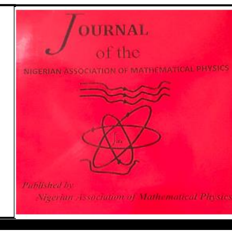
Journal of NAMP 68, 1 (2024) 151-156

The Nigerian Association of Mathematical Physics

Journal homepage: https://nampjournals.org.ng



GENERALIZATION OF NEWTON'S DYNAMICAL GRAVITATIONAL POTENTIAL USING GRAVITATIONAL TIME DILATION AND GRAVITATIONAL LENGTH CONTRACTION IN SCHWARZSCHILD SPACETIME

PATRICK AGWU OKPARA, SUNDAY NWOKPOKU ALOKE*, NELSON EZIEKE, NNAEMEKA MAJINDU,

Department of Industrial Mathematics and Health Statistics, International Institute for machine learning, robotic and Artificial intelligent of David Umahi Federal University of Health Sciences, Uburu, Nigeria

ARTICLE INFO

Article history:
Received xxxxx
Revised xxxxx
Accepted xxxxx
Available online xxxxx

Keywords:
Gravitational scalar potential, Gravitational time dilation, Gravitational length contraction, Schwarzschild space time.

ABSTRACT

In this article, we employed gravitational time dilation and length contraction within Schwarzschild spacetime to formulate a generalized gravitational field equation. This dynamic field equation was then used for static, homogeneous spherical massive bodies to derive the generalized exterior gravitational scalar potential. The results indicate that the generalized dynamic gravitational scalar potential includes an additional correction term proportional to c^{-2}*, which is absent in both Newton's equations of motion and Einstein's geometrical equations of motion.*

1. Introduction

In 1686, Newton introduced his dynamical theory of gravitation. According to Newton's universal law of gravitation, every particle exerts a force on every other particle along the line connecting their centers. The space or region where this force exists is referred to as the gravitational field [1, 4]. It was found that this force controls the motion of moons, planets, and galaxies in their respective orbits.

*Corresponding author: SUNDAY NWOKPOKU ALOKE
E-mail address: alokesundayn@gmail.com
https://doi.org/10.60787/10.60787/jnamp.v68no1.429

The importance of the laws of motion and gravity lies in their ability to explain the observed facts about the solar system's existence. The theory of universal gravitation within Earth's atmosphere assumes that Earth is a perfect sphere [2, 3]. Similarly, within the solar system, the motion of bodies such as planets and stars is also regarded as that of perfect spheres. Consistently, the general theory of relativity posits that planets and photons are perfect spheres. In Einstein's theory of gravitation, the movement of bodies and particles is analyzed under the assumption that Schwarzschild spacetime forms a perfect sphere [3]. In reality, all celestial bodies, including planets, stars, black holes, and galaxies, are perfect spheres. The spherical geometry of celestial bodies clearly has corresponding effects on the motion of particles within their gravitational fields. These effects are present in both Newton's dynamical gravitational theory and Einstein's geometrical gravitational theory. Consequently, the groundwork is laid for understanding the gravitational fields of static, homogeneous spherical bodies. It is evident that the more massive the spherical body, the greater the curvature it imposes on spacetime. A stronger curvature of spacetime results in a stronger gravitational field created by the spherical body. In a paper titled "Generalization of Newton’s Dynamical Gravitational Scalar Potential for Static Homogeneous Spherical Distribution of Mass," the author utilized the Golden Laplacian operator method. The results revealed a generalized gravitational scalar potential, including additional correction terms absent in Newton’s earlier work.

There are two main gravitational theories in physics: Newton’s dynamical gravitational theory and Einstein’s geometrical gravitational theory. Newton’s theory observed that all interactions in nature involve forces. Hence, for a body to move from one point to another, a force must act upon it. Newton’s theory was successful in explaining gravitational phenomena on Earth and the observational facts of the solar system [6]. However, it failed to account for the anomalous orbital precession of planets and the gravitational redshift caused by the Sun.

In 1915, Albert Einstein introduced his geometric theory of gravitation, known as the general theory of relativity. Einstein’s theory posits that gravitation is not the result of a force but rather a manifestation of the curvature of space and time [4, 5]. He used geometrical constructs (tensors) to describe gravitation, instead of the dynamical quantities like force and potential. Notably, general relativity unified special relativity with Newton’s law of universal gravitation by providing the insight that gravity arises from the curvature of spacetime, shaped by the mass-energy and momentum within it [7]. In 1908, Hermann Minkowski gave a mathematical framework for special relativity, which applied in the absence of gravity. He extended the three-dimensional Euclidean spacetime by taking the three spatial dimensions given as (t, x, y, z) and $(t+dt, x+dx, y+dy, z+dz)$. He transformed the dimensions into the Minkowski flat spacetime given by the square of the infinitesimal interval between the points which is the line element with Cartesian coordinates [8] given as:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \qquad 1.1$$

The Minkowski flat spacetime in equation (1.1) can be converted to flat four-dimensional pseudo Euclidean spacetime of the spherical coordinate given by the line element as

$$ds^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\theta^2 \qquad 1.2$$

However, in searching for a geometric theory of gravity, Einstein had to generalized the Minkowski spacetime of special relativity in equation (1.2) as

$$ds^2 = \sum_{\alpha\beta=0}^{3} \bigcap_{\alpha\beta} dx^\alpha dx^\beta \quad 1.3$$

In Einstein general theory of relativity, it follows that mass curves the one time dimension and three space dimensions of spacetime. Therefore, the spacetime curvature is greatest near the mass and vanishes at a distance [9]. In general theory of relativity, Einstein relates the metric tensor components which describe the curvature of spacetime to the distribution of matter throughout spacetime. Therefore, he described spacetime as a curved four-dimensional pseudo-Riemannian manifold. In general relativity, spacetime can be represented using non-Cartesian coordinate systems, such as spherical coordinates. [10].

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \quad 1.4$$

Where $g_{\alpha\beta}$ is the fundamental or metric tensor. Schwarzschild metric is a solution of Einstein gravitational field equations. This metric describes the spacetime curvature around static massive objects. This metric represents the gravitational field around a non-rotating symmetrically spherical object. The gravitational field described by Schwarzschild metric is known as the Schwarzschild geometry as described in equation (1.5) below

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \quad 1.5$$

2. Methods

In this study, we apply the generalized dynamic gravitational field equation to static, homogeneous spherical massive bodies by finding the general solution to the exterior field equation. The line element of Schwarzschild metric is given by

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(cdt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \quad 2.1$$

Recall that in a weak static field, the equation of motion for a particle in a gravitational field according to Newtonian mechanics is retrieved. [8]. Taylor series expansion derived by Mungan was applied to the Schwarzschild spacetime metric given by equation (2.1) above. The application of Taylor series expansion using first few terms can make difficult problems possible with the approximation of few terms. The Taylor series expansion is truncated to the third order given by [11] below.

$$(1+t)^p = 1 + \frac{t^p}{1!} + \frac{p(p-1)}{2!}t^2 \quad 2.2$$

3. Theoretical Analysis

Herein, the research was to construct gravitational time dilation in the Schwarzschild spacetime. The generalized gravitational scalar potential outside the body, influencing a particle in spherical polar coordinates r, θ, ϕ where the differential coordinates are zero. Let's examine a clock that remains stationary at a specific location within the Schwarzschild gravitational field surrounding a spherical body. Furthermore, since the two events occur at the same place then they are separated by a differential in coordinate given by

$$dr = d\theta = d\phi = 0 \qquad 3.1$$

We substituted equation (3.1) into the Schwarzschild metric line element given in equation (2.1). The new line element formed is given by

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 \qquad 3.2$$

Let us relate proper time between two events for time like intervals. Then the proper time between the events measured by a clock at rest at the location of the events given by

$$ds^2 = -c^2 d\tau^2 \qquad 3.3$$

Substituting equation (3.3) into equation (3.2) gives a line element. The proper time between the events, as would be measured by clock at rest at the location of the events is given in equation (3.3). The comparison of equation (3.2) and equation (3.3) for a distant observer at rest shows that the proper time is given by

$$d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right)dt^2 \qquad 3.4$$

Taking the square root of equation (3.4) gives

$$d\tau = \left(1 - \frac{2GM}{c^2 r}\right)^{\frac{1}{2}} dt \qquad 3.5$$

In equation (3.5), we divided through with the coefficient of the time coordinate to obtain

$$dt = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} d\tau \qquad 3.6$$

The Taylor series expansion was applied on equation 3.6, which is the reduced Schwarzschild line element. The new line element for Schawrzschild spacetime is given by

$$dt = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} d\tau \cong \left(1 + \frac{GM}{c^2 r} + \frac{3G^2M^2}{2r^2c^4}\right)d\tau \qquad 3.7$$

It follows that

$$dt = \left(1 + \frac{GM}{c^2 r} + \frac{3G^2M^2}{2r^2c^4}\right)d\tau \qquad 3.8$$

The construction of gravitational length contraction in Schwarzschild spacetime was obtained by applying the Taylor series expansion approach to static homogeneous spherical massive bodies as given in equation (2.1). In order to determine the radial distance within the Schwarzschild spacetime, let (t,θ,ϕ) be constants, then their derivatives are given by

$$dt = d\theta = d\phi = 0 \qquad 3.9$$

We substitute equation (3.9) into equation (2.1) which represents the Schwarzschild line element for determining an interval of radial distance given by

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right)^{-1} d\tau^2 \qquad 3.10$$

Taking the square root of equation (3.10) which is the reduced line element in equation (2.1) to obtain the equation given below

$$ds = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} d\tau \qquad 3.11$$

The Taylor series expansion in equation (2.2) was applied to equation (3.11) given by

$$ds\left(1 + \frac{GM}{c^2 r} + \frac{3G^2M^2}{2r^2c^4}\right)d\tau \qquad 3.12$$

In the Schwarzschild field, within the neighborhood of a massive body, two points with the same angle θ and ϕ nowhaveaseparationwhichisdifferentfromthecorrespondingseparationinemptyspaceis represented by

$$ds = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} d\tau \cong \left(1 + \frac{GM}{c^2 r} + \frac{3G^2M^2}{2r^2c^4}\right)d\tau \qquad 3.13$$

Discussion of Results

Newton's dynamical gravitational potential lacks extra correction terms of all orders of, which are added to the generalized dynamical and geometrical gravitational scalar potentials outside the body in equations (3.8) and (3.13). Similarly, in a weak gravitational field, the proper distance and the coordinate distance are equivalent, and their result follows the Equivalents principle in physics, and the proper term of an observer will coincide with the coordinate time and its consequence predicts the weak gravitational field that follows it. The outcome of equation (3.8) suggests that the observer's proper time is less than the coordinate time since the dilated coordinate time is higher than the function of the proper time.

Conclusion

An observer is seen as being at a fixed place around the huge body in gravitational time dilation. The spacetime, or reduced Schwarzschild line element, was subjected to the Taylor series expansion. The results of this study suggest that post-Newtonian and post-Einstein correction terms of all orders c^{-2} can be predicted for the gravitational field of static, homogeneous, spherical heavy masses by generalizing Newton's dynamical gravitational theory. We showed how to create generalized dynamical equations of motion for these bodies by deriving extra correction terms c^{-2} that are absent from both Einstein's and Newton's equations of motion through the use of gravitational time dilation and length contraction. It was mentioned that post-Newtonian and post-Einstein corrective terms of order c^{-2} are included in all of the study's outcomes.

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