wall interaction is found to horizontally shift the velocity profile to the cooler wall side and to reduce the fluid velocity.
2. Increase in Hartmann number reduced the influence of fluid velocity for both symmetric and asymmetric heating.
3. It is found that temperature and velocity are decreasing function of heat absorption parameter while increasing function of heat generation parameter.
4. Skin friction increases with increase in heat generation parameter at microchannel walls while the reverse trend is observed in the case of heat absorption parameter.
5. Finally, the rate of heat transfer increases with increase of heat generation parameter at microchannel walls while the reverse trend in the case of heat absorption parameter.

Acknowledgement

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Appendix

$$F_1=\frac{A}{\delta^2+M^2},\ F_2=\frac{1}{\delta^2+M^2}\left[A\cos(\delta)+B\sin(\delta)\right],\ F_3=\frac{B\delta}{\delta^2+M^2}, F_4=M\exp(M),$$

$$F_5=M\exp(-M),\ F_6=\frac{1}{\delta^2+M^2}\left[B\delta\cos(\delta)-A\delta\sin(\delta)\right],\ F_7=1-M\beta vKn, F_8=1+M\beta vKn,$$

$$F_9=F_3\beta vKn-F_1,\ F_{10}=\exp(M)+\beta vKnF_4, F_{11}=\exp(-M)-\beta vKnF_5,\ F_{12}=-\beta vKnF_6-F_2,$$

$$F_{13}=1-\beta vKn\ln\delta,\ F_{14}=1+\beta vKn\ln\delta,\ F_{15}=\exp(\delta)+\beta vKn\ln\delta\exp(\delta),$$

$$F_{16}=\exp(-\delta)-\beta vKn\ln\delta\exp(-\delta),\ F_{17}=\frac{1}{\delta^2-M^2}\left[E+F\right],$$

$$F_{18}=\frac{1}{\delta^2-M^2}\left[E\exp(\delta)+F\exp(-\delta)\right],\ F_{19}=\frac{1}{\delta^2-M^2}\left[E\delta-F\delta\right],$$

$$F_{20}=\frac{1}{\delta^2-M^2}\left[E\delta\exp(\delta)-F\delta\exp(-\delta)\right],\ F_{21}=F_{17}-\beta vKnF_{19},$$

$$F_{22}=\exp(M)+\beta vKnM\exp(M),\ F_{23}=\exp(-M)-\beta vKnM\exp(-M),\ F_{24}=F_{20}\beta vKn+F_{18},$$

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