Journal of NAMP 68, 1 (2024) 157-162

The Nigerian Association of Mathematical Physics

Journal homepage: https://nampjournals.org.ng

COMPARATIVE STUDY OF GENERALIZED NEWTON DYNAMICAL GRAVITATIONAL SCALAR POTENTIAL WITH THE GOLDEN RIEMANIAN DYNAMICAL GRAVITATIONAL SCALAR POTENTIAL

PATRICK AGWU OKPARA, SUNDAY NWOKPOKU ALOKE*, NELSON EZIEKE, NNAEMEKA MAJINDU,

Department of Industrial Mathematics and Health Statistics, International Institute for machine learning, robotic and Artificial intelligent of David Umahi Federal University of Health Sciences, Uburu, Nigeria

ARTICLE INFO

Article history:
Received xxxxx
Revised xxxxx
Accepted xxxxx
Available online xxxxx

Keywords:
Gravitational scalar potential,
Taylor series,
Golden Riemanian scalar potential,
Spherical massive bodies
Additional correction terms.

ABSTRACT

Over the years, there has been a growing need to generalize both Newton's dynamical theory of gravitation and Einstein's geometrical theory of gravitation to achieve better consistency with all physical theories. In this article, a Taylor series expansion approach was utilized to extend Newton's dynamical gravitational field, resulting in the construction of a generalized dynamical gravitational field equation. This generalized equation was then applied to static, homogeneous spherical massive bodies to derive generalized exterior gravitational scalar potentials. The generalized dynamical gravitational scalar potential was utilized to analyze the motion of planets within the solar system. The findings reveal that this potential is enhanced by additional correction terms of all orders of c^{-2} which are not present in Newton's dynamical equation of motion. Additionally, the generalized dynamical gravitational scalar potential includes a c^{-4} post-Newtonian correction term. These results were compared with those obtained using the Golden Riemannian dynamical gravitational scalar.

1. Introduction

The theory of gravitation was first postulated by Isaac Newton more than three hundred years ago. In his postulate, he stated that the force that keeps an object on the ground and the force that keeps the planets in their orbit around the sun are both the same [2]. Shortly after Newton discovered and developed gravitation, it was observed that it could not account for the anomalous orbital precession of the planets as well as the gravitational shift caused by the Sun, as a result of that, Einstein sought for appropriate geometrical quantities for the description of gravitation [1].

*Corresponding author: SUNDAY NWOKPOKU ALOKE
E-mail address: alokesundayn@gmail.com
https://doi.org/10.60787/10.60787/jnamp.v68no1.429

He chose the metric tensor as the fundamental quantity for the description of gravitation. In Newton's dynamical theory, it is a well-known fact that all interactions in nature manifest through force [3]. This Newton's dynamical theory was successful in explaining the gravitational phenomena on earth and the experimental fact of the solar system [4]. The limitation of this theory is that, it could not explain the anomalous orbital procession of the orbital of the planets as well as the gravitational shift by the sun [6]. At the end of 19th century, there were several attempts to generalize or extend Newton's dynamical theory of gravitation in order to provide better agreement with the experimental data or better consistency to all physical theories [9]. In 1915, Albert Einstein published his geometrical theory of gravitation, which is known as general relativity theory. According to Einstein's geometrical theory of gravitation, gravitation is not due to force, but a manifestation of geometrical curving of space and time [3]. Many physicists had continued to hold on to the view that Newton's dynamical theory of gravitation can be extended in such a way as to account satisfactorily for the experimental data and phenomena [5]. In this article, the comparative study of Generalized Newton dynamical gravitational scalar potential with the Golden Riemanian dynamical gravitational scalar potential is evaluated by means of Taylor series expansion method.

2. Theoretical Analysis

The metric that characterizes the curvature of spacetime around a static massive object is represented by

$$ds^2 = -\left(1-\frac{2GM}{c^2r}\right)(cdt)^2 + \left(1-\frac{2GM}{c^2r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad 2.1$$

Where

M is the mass of gravitating object
G is the universal gravitational constant
c is the speed of light
θ is the polar coordinate
ϕ is the azimuthal coordinate t is the coordinate of time
r is the radial coordinate ds is element of the proper distance
dr is an element of the coordinate distance
dt is an element of the coordinate time

The metric in equation (2.1) can be written in the exponential form as

$$ds^2 = -e^{\frac{2\phi}{c^2}}(cdt)^2 + dl^2 \qquad 2.2$$

Where dl in equation (2.2) represent the whole spatial part of the metric. The relativistic gravitational potential is obtained by the comparison of time coefficients of equations (2.1) and (2.2) given by

$$-\left(1-\frac{2GM}{c^2r}\right)(cdt)^2 = -e^{\frac{2\phi}{c^2}}(cdt)^2 \qquad 2.3$$

Herein equation (2.3) i given as

$$\left(1-\frac{2GM}{c^2 r}\right)=-e^{\frac{2\phi}{c^2}} \qquad 2.4$$

Taking the natural logarithm of the equation (2.4) yields

$$\ln\left(1-\frac{2GM}{c^2 r}\right)=\frac{2\phi}{c^2} \qquad 2.5$$

By simplification of equation (2.5) above, the gravitational scalar potential ϕ is given by

$$\phi=\frac{c^2}{2}\ln\left(1-\frac{2GM}{c^2 r}\right) \qquad 2.6$$

The equation (2.6) represents the exact expression of the relativistic gravitational scalar potential equivalent to Schwarzchild curved spacetime. The logarithm function in equation (2.6) is expanded using Taylor series expansion approach. The expansion of the natural logarithm given by the Taylor series expansion approach [10]. The approximated natural logarithm in equation (2.6) is given by

$$\ln(1-p)=p-\frac{1}{2}p^2+\frac{1}{3}p^3+\ldots \qquad 2.7$$

Substituting $p=-\frac{2GM}{c^2 r}$ into equation (2.7), then expansion of equation (2.6) is given by

$$\phi=\frac{c^2}{2}\left[-\frac{2GM}{c^2 r}-\frac{1}{2}\left(\frac{2GM}{c^2 r}\right)^2-\frac{1}{3}\left(\frac{2GM}{c^2 r}\right)^3\right] \qquad 2.8$$

The gravitational potential in equation (2.8)

$$\phi=\left[-\frac{GM}{r}-\frac{1}{c^2}\left(\frac{GM}{r}\right)^2-\frac{4}{3c^4}\left(\frac{GM}{r}\right)^3\right] \qquad 2.9$$

The equation (2.9) yields as

$$\phi=-\frac{GM}{r}\left[1+\frac{GM}{c^2 r}+\frac{4G^2M^2}{3c^4 r^2}\right] \qquad 2.10$$

In this article, we derive the generalized dynamical gravitational scalar potential outside a spherical massive body using a novel dynamical approach. The generalized golden gravitational field equation for a static, homogeneous spherical massive body, as referenced in [7], [8], and [9], is given by

$$f''+\frac{2}{c^2}ff''+\frac{2}{r}f'+\frac{4}{c^2 r}ff'=\begin{cases}0, & r>R\\ 4\pi G\ell_0, & r<R\end{cases} \qquad 2.11$$

Where f' is differentiation once with respect to r and f'' is differentiation twice with respect to r.

We want to seek the exterior solution of equation (2.11) as given below:

$$f(r) = \frac{A_1}{r} + \frac{A_2}{r^2} + \frac{A_3}{r3} + \ldots \qquad 2.12$$

Where A_1, A_2 and A_3 are arbitrary constant.

We differentiate equation (2.12) once and twice and substitute the solution to equation (2.11). The evaluation of all these equations, it follows that the generalized dynamical gravitational scalar potential outside the body is expressed as

$$\phi = -\frac{GM}{r}\left[1 - \frac{3GM}{5c^2R} - \frac{2GM}{c^2r}\right] \qquad 2.13$$

In this comparative study of the generalized Newtonian dynamical gravitational scalar potential (GNDGSP) and the golden Riemannian dynamical gravitational scalar potential (GRDGSP), the validity of the research article is assessed through the application to planetary bodies in the scalar system. Table (1) below illustrates that the results obtained in the ratio are effective.

Table 1: Comparative analysis of the Generalized Newtonian Dynamical Gravitational Scalar Potential (GNDGSP) with the Golden Riemannian Dynamical Gravitational Potential (GRDGSP)

Body	Mass (kg)	Radius (m)	Mean Distance from the sun (km)	GRDGSP $-\frac{GM}{r}\left[1-\frac{3GM}{5c^2R}-\frac{2GM}{c^2r}\right]$	GNDGSP $-\frac{GM}{r}\left[1+\frac{GM}{c^2r}+\frac{4G^2M^2}{3c^4r^2}\right]$	Ratio of GRDGSP to GNDGSP
MERCURY	3.30×10^{23}	2.44×10^{6}	57.9×10^{6}	-380.16	-380.16	1.0000
VENUS	4.87×10^{24}	6.05×10^{6}	108.2×10^{6}	-3002.12	-3002.12	1.0000
EARTH	5.97×10^{24}	6.378×10^{6}	149.6×10^{6}	-2661.7599	-2661.76	0.9999
MARS	6.42×10^{23}	3.397×10^{6}	227.9×10^{6}	-187.90	-187.90	1.0000
JUPITER	1.90×10^{27}	271.49×10^{6}	778.6×10^{6}	-162766.498	-162766.50	1.0000
SATURN	5.69×10^{26}	60.268×10^{6}	1427×10^{6}	-26595.8698	-26595.87	1.0000
URANUS	8.66×10^{25}	25.559×10^{6}	2871×10^{6}	-2011.9199	-2011.92	1.0000
NEPTUNE	1.03×10^{26}	24.746×10^{6}	4497×10^{6}	-1527.7099	-1527.71	1.0000
PLUTO	1.31×10^{22}	1.160×10^{6}	7376×10^{6}	-0.1185	-0.1185	1.0000

Remarks and Conclusion

In this research, we successfully derived the generalized dynamical gravitational scalar potential outside static homogeneous spherical massive bodies using a Taylor series expansion approach, as presented in equation (2.10). The Golden Riemannian dynamical gravitational scalar potential for a static homogeneous spherical massive body is described in equation (2.13). In both equations (2.10) and (2.13), the leading term on the left-hand side corresponds to the well-known Newtonian dynamical gravitational scalar potential. As c approaches 0, these equations reduce to the corresponding pure Newtonian gravitational field equation, demonstrating consistency with the established equivalence principle in physics. The results for the generalized dynamical gravitational scalar potential include additional correction terms of all orders of c^{-4} which are absent in both Newton's and the Golden Riemannian dynamical gravitational scalar potentials. This study indicates that the relationship between the Golden Riemannian dynamical gravitational scalar potential and that predicted by the generalized Newtonian dynamical gravitational scalar potential can be analyzed using the Taylor series expansion, as summarized in Table 1. The validity of this research is assessed through the comparison of equations (2.10) and (2.13) when applied to all the planetary bodies in the solar system. The comparison of the generalized Newtonian dynamical gravitational scalar potential with the Golden Riemannian dynamical gravitational scalar potential, presented in the accompanying table, demonstrates that the results obtained in the ratio are effective.

References

[1] Okpara, P. A., Umahi, E. A. and Oboma, D. N. (2018). Generalized Dynamical gravitational scalar potential for static homogeneous spherical distribution of mass using Taylor series approach. Journal of the Nigeria Association of Mathematical Physics, vol 48, 243-246.

[2] Nura, Y., Howusu, S. X. K, Nwagbara, O. and Nuhu, I. (2017). Solution of Newton gravitational field equation of a static homogeneous prolate spheoidal massive body. International Journal of Engineering Science and Innovation Technology (IJESIT) 6(1), 67-76.

[3] Ebenezer, N. I. (2012). Gravitational fields Exterior to homogeneous spheroidal masses. The Abraham Zelmanov Journal, vol 5, 1-67.

[4] Howusu, S. X. K. (2004). Einstein's equation of motion in the gravitational field of an oblate speroidal body. Journal of the Nigerian Association of Mathematical Physics, 8(1), 251-260.

[5] Chifu, E.N., Usman, A. and Meludu, O. C. (2010). Gravitational time dilation and length contraction in fields exterior to static oblate spheroidal Mass distributions. Journal of the Nigerian Association of Mathematical Physics, vol. 15, 247-252

[6] Lumbi, W. L. and HOwusu, S. X. K. (2014). Generalized Dynamical Equation of motion for particles of non-zero mass for static Homogeneous spherical. Journal of the Nigerian Association of Mathematical Physics, 27(7), 365-368

[7] Maisalatee, A. U., Lumbi, W. L. Ewa, I. I., Mohammed, M. and Kaika, Y. K. (2020). Generalization of Quantum Mechanical Wave Equation in Spherical coordinate using Great

Metric Tensors and a Variable Gravitational Scalar Potential. Journal of the Nigerian Association of Mathematical Physics, 2(1), 139-147.

[8] Lumbi, W. L., Howusu, S. X. K., Nwagbara, O. and Gurku, U. M. (2016). Generalization of Newton's Dynamical Gravitational Scalar Potential for Static Homogeneous Spherical Distribution of Mass using Golden Laplacian Operator. International Journal of pure and Applied Sciences. 6(1), 105-111

[9] Lumbi, W. L., Nwagbara, O. Ewa, I .I., Yakubu, N. and Hassanu, M. (2017). A Generalization of Newton's Planetary Gravitational Equation of Motion for Static Homogeneous Spherical Massive Bodies. International Journal of Theoretical and Mathematical Physics. 7(1), 1-3

[10] Mungan, C. E. (2009). Three Important Taylor Series for Introductory physics. Lat. Am. J. Phys. Educ. 3(3):535-538