Journal of the Nigerian Association of Mathematical Physics
Volume 65, (October 2022– August 2023 Issue), pp207 – 216
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PARAMETER ESTIMATION OF LINEAR REGRESSION MODEL WITH MULTICOLLINEARITY AND HETEROSCEDASTICITY PROBLEMS

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Abstract

It is very obvious that the assumption of the classical linear regression model are rarely fulfilled in real life situation. The violation of assumption of independent regressors and equal error variances leads to the problems of multicollinearity and heteroscedasticity respectively. In practice, both problems do exist together in a data set. Most of the developed existing estimators addressed each problem separately. Usually, one of the problems is handled while the other is left uncared for. Estimators to handle the two problems jointly are hardly common. There is therefore a need to develop estimators that can handle parameter estimation even when there is multicollinearity and heteroscedasticity. Consequently, this paper proposed estimators to handle parameter estimation of linear regression model having both multicollinearity and heteroscedasticity problems with the aim of identifying the most efficient (best) when both are in existence. The Ordinary least squares (OLS) estimators resulted to the weighted Least Squares model with Heteroscedasticity measures by real weight (OLSRW) and three other weights (OLSW1, OLSW2, OLSW3. Similarly, the Generalized Ridge Esimator (GRE) and the Ordinary Ridge Estimator (ORE) respectively resulted into proposed estimators GRERW, GREW1, GREW2, GREW3, ORERW, OREW1, OREW2, and OREW3. Monte carlo simulation were conducted one thousand (1000) times on a linear regression model exhibiting different levels of multicollinearity (ρ = 0.6, 0.8, 0.9, 0.99, 0.999, 0.9999) with various known natures of heteroscedasticity, error variances ($\sigma_i^2 = \mathbf{0.01, 1.0, 25, 100, 625}$) at seven levels of sample size (n=15, 20, 30, 50, 100, 250, 500). The comparison of the estimators were done based on the their finite sampling properties especially the mean squares error, and were compared at each level of multicollinearity, heteroscedasticity, error variance and sample sizes. Ranking of the estimators were also conducted on the basis of their performances using the criteria. The results of investigation revealed that with known Heteroscedasticity structures present with multicollinearity problem, the proposed GRERW estimator is best. Also, when there is problem of multicollinearity with known natures of heteroscedasticity but assumed to be unknown, the proposed estimator GREW2 performed better.

Keywords: Linear regression model, Multicollinearity, Heteroscedasticity, Error variance, Proposed estimators, Sample size.

1.0 INTRODUCTION

Linear regression model is recognized as the most widely used statistical techniques for solving functional relationship problems among variables, it explains observations of a dependent variable with observed values of one or more independent variables. A classical linear regression model is commonly used for prediction tool. The k- variable linear regression model used to study the relationship between a dependent variable and k-independent variables is represented by:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \varepsilon_i, \qquad i = 1, \dots, n \qquad (1)$$

Where y is the dependent variable, $x_1, \dots, x_k$ are the independent or explanatory variables, $\beta_0, \beta_1, \dots, \beta_k$ are the unknown parameters to be estimated, ε_i is the random or disturbance or stochastic term and k is the number of explanatory variables excluding the constant term.

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In vector form, the equation (1) becomes

$$Y = X\beta + \varepsilon \qquad (2)$$

Such that

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & \cdots & . \\ . & . & \cdots & . \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \vdots \\ \beta_k \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \vdots \\ \varepsilon_n \end{bmatrix} \qquad (3)$$

Where y is an n×1 vector of observations on a response variable, X matrix is an n×(k+1) full rank of independent variables, β is a (k+1)×1 vector of unknown parameters to be estimated, ε is n×1 vector of random error [1]

The parameter β in a linear regression model commonly estimated using Ordinary least squares estimator (OLSE). The OLSE of β is given as:

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'Y \qquad (4)$$

$\hat{\beta}_{OLS}$ is unbiased estimator of β. The estimator is generally preferred, provided all the underlying classical linear regression model assumptions are satisfied [2]. It had been observed that some of these assumptions in the model are hardly satisfied in real life situation. Many authors including [3], Fomby [4], Ayinde *et al* [5] have suggested various situations and instances where these assumptions may be violated and itemized their consequences on the OLS estimator when used to estimate the model parameter. The violation of assumption of independent regressors leads multicollinearity as found in business and economic data. Multicollinearity is the term used to explain cases in which the explanatory variables or regressors are correlated (Lukman [6]). Multicollinearity has been a serious problem in most economic variables that required urgent attention. If there is perfect multicollinearity, the regression coefficients are indeterminate and their standard error will be infinite, if the multicollinearity is not perfect but high, estimates of the regression coefficients are possible but they do have large standard errors which affect both the inferences and forecasting that is based on the model [7].

The Ordinary least squares (OLS) estimates remain unbiased and inefficient when multicollineariy is present [8]. From the literature reviewed, several authors have worked on the methods of detecting the presence of multicollinearity and developed the alternatives estimators to estimate the parameters in the linear regression model. These include Ridge regression estimator developed by Hoerl and Kennard [9], [10], [11], [12], [13], [14], [15]. There is another method based on principal component regression suggested by [16] to handle multicollinearity by eliminating the model instability and reduces the variances of the regression coefficients. Some authors adopted the method of partial least squares [17] and [18] which generalizes and combine attributes from principal component analysis and multiple linear regression.

However, the concept of ridge was introduced by [9]. Ridge regression is biased method of estimation which has been shown to be more efficient than OLS estimator when data exhibit multicollineatrity. It is obtained by adding a ridge parameter k, to the main diagonal element of $X'X$, the correlation matrix.

The ridge estimator is given by:

$$\hat{\beta}_R = (X'X + KI)^{-1}X'Y \qquad (5)$$

Where k is a non-negative constant called the biasing or ridge parameter. It is observed that when k=0, the equation (5) returns to ordinary least square estimator. In ridge regression, different estimation techniques had been proposed for finding the optimal biasing parameter k, among authors are [9], [19, [20], [13], [21], [22] and recently [15], [23], [24], [25].

In addition, with different k values , the estimator becomes Generalized Ridge Estimator (GRE) and with constant k value, the estimator is Odinary ridge estimator (ORE). In considering the properties of the ridge estimator in (5), the mean is obtained by taking the expectation in (5) to give

$$E(\hat{\beta}_R) = Z\beta \qquad (6)$$

Where $= (X'X + KI)^{-1}X'X$.

The bias of the estimator is defined as:

$$Bias(\beta_R) = E(\hat{\beta}_R) - \beta \qquad (7)$$

Then, by substituting (6) into (7) yields

$$\text{Bias}(\hat{\beta}_R) = -(X'X + KI)^{-1}K\beta \qquad (8)$$

$$\text{Bias}^2(\hat{\beta}_R) = [(X'X + KI)^{-1}]^2 K^2 \beta^2 \qquad (9)$$

The variance of the ridge estimator is derived to yield:

$$V(\hat{\beta}_R) = \sigma^2[(X'X + KI)^{-1}]^2 X'X \qquad (10)$$

While the mean square error (MSE) of the ridge estimator is obtained as:

$$\text{MSE}(\hat{\beta}_R) = \sigma^2[(X'X + KI)^{-1}]^2 X'X + [(X'X + KI)^{-1}]^2 K^2 \beta^2 \qquad (11)$$

Since X′X is a positive definite matrix, there exists an orthogonal matrix Q such that $X'QX = T$, where $T = diag(t_1, t_2, \ldots\ldots, t_p)$ and $t_1, t_2, \ldots\ldots, t_p$ are the eigenvalues of X′X. Now let $\alpha = Q'\beta$, then,

$$MSE(\hat{\beta}_R) = \sigma^2 \sum_{i=1}^{p} \frac{t_i}{(t_i+k)^2} + K^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(t_i+k)^2} \quad (12)$$

Where α_i is the i^{th} element of the vector $\alpha = Q'\beta$. In view of the above, the MSE of the OLS estimator is the trace of the variance-covariance matrix given as:

$$\text{MSE}(\hat{\beta}_{OLS}) = \sigma^2 \text{trace}(X'X)^{-1} = \sigma^2 \sum_{i=1}^{p} \frac{1}{t_i} \quad (13)$$

Several biasing ridge parameter k, for Generalized ridge estimator exist in literature. These include the one proposed by [9]. They gave the optimum value of k as:

$$K_i = \frac{\sigma^2}{\alpha_i^2}, \ i = 1, 2, \ldots, p \quad (14)$$

Since σ^2 and α_i^2 are generally unknown, they were suggested by Hoerl and Kennard to be replaced by their corresponding unbiased estimates $\hat{\sigma}^2$ and $\hat{\alpha}_i^2$, therefore,

$\hat{K}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$, where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \varepsilon_i^2}{n-p}$.

Several biasing ridge parameter k, for Ordinary ridge estimator also exist in literature. These include the one proposed by [9] given as:

$$\hat{K}_{HK} = \frac{\hat{\sigma}^2}{Max(\hat{\alpha}_i^2)} \quad (15)$$

When all the assumptions of the classical linear regression model hold except that the error terms are not homoscedastic i.e $E(\varepsilon'\varepsilon) \neq \sigma^2 I$ which implies that $E(\varepsilon'\varepsilon) \neq \sigma^2\Omega$. the resulting model is the Generalized least squares (GLS) with Ω [26]. Assume P to be a non-singular symmetric matrix such that $\Omega = P'P$ as positive definite. Then P^{-1} is introduced to a linear regression model in (2) to yield $P^{-1}Y = P^{-1}X\beta + P^{-1}\varepsilon$

and the transformed model becomes:

$$Y^* = X^*\beta + \varepsilon^* \quad (16)$$

Thus, the variance of the transformed disturbance term (ε_i^*) is homoscedastic. Consequently, the OLS estimates of the transformed model have all the optimal properties of OLS and the usual inferences are valid. By Gauss-markov theorem [27], the best linear unbiased estimator of β via the transformed (16) is defined as

$$\hat{\beta}_{GLS} = \left(X^{*'}X^*\right)^{-1} X^{*'}Y^* \quad (17)$$

which is equivalent to:

$$\hat{\beta}_{GLS} = \left[X'\Omega^{-1}X\right]^{-1} X'\Omega^{-1}Y \quad (18)$$

Where $\Omega^{-1} = P^{-1'}P^{-1}$, the Aitken has shown that GLS estimator $\hat{\beta}$ of β as defined in (17) is efficient among the class of linear unbiased estimators of β with variance-covariance of β given by:

$$Var(\hat{\beta}_{GLS}) = \sigma^2 \left(X'\Omega^{-1}X\right)^{-1} \quad (19)$$

When heteroscedasticity exists in a data set. The predicted values are unbiased and inefficient, and the sampling variances of the error term are known to be underestimated causing the t and F tests to be invalid [4]; [8]; [28]. To compensate for the lost of efficiency. Several methods have been developed, these include the estimators provided by [29], [30], [31], [32], [33], [34] and [35]. In most cases, when both problems do present in a data set. The use of ridge regression estimator or weighted least squares estimator becomes inappropriate to hand both problems simultaneously. Consequently, this paper proposed and examined estimators to handle both multicollinearity and heteroscedasticity jointly.

2.0 MATERIALS AND METHODS

2.1 The Proposed Estimator Derivative

Having obtained the Ridge regression estimator as earlier stated in (5) as $\hat{\beta}_R = (X'X + KI)^{-1}X'Y$, then, applying OLS Estimator in the transformed model (16) resulted into Generalized least squares estimator $\hat{\beta}_{GLS} = (X^{*'}X^*)^{-1}X^{*'}Y^*$ which is earlier stated in (17). The proposed Estimator with the conclusion of (5) and (17) becomes Weighted ridge estimator which is derived as:

$$\hat{\beta}_{Proposed} = \left(X^{*'}X^* + KI\right)^{-1} X^{*'}Y^* \quad (20)$$

Then, solving equation (20) resulted into:

$$\hat{\beta}_{proposed} = [X'\Omega^{-1}X + KI]^{-1}(X'\Omega^{-1}Y) \quad (21)$$

Where Ω^{-1} is assumed to be known, however, Ω^{-1} is not always known in practice. It is often estimated but to have model corrected for heteroscedasticity, weight variables are required [36]; [37].

The following estimators, both existing and proposed are used in the study. Applying OLS estimator (16) resulted into weighted least square estimator which includes OLSRW, OLSW1, OLSW2, OLSW3, furthermore, if GRE and ORE are applying into (16), it resulted into the Proposed estimators: GRERW, GRERW1,GREW2, GREW3, ORERW, OREW1, OREW2 and OREW3.

2.2 Model Formulation for Monte Carlo Study

To examine the proposed and existing estimators, a regression model of the form is considered

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i \quad (22)$$

Where ε_i is the error term assumed to be normally distributed with mean zero and variance σ_i^2, i.e $\varepsilon_i \sim N(0, \sigma_i^2)$. The Xs are fixed independent variables exhibiting different degrees of multicollinearity and y_i is the response variable, βs are known values.

2.3 Procedure for the Explanatory variables

The specified intercorrelated normally distributed variables were generated using the equations provided by [10] and used by [7], [12], [13], [38], [39] for three (3) explanatory variables. This is given as:

$$X_{ti} = (1-\rho^2)^{\frac{1}{2}} Z_{ti} + \rho Z_{tp} \qquad (23)$$

t = 1, 2, 3. i= 1, 2, …., p, where Z_{ti} is the independent standard normal distribution with mean zero and unit variance, ρ is the correlation among the explanatory variables. The values of ρ were taken as 0.6, 0.8, 0.9, 0.99, 0.999, and 0.9999. In this study, the number of explanatory variables (p) was taken to be three (3).

2.4 Procedure for Generating the Error Term

The error term were generated using the distribution of standard normal variate stated below in (24) to exhibit different form of Heteroscedasticity and the various heteroscedasticity structures considered in this paper includes $\sigma_i^2 = \sigma^2 ABS(X_{i1})$ [40], $\sigma^2 X_{i1}^2$ [30], $\sigma^2(1+X_{i1})^2$ [41], $\sigma^2 exp(X_{i1})$ [42] and $\sigma^2[E(Y_i)]^2$. Following the distribution of the standard normal variate, $\varepsilon_i \sim N(0, \ \sigma_i^2)$

$\varepsilon_i = Z\sigma_i$, where $Z \sim N(0,1)$

$$= Z\sigma\sqrt{\Omega} \qquad (24)$$

2.5 Procedure for Generating the Response variable

The true values of the regression coefficient of model (22) are fixed as $\beta_0 = 4.0$, $\beta_1 = 3.4$, $\beta_2 = 4.5$ and $\beta_3 = 6.0$. Having generated X_i with different level of multicollinearity and error terms with various natures of heteroscedasticity. The values of the dependent variable is generated using (22). Monte Carlo simulation experiments were carried out one thousand (1000) times at seven sample sizes (n= 15, 20, 30, 50, 100, 250, 500).

2.6 Criterion for Investigation and Performance of Proposed Estimator.

Evaluation, examination and comparison of the estimators were done based on the finite sampling properties especially the mean squares error (MSE) which comprises variance and square of bias of the estimator.

$$MSE(\hat{\beta}_i) = \frac{1}{1000}\sum_{j=1}^{1000}(\hat{\beta}_{ij} - \hat{\beta}_i)^2 \qquad (25)$$

For each replicate, the estimated MSE for each of the estimators $(\hat{\beta})$ is obtained. The estimator with the smallest estimated MSE is considered best.

3.0 RESULTS

The full summary of the simulated results under the mean square error criterion at various sample sizes, multicollinearity levels, known natures of heteroscedasticity, error variances are pictorially presented in figures 1, 2, 3, 4, 5, 6 while with known but assumed to be unknown heteroscedasticity structures equally presented in figures 7, 8, 9, 10, 11 and 12.

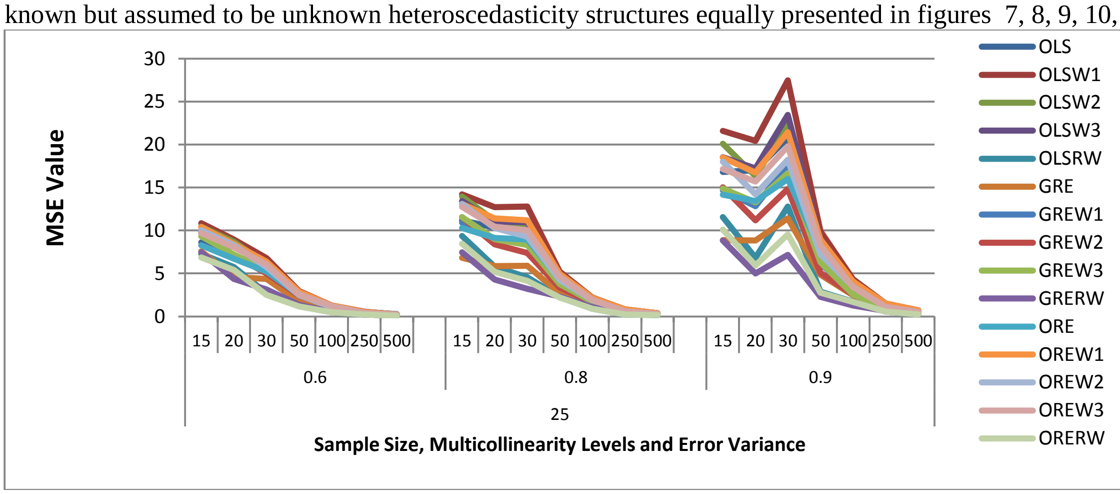

Figure 1: Graphical Representation of the Mean Square Error of the Estimators at Different Sample Sizes When Multicollinearity is High with Known Nature of Heteroscedasticity of the Form ABS(X) and Error Variance of 25.

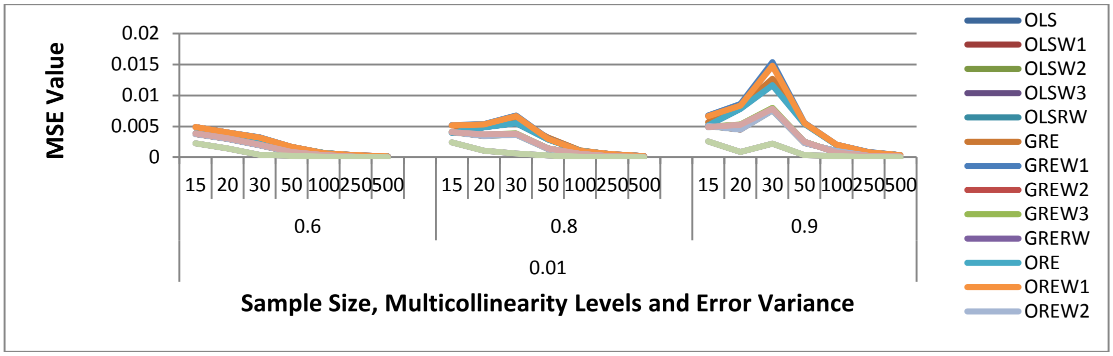

Figure 2: Graphical Representation of the Mean Square Error of the Estimators at Different Sample Sizes When Multicollinearity is High with Known Nature of Heteroscedasticity of the Form $(X)^2$ and Error Variance of 0.01.

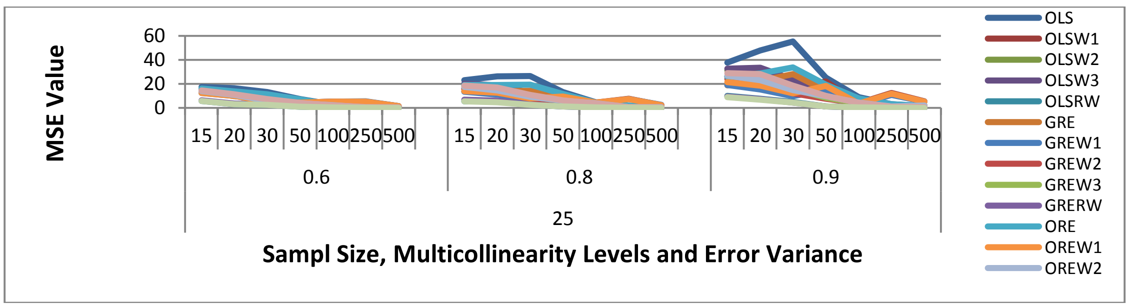

Figure 3: Graphical Representation of the Mean Square Error of the Estimators at Different Sample Sizes When Multicollinearity is High with Known Nature of Heteroscedasticity of the Form $(1 + X)^2$ and Error Variance of 25.

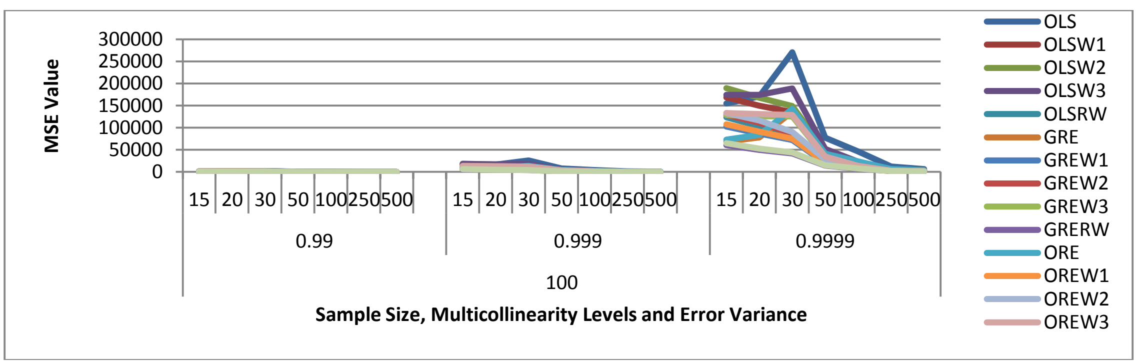

Figure 4: Graphical Representation of the Mean Square Error of the Estimators at Different Sample Sizes When Multicollinearity is Severe with Known Nature of Heteroscedasticity of the Form EXP(X) and Error Variance of 100.

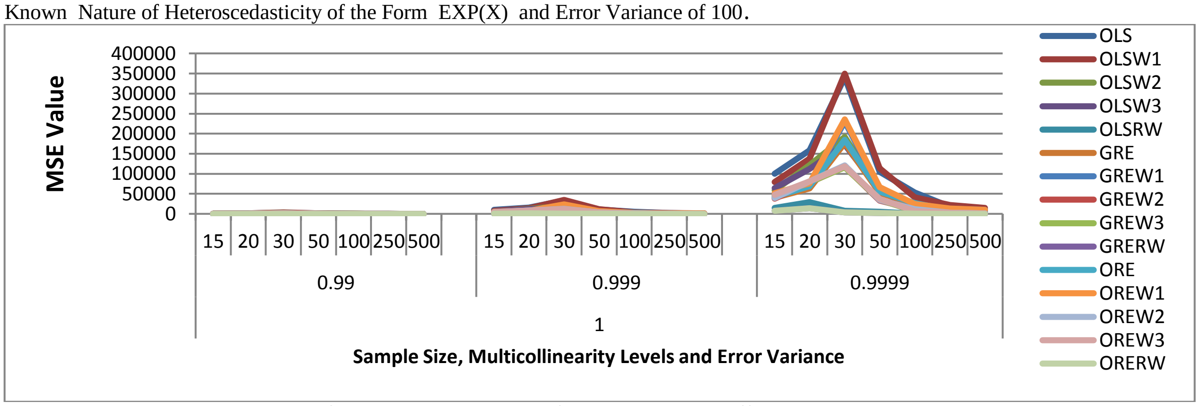

Figure 5: Graphical Representation of the Mean Square Error of the Estimators at Different Sample Sizes When Multicollinearity is Severe with Known Nature of Heteroscedasticity of the Form $[E(Y)]^2$ and Error Variance of 1.

Table 1 : Number of Times Each Estimator Produced Minimum Mean Square Error When Counted Over Level of Multicollinearity , Known Natures of Heteroscedasticity and Error Variance.

Estimators	Sample Size							TOTAL	RANK
	15	20	30	50	100	250	500		
OLS	0	0	0	0	0	0	0	0	12.5
OLSW1	2	0	0	0	1	0	0	3	7
OLSW2	0	0	0	0	0	0	0	0	12.5
OLSW3	0	0	0	0	0	0	0	0	12.5
OLSRW	15	2	5	12	17	20	24	95	3
GRE	5	1	0	0	0	0	0	6	5
GREW1	2	0	0	0	3	1	2	8	4
GREW2	0	0	4	0	0	0	0	4	6
GREW3	0	0	0	0	0	0	0	0	12.5
GRERW	**96**	**107**	**95**	**90**	**75**	**72**	**62**	597	**1**
ORE	0	0	0	0	2	0	0	2	8
OREW1	0	0	0	0	1	0	0	1	9
OREW2	0	0	0	0	0	0	0	0	12.5
OREW3	0	0	0	0	0	0	0	0	12.5
ORERW	30	40	46	48	51	57	**62**	334	**2**
TOTAL	150	150	150	150	150	150	150	1050	

NOTE : Estimator with highest frequency is bolded

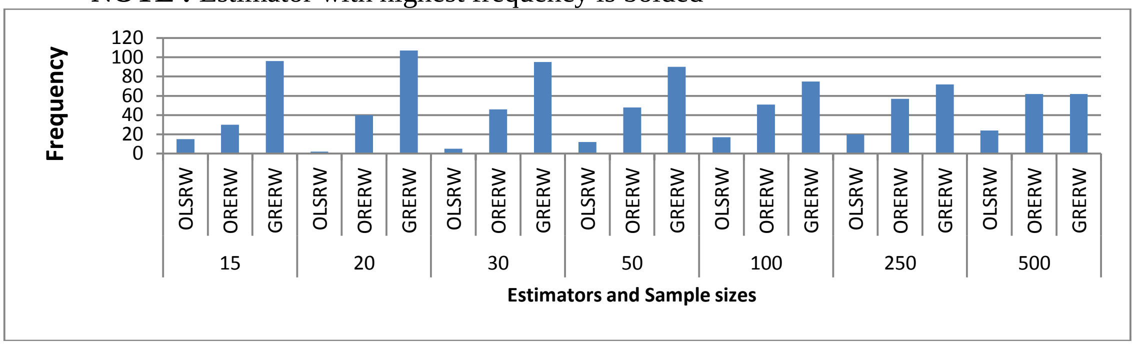

Figure 6 : Graphical Representation of the Frequency of the Best Estimators Under Mean Square Error Criterion at Different Sample Sizes When There is Multicollinearity With Known Natures of Heteroscedasticity in the Model.

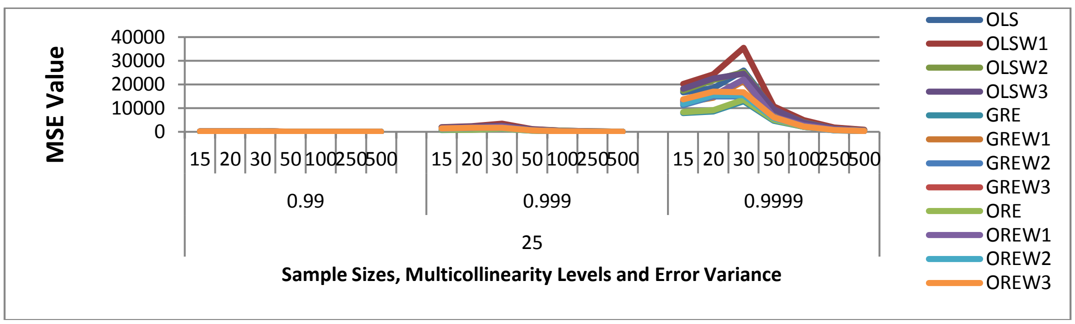

Figure 7: Graphical Representation of the Mean Square Error of the estimators at Different Sample Sizes When Multicollinearity is Severe with known but Assumed to be Unknown Nature of Heteroscedasticity of the form $ABS(X)$ and Error Variance of 25.

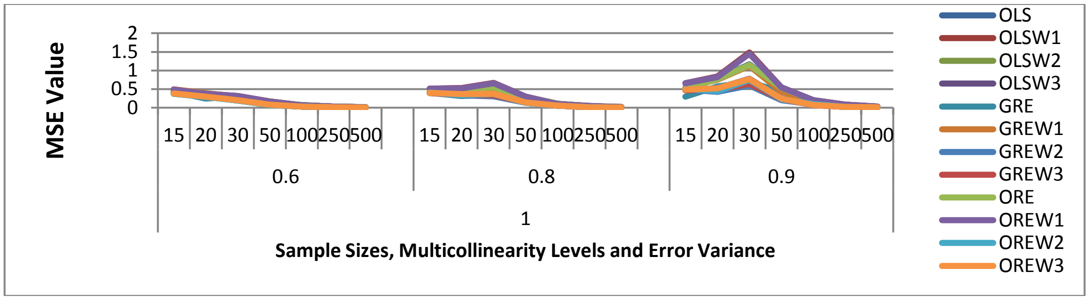

Figure 8: Graphical Representation of the Mean Square Error of the estimators at Different Sample Sizes When Multicollinearity is High with known but Assumed to be Unknown Nature of Heteroscedasticity of the form $(X)^2$ and Error Variance of 1.

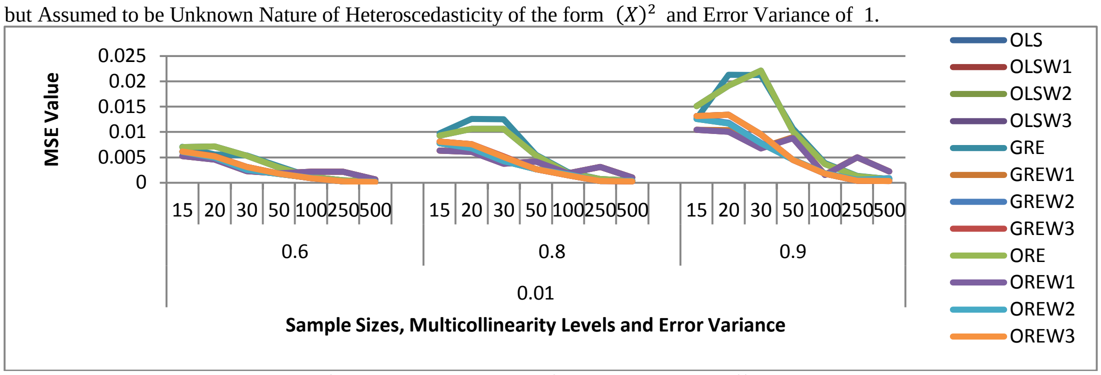

Figure 9: Graphical Representation of the Mean Square Error of the estimators at Different Sample Sizes When Multicollinearity is High with known but Assumed to be Unknown Nature of eteroscedasticity of the form $(1 + X)^2$ and Error Variance of 0.01.

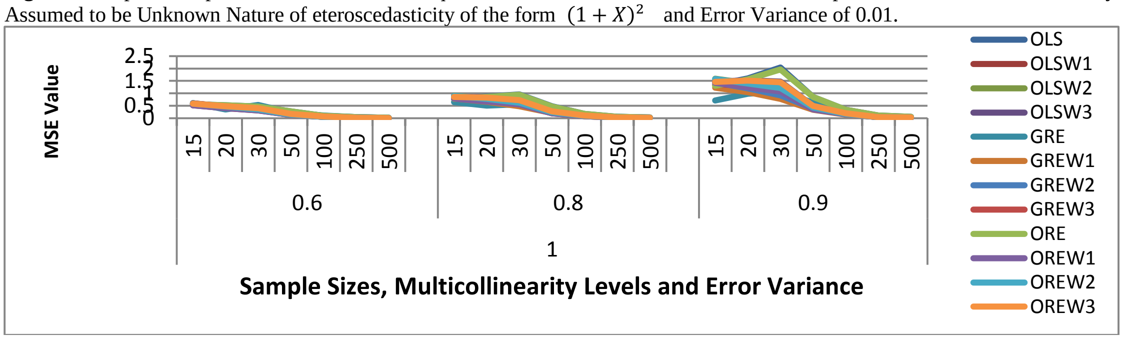

Figure 10: Graphical Representation of the Mean Square Error of the estimators at Different Sample Sizes When Multicollinearity is High with known but Assumed to be Unknown Nature of Heteroscedasticity of the form $EXP(X)$ and Error Variance of 1.

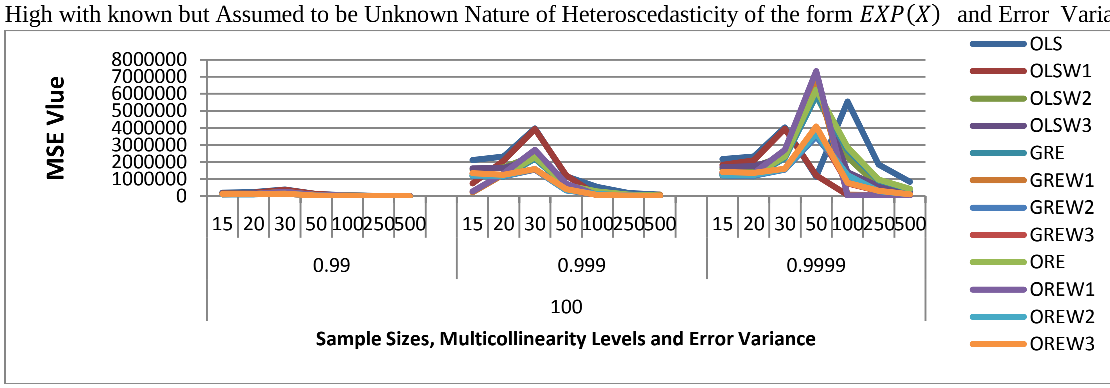

Figure 11: Graphical Representation of the Mean Square Error of the estimators at Different Sample Sizes When Multicollinearity is Severe with known but Assumed to be Unknown Nature of Heteroscedasticity of the form $[E(Y)]^2$ and Error Variance of 100.

Table 2: Number of Times Each Estimator Produces Minimum Mean Square Error When Counted Over Levels of Multicollinearity, Known But Assumed to be Unknown Natures of Heteroscedasticity And Error Variance.

Estimators	Sample Size							TOTAL	RANK
	15	20	30	50	100	250	500		
OLS	3	0	0	2	0	0	0	5	10
OLSW1	6	0	0	0	4	3	2	15	8
OLSW2	1	0	0	0	1	1	0	3	11
OLSW3	0	0	0	0	0	1	0	1	12
GRE	**97**	**88**	24	23	23	0	0	255	**1**
GREW1	11	25	**50**	22	35	18	18	179	**3**
GREW2	15	19	38	**56**	14	**65**	**47**	254	**2**
GREW3	2	0	18	23	**38**	15	30	126	**4**
ORE	5	3	4	0	1	0	0	13	9
OREW1	5	9	10	8	12	12	13	69	6
OREW2	3	6	4	13	8	29	23	86	**5**
OREW3	2	0	2	3	14	6	17	44	7
TOTAL	150	150	150	150	150	150	150	1050	

NOTE : Estimator with highest frequency is bolded.

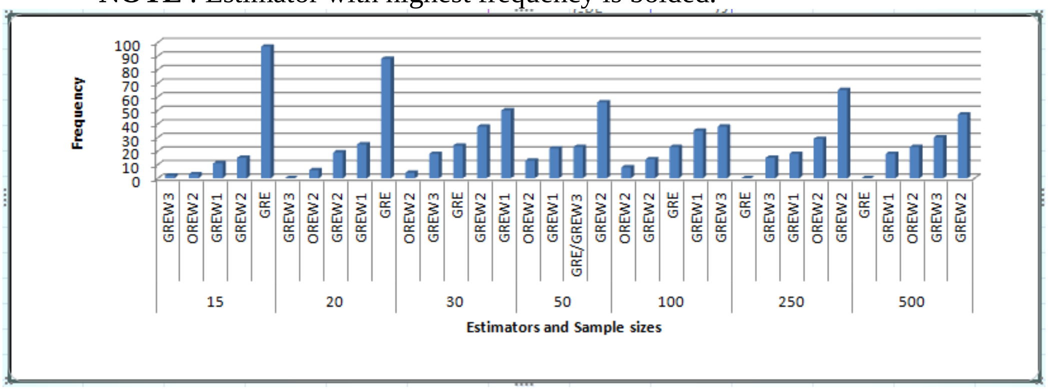

Figure 12: Graphical Representation of the Frequency of the Best Estimators Under Mean Square Error Criterion at Different Sample Sizes When There is Multicollinearity With Known But Assumed to be Unknown Natures of Heteroscedasticity in the Model.

4.0 DISCUSSION

Based on Figures1, 2, 3, 4, 5 and 6 pictorially presented. It can be generally observed that, as the sample size increases, the mean square error generally reduces. However, except for a particular known nature of heteroscedasticity of the form $(1+X)^2$, the mean square error of some of the estimators do converge to zero as pictorially presented in figures 1, 2, 4 and 5. As the multicollinearity increases with known natures of heteroscedasticity, the mean square error of the estimators increases. Also, as error variance increases, the mean square error of the estimators increases. Having counted the number of times each estimator has minimum mean square error over the six (6) levels of multicollinearity, five (5) known natures of heteroscedasticity and five (5) levels of error variance, Table 1 was observed. Thus, the maximum frequency is expected to be one-hundred and fifty (150) and so the closer the frequency of an estimator to one-hundred and fifty (150), the better the estimator.

The following are observed from Table 1. The Generalized ridge estimator real weight (GRERW) is best estimator with highest frequency at all level of sample sizes to handle the problem of multicollinearity with known forms of heteroscedasticity jointly. Morever, ORERW generally fair when $n \leq 500$. Although, ORERW performs equally with GRERW when $n = 500$. More generally, the best three (3) estimators in terms of mean square error are GRERW, ORERW and OLSRW. Figure 6 presents their frequency of counts at different sample sizes. The simulated results pictorially presented in figures 7, 8, 9, 10 and 11 under the mean square error criterion at various sample sizes, multicollinearity levels, known but assumed to be unknown heteroscedasticity structures and error variances, it can be observed that as the sample size increases, the mean square error of the estimators generally decreases.

However, with the exception of a particular known but assumed to be unknown nature of heteroscedasticity of the form $(1+X)^2$, the mean square error of some of the estimators do converge to the same value as pictorially presented in figures 7, 8, 10 and 11. As multicollinearity increases with known but assumed to be unknown natures of heteroscedasticity, the mean square error of the estimators increases. Table 2 result was obtained based on number of times each estimator produced minimum mean square error when counted over levels of multicollinearity, known but assumed to be unknown natures of heteroscedasticity and error variances. The following are observed form Table 2, the Generalized ridge estimator (GRE) is best estimator when ≤ 20 , while GREW1 is best when $n = 30$ to remedy the problem of multicollinearity with known but assumed to be unknown forms of heteroscedasticity jointly. GREW2 is best estimator when sample size is between 50 and 500, except when $n = 100$, at this instance, GREW3 is best. More generally, the five (5) best estimators in terms of mean square error are GRE, GREW2, GREW1, GREW3 and OREW2. Figure 12 presents their frequency of counts at different sample sizes.

5.0 CONCLUSION

The study has proposed estimators for the estimation of parameter linear regression with multicollinearity and heteroscedasticity problems. When there is multicollinearity with known natures of heteroscedasticity problem , the proposed estimator GRERW performs more efficiently than the existing ones to remedy both problems simultaneously. Since the natures of heteroscedasticity are rarely known in reality, which resulted to multicollinearity with known but assumed to be unknown natures of heteroscedasticity in the model. The proposed estimator GREW2 performs better and more efficient than the existing estimators to handle the problem of multicollinearity with known but assumed to be unknown natures of heteroscedasticity jointly.

REFERENCES

[1] Ekum, M.I., Akinmoladun, O.M., Aderele, O.R. and Esan, O.A. (2015). Application of Multivariate Analysis on the effects of World Development Indicators on GDP per capita of Nigeria (1981-2013). *International Journal of Science and Technology (IJST)*, 4(2):254-534.

[2] Gujarati, N. D, Porter, C.D. and Gunasekar S. (2012) ' Basic Econometrics.' (Fifth Edition). New Delhi: Tata McGraw-Hill.

[3] Neter, J. and W. Wasserman.(1974) 'Applied Linear Model.' Richard D. Irwin, Inc

[4] Fomby, T. B., R. C. Hill and S.R. Johnson. (1984) 'Advanced econometric methods.' Springer- Verlag, New York, Berlin, Heidelberg, London, Paris, Tokyo.

[5] Ayinde, K., Lukman, A. F. and Arowolo, O.T. (2015) 'Combined parameters estimation methods of linear regression model with multicollinearity and autocorrelation. Journal of Asian Scientific Research, 5 (5), 243 - 250.

[6] Lukman, A. F., Osowole, O. L. and Ayinde, K. (2015) ' Two stage robust method in a linear regression model'. Journal of Modern Applied Statistical Methods, 14 (2), 53-67.

[7] Lukman, A. F. and Ayinde, K. (2016) ' Some improved classification-based ridge parameter of Hoerl and Kennard estimation techniques'. West African Journal of Industrial and Academic Research, Vol. 16, (1), 72-79.

[8] Chatterjee, S., A. S. Hadi and B. Price. (2000) 'Regression Analysis by Example.' 3rd Edition, A Wiley-Interscience Publication. John Wiley and Sons.

[9] Hoerl, A. E and Kennard, R.W. (1970) 'Ridge regression: Biased estimation for non-orthogonal problems.' Technometrics, 12, 55-67.

[10] McDonald, G. C. and Galarneau, D. I. (1975) 'A monte Carlo evaluation of some ridge-type estimators'. Journal of the American Statistical Association, 70 (350), 407-412.

[11] Lawless, J. F. and Wang, P. (1976) 'A simulation study of Ridge and other Regression Estimators'. Communications and Statistics, A5, 307-323.

[12] Wichern, D. and Churchill, G. (1978) 'A comparison of ridge Estimators'. Technometrics, 20, 301-311.

[13] Kibria, B. M. (2003) 'Performance of Some New Ridge Regression Estimators'. Communications in Statistics-Simulation and Computation, 32 (2), 419-435.

[14] Kibra, B. M. G. and Shipra, B. (2016) ' Some ridge regression estimators and their performances'. Journal of Modern Applied Statistical Methods, 15 (1), 206- 231.

[15] Kibra, B. M. G. and Lukman, A. F. (2020) 'A new ridge-type estimator for the linear regression model: Simulations and Applications. Hindawi Scientifica.

[16] Massy, W. F. (1965) 'Principal component regression in exploratory statistical research.' Journal of the American Statistical Association, 60, 234-246.

[17] Tabakan, G. and Akdeniz, F. (2010) 'Difference-based ridge estimator of parameters in partial linear model.' Statist,.Pap., 51, 357-368

[18] Tabakan, G. (2013) 'Performance of the difference-based estimators in partially linear models.' Statistics, 47, (2), 329-347.
[19] Naes, T. and Marten, H. (1988) 'Principal component regression in NIR analysis : View point, background details selection of components.' Journal of Chemometrics, 2, 155-167.
[20] Liu, K. (1993) 'A new class of biased estimate in linear regression.' Communications in Statistics- Theory and Methods, 22, 393-402, MR1212418.
[21] Khalaf, G. and Shukur, G. (2005) ' Choosing ridge parameters for regression problems. Communications in Statistics-Theory and Methods, 34, 1177-1182.
[22] Dorugade, A. V. and Kashid, D. N. (2010) ' Alternative method for choosing ridge parameter for regression.' International Journal of Applied Mathematical Sciences, 4 (9), 447- 456.
[23] Lukman, A. F., Ayinde, K. and Ajiboye, S. A. (2017) ' Monte- Carlo study of some classification-based ridge parameter estimators'. Journal of Modern Applied Statistical Methods, 16 (1), 428-451.
[24] Ayinde, K., Lukman, F., Samuel, O. O. and Ajiboye, S. A. (2018) ' Some new adjusted ridge estimators of linear regression model'. Internation Journal of Civil Engineering and Technology, 9 (11), 2838- 2852.
[25] Fayose, T. S. and Ayinde, K. (2019) 'Different forms biasing parameter for generalized ridge regression estimators' International Journal of Computer Applications, Vol. 181, 37, 21- 29.
[26] Aitken, A .C . (1935) 'On least squares and linear combination of observations. 'Proceedings of the royal statistical society of Edinburgh, 55, 42-48.
[27] Markov, A. A. (1900) 'Wahrscheinlichkeitsrechnug.'' Leipzig; Tuebner.
[28] Maddala, G. S. (2002) 'Introduction to Econometrics.'' 3rd Edition, JohnWiley and Sons Ltd., England.
[29] Cochran, W. G. and Carroll, S.P. (1953) 'A sampling investigation of the efficiency of weighting inversely as the estimated variance.' Biometrics, 9 (4), 447-459.
[30] Park, R. E. (1966) ' Estimation with heteroscedastic error terms.'' Econometrica, 34, 888-892.
[31] Rao, C.R. (1970) 'Estimation of heteroscedasticity variances in linear models'. Journal of the American Statistical Association 65, 161-172.
[32] Horn, S. D, Horn, R.A. and Duncan, D.B. (1975) 'Estimating heteroscedastic variances in linear model.' Journal of the American Statistical Association, 70, 380-385.
[33] Magnus, J. R. (1978) 'Maximum likelihood estimation of the GLS model with unknown parameter in the disturbance covariance matrix.'' Journal of Econometrics, 7, 281-312.
[34] Balasiddamuni, P., Prakash, K., Rao, K., Prasad, A., Abbaiah, R. and Rayalu, G. (2013) 'A minimum quadratic unbiased estimation (MINQUE) of parameters in a linear regression model with spherical disturbances.' International Journal of Scientific and Technology Research, 2 (5), 135-138.
[35] Shin, H. C .and Jibum Kim. (2014) 'Weighted least squares estimation with simultaneous consideration of variances and sampling weights.' Proceedings of the survey research methods section, the American Statistical Association, 2972- 2978.
[36] Fuller, W. A. and Rao, J .N. K. (1978) 'Estimation for a linear regression model with unknown diagonal covariance matrix'. Annals of Statistics, 6 (5), 1149-1158.
[37] Carroll, R. J. and Ruppert, D.(1988) 'Transformation and weighting in regression.' New York, NY: Chapman & Hall.
[38] Gibbons, D. G. (1981) 'A Simulation Study of Some Ridge Estimators'. Journal of the American Statistical Association, 76 (373), 131-139.
[39] Lukman, A. F., Ayinde, K., Binuomote, S. and Clement, O. A. (2019b) ' Modified ridge-type estimator to combat multicollinearity: Application to chemical data. Journal of Chemometrics, 33 (5), 3125- 3132.
[40] Glesjer, H. (1969) 'A new test for heteroscedasticity.' Journal of the American Statistical Association, 64, 316- 323.
[41] Bruesch, T. S. and Pagan, A. R. (1979)' A Simple test of heteroscedasticity and random coefficient variation.' Econometrica, 47, 1287 – 1294.
[42] Box, G. E. P. and Hill, W. J. (1974) 'Correction inhomogeneity of variance with power transformation weighting'. Technometrics, 16 (3), 385-389.