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DYNAMIC BEHAVIOR OF A DAMPED SHEAR BEAM RESTING ON A VLASOV FOUNDATION

Ajijola, O. O[1], Ogunbamike, O. K.[2*] and Adedowole, A[3].

[1,3] Department of Mathematical Sciences, Adekunle Ajasin University, Akungba-Akoko, Nigeria

[2] Department of Mathematical Sciences, Olusegun Agagu University of Science and Technology, Okitipupa, Nigeria..

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ABSTRACT

The response behavior of a damped shear beam resting on Vlasov foundation when traversed by moving load travelling at constant velocity is investigated in this study. The governing equations are coupled second order partial differential equations. The Finite Fourier integral transform technique was adopted to reduce the governing the motion of this class of dynamical problem to sequence of coupled second order ordinary differential equations. Thereafter, the simplified equations of the beam-load system were then solved by Laplace transformation. The solution obtained was analyzed to obtain the conditions under which resonance will take place and speeds at which this may occur. Also, the displacement response for the dynamical problem was calculated for various values time t and the effects of pertinent structural parameters on the response of prestressed shear beam when under the action of the moving load were presented in plotted curves.

1. INTRODUCTION

The dynamic behavior of elastic beams subjected to moving loads is a subject of paramount importance in engineering and applied mechanics [1-10]. Such studies find relevance in diverse fields, including transportation infrastructure, mechanical systems, and structural engineering. Beams resting on subgrades are importance components in a wide range of industrial applications such as bridges, decking slabs and road ways. In these applications, such structures are repeatedly exposed to dynamic loads serving as essential components where they are often required to carry loads such as vehicles, trains, or moving machinery. Generally, the response of an elastic beam to dynamic loads is essential for ensuring the safety, reliability, and durability of the structures and systems they support. The concept of moving loads has long been studied in the context of structural dynamics [11-15].

[*]Corresponding author: OGUNBAMIKE, O. K.

E-mail address: ok.ogunbamike@oaustech.edu.ng

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Early research focused on static loading conditions, which simplified the analysis but overlooked the dynamic interactions that arise when loads traverse the beam at finite speeds. Moving loads induce time-dependent variations in the stress, strain, and displacement fields of the beam, which can lead to phenomena such as resonance, vibration amplification, and dynamic instability. These effects are particularly critical for high-speed applications, such as railway bridges and high-speed conveyors, where the interaction between the load velocity and the natural frequencies of the beam becomes significant. Also, elastic beams, characterized by their ability to undergo both bending and shear deformations, are widely used in structural designs due to their flexibility and capacity to withstand various types of loading [16]. When loads move on a structure, the dynamic behavior of these beams depends on several factors, including the load velocity, the beam's material and geometric properties, and the boundary conditions which are pivotal in understanding and predicting their behavior under various loading conditions. Additionally, external influences such as damping, foundation interactions, and environmental conditions can further complicate the response. A damped shear beam resting on a Vlasov foundation provides a particularly interesting case due to the complex interactions between the beam's internal properties and the foundation's support characteristics. The shear beam theory serves as a fundamental approach to analyzing beams subjected to dynamic loads. Unlike the Euler-Bernoulli beam theory, which assumes pure bending, the shear beam model incorporates transverse shear deformations, making it more suitable for short and thick beams. Timoshenko [17] first introduced the concept of shear deformation in beam theory, laying the groundwork for advanced dynamic analyses. Later refinements, such as those by Cowper [18], incorporated rotary inertia and more accurate shear deformation effects, improving the prediction of dynamic responses. These foundational theories form the basis for understanding the behavior of beams in dynamic scenarios, particularly when supported on elastic foundations. On the other hand, the inclusion of damping in dynamic analysis significantly enhances the realism of models. Rayleigh [19] introduced proportional damping models, while Caughey and O'Kelly [20] developed more generalized theories. Modern studies, such as those by Liu, et al. [21], focus on the application of viscous and hysteretic damping models to beams and Ogunbamike [22] determines the effect of a simply supported beam subjected to partially distributed loads and with damping due to resistance to the transverse displacement. In a recent development, the effects of viscous damping and damping due to strain resistance is investigated by Ogunbamike [23]. He used the generalized finite integral transform and the Struble asymptotic techniques to solve the beam problem. It will be recalled that, in the context of shear beams, damping does not only influence natural frequencies but also dictates the amplitude decay of oscillations, crucial for applications in seismic and vibration isolation systems. The incorporation of damping mechanisms which can arise from material properties, structural interfaces, or external devices, into these beams accounts for energy dissipation during dynamic loading, which is vital for reducing vibrations and enhancing structural stability [24, 25]. However, the behavior of shear beams when subjected to dynamic loads on elastic foundations, such as Vlasov foundations, becomes complex and requires advanced analytical techniques. The Vlasov foundation model is a popular model for representing the behavior of elastic foundations due to its ability to capture the effect of foundation stiffness and damping in a dynamical system. In this paper, the dynamic behavior of shear beams on Vlasov foundations has been studied to predict the response of shear beams under different loading conditions.

2. Problem Statement

The equations governing transverse displacement of shear beam on elastic foundation and under the action of moving load are based on the following assumptions.

(a) The beam is homogeneous at any cross - section (prismatic) and material is linearly elastic.
(b) The principal plane is the x - y plane.
(c) There is an axis of the beam that undergoes no extension or contraction. The x- axis is located along this neutral axis.
(d) Plane section remains plane after bending but are no longer normal to the longitudinal axis.
(e) The effect of Shear deformation is considered.
(f) The beam is simply supported end condition.
(g) The applied moving load is concentrated.
(h) The prestressed and foundation parameters are all linear.

3. Mathematical formulation of the problem

The governing equations of motion describing the transverse translation $V(x,t)$ and angular rotation $\phi(x,t)$ of a finite damped shear beam resting on Vlasov foundation and subjected to moving load travelling at constant velocity are second order simultaneous partial differential equations given by

$$M\frac{\partial^2 V(x,t)}{dt^2}+K^*GA\left[\frac{\partial\Phi(x,t)}{\partial x}-\frac{\partial^2 V(x,t)}{dx^2}\right]-N\frac{\partial^2 V(x,t)}{dx^2}-C\frac{\partial V(x,t)}{dt}+F(x,t)=P(x,t) \quad (1)$$

and

$$EI\frac{\partial^2\Phi(x,t)}{dx^2}-K^*GA\left[\Phi(x,t)-\frac{\partial V(x,t)}{dx}\right]=0 \quad (2)$$

where M is the mass per unit length of the beam, K^* is the shear correction factor, G is the shear parameter of the beam, A is the cross-sectional area of the beam, N is the axial force, E is the Young's modulus of elasticity of the beam material, I is the moment of inertia, EI is the flexural stiffness / rigidity, x is the spatial coordinate, t is the time coordinate, $F(x,t)$ is the foundation reaction and $P(x,t)$ is the moving load acting on the beam per unit length. The relationship between the foundation reaction $F(x,t)$ and lateral deflection $V(x,t)$ is given by

$$F(x,t)=KV(x,t)-G\frac{\partial^2 V(x,t)}{dx^2} \quad (3)$$

Where K and G are two parameters of the foundation. Specifically, K is the footing stiffness and G is the shear coefficient and the load function $P(x,t)$ is assumed in the form

$$P(x,t)=P_0\delta(x-vt) \quad (4)$$

The Dirac delta function $\delta(\cdot)$ is given as

$$\int_b^a \delta(x-vt)f(x)dx=\begin{cases}0; & vt<a<b\\ f(vt); & a<vt<b\\ 1; & a<b<vt\end{cases} \quad (5)$$

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x = 0 \; and \; x = l$. Thus, boundary conditions are given as

$$V(0,t) = V(l,0) = 0; \qquad \frac{\partial V(0,t)}{\partial x} = \frac{\partial V(l,t)}{\partial x} = 0 \tag{6}$$

$$\Phi(0,t) = \Phi(l,0) = 0; \qquad \frac{\partial \Phi(0,t)}{\partial x} = \frac{\partial \Phi(l,t)}{\partial x} = 0 \tag{7}$$

with the initial conditions

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial x} \tag{8}$$

$$\Phi(x,0) = 0 = \frac{\partial \Phi(x,0)}{\partial x} \tag{9}$$

Substituting equations (3) and (4) into (1), after some simplifications and re-arrangements equations (1) and (2) become

$$M\frac{\partial^2 V(x,t)}{dt^2} + K^*GA\left[\frac{\partial \Phi(x,t)}{\partial x} - \frac{\partial^2 V(x,t)}{dx^2}\right] - N\frac{\partial^2 V(x,t)}{dx^2} - C\frac{\partial V(x,t)}{dt} + F(x,t) = P_0\delta(x - vt) \tag{10}$$

and

$$EI\frac{\partial^2 \Phi(x,t)}{dx^2} + K^*GA\left[\frac{\partial V(x,t)}{dx} - \Phi(x,t)\right] = 0 \tag{11}$$

3. Solution procedures

The shear beam investigated in the present study is uniformly finite. In order to obtain analytical solution of the initial boundary value problem in equations (10) and (11), we used the method of finite Fourier transformation. That is

$$V(n,t) = \int_0^l V(x,t)\sin\frac{n\pi x}{l}dx$$

(12)

with the inverse

$$V(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} V(x,t)\sin\frac{n\pi x}{l} \tag{13}$$

In the same manner, the finite Fourier cosine integral

$$\Phi(x,t) = \int_0^l W(n,t)\cos\frac{n\pi x}{l}dx \tag{14}$$

with the inverse

$$\Phi(x,t)=\frac{2}{l}\sum_{n=1}^{\infty}W(n,t)\cos\frac{n\pi x}{l} \tag{15}$$

is used. Thus, equations (12) and (14) when substituted into the governing equations (10) and (11) accordingly and after some simplifications and rearrangements yield

$$V_{tt}(n,t)+\mu_1 V_t(n,t)+\mu_2 V(n,t)+\mu_3\Phi(n,t)=Q_1\sin\theta_n t \tag{16}$$

and

$$\in_1 \Phi(n,t)+\in_2 V(n,t)=0 \tag{17}$$

where

$$\begin{aligned}&\mu_1=\frac{C}{M}, \qquad \mu_2=-K^*GA\left(\frac{n^2\pi^2}{Ml^2}\right)-\frac{Nn^2\pi^2}{Ml^2}+\frac{K}{M}-\frac{n^2\pi^2}{Ml^2}, \qquad \mu_3=-K^*GA\left(\frac{n\pi}{l}\right)\\ &Q_1=\frac{P_0}{M}, \qquad \in_1=EI\left(\frac{n^2\pi^2}{l^2}\right)-K^*GA, \qquad \in_3=K^*GA\left(\frac{n\pi}{l}\right)\end{aligned} \tag{18}$$

Hence, equations (17) and (18) are now the fundamental equations governing the motion of a finite shear beam on bi-parametric elastic foundation with the moving load travelling at a constant velocity. Now, solving equations (16) and (17) simultaneously, it is straight forward to show that

$$V_{tt}(n,t)+\mu_1 V_t(n,t)+\mu_5 V(n,t)=Q_1\sin\theta_n t \tag{19}$$

where

$$\in_3=-\frac{\in_2}{\in_1}, \qquad \mu_4=\mu_3\in_3, \qquad \mu_5=\mu_2+\mu_4 \tag{20}$$

The equation (19) is subjected to Laplace transformation defined as

$$F(s)=\int_0^{\infty}f(t)e^{-st}\,dt \tag{21}$$

where s is the Laplace parameter. After simplification and rearrangement, one obtains the simple algebraic equation given by

$$V(n,s)=Q_1\left(\frac{1}{s^2+\alpha_{a1}+\alpha_{a2}}\right)\left(\frac{\theta_n}{s^2+\theta_n^2}\right) \tag{22}$$

which is further simplified to give

$$V(n,s) = Q_1\left(\frac{1}{\left(s+\frac{\alpha_{a1}}{2}\right)^2+\rho^2}\right)\left(\frac{\theta_n}{s^2+\theta_n^2}\right) \tag{23}$$

where

$$\rho^2 = \alpha_{a5} - \left(s+\frac{\alpha_{a1}}{2}\right)^2 \tag{24}$$

At this point, in order to obtain the Laplace inversions of equation (23), the following representations are made

$$f(s) = \frac{1}{\left(s+\frac{\alpha_{a1}}{2}\right)^2+\rho^2} \tag{25}$$

and

$$g(s) = \left[\frac{\theta_n}{s^2+\theta_n^2}\right] \tag{26}$$

The Laplace inversion (23) is the convolution of $f(s)$ and $g(s)$ defined by

$$f(s)*g(s) = \int_0^t f(t-u)g(u)du \tag{27}$$

Using (25) and (26) in (27), Laplace inversion of (23) is given as

$$V(n,t) = \frac{R_0 e^{-\frac{\alpha_{a1}}{2}}}{P(\beta_1-\beta_0)(\beta_2-\beta_0)}\left\{\beta_2\left[Pe^{-\frac{\alpha_{a1}}{2}t}\sin\theta_n t - \theta_n \sin Pt\right] + \beta_0\left[Pe^{-\frac{\alpha_{a1}}{2}t}\sin\theta_n t + \theta_n \sin Pt\right]\right.$$
$$\left. - \alpha_{a1} P\theta_n\left[Pe^{-\frac{\alpha_{a1}}{2}t}\cos\theta_n t - \cos Pt\right]\right. \tag{28}$$

where

$$\beta_1 = (P+\theta_n)^2; \qquad \beta_2 = (P-\theta_n)^2; \qquad \beta_0 = -\left(\frac{\alpha_{a1}}{2}\right)^2 \tag{29}$$

Using (13), we have

$$V(x,t)=\sum_{n=1}^{\infty}\frac{2R_0e^{-\frac{\alpha_{a1}}{2}}}{LP(\beta_1-\beta_0)(\beta_2-\beta_0)}\left\{\beta_2\left[Pe^{-\frac{\alpha_{a1}}{2}t}\sin\theta_nt-\theta_n\sin Pt\right]+\beta_0\left[Pe^{-\frac{\alpha_{a1}}{2}t}\sin\theta_nt+\theta_n\sin Pt\right]-\alpha_{a1}P\theta_n\left[Pe^{-\frac{\alpha_{a1}}{2}t}\cos\theta_nt-\cos Pt\right]\right\}\sin\frac{n\pi x}{l} \quad (30)$$

which represents the transverse displacement to moving load of damped shear beam.

Similarly, in view of (15), we have

$$V(x,t)=\sum_{n=1}^{\infty}\frac{2R_0e^{-\frac{\alpha_{a1}}{2}}}{LP(\beta_1-\beta_0)(\beta_2-\beta_0)}\left\{\beta_2\left[Pe^{-\frac{\alpha_{a1}}{2}t}\sin\theta_nt-\theta_n\sin Pt\right]+\beta_0\left[Pe^{-\frac{\alpha_{a1}}{2}t}\sin\theta_nt+\theta_n\sin Pt\right]-\alpha_{a1}P\theta_n\left[Pe^{-\frac{\alpha_{a1}}{2}t}\cos\theta_nt-\cos Pt\right]\right\}\cos\frac{n\pi x}{l} \quad (31)$$

which represents the angular displacement to moving load of damped shear beam.

4. Comments on the closed-form solution

In this section, discussion of the analytical solution is based on transverse displacement only. It important to determine the condition under which resonance will take place. The effects of resonance in a dynamical system is of a great concern in engineering design and engineering analysis. Resonance takes place when the motion of the vibrating system becomes unbounded. That is, the point at which transverse displacement of an elastic beam increase without limit. In actual practice, when this happens, the structure would collapse as the intensive vibration causes cracks or permanent deformation in the vibrating structures. It is clearly seen from equation (31) that the beam on bi-parametric elastic foundation with a moving load is considered in this study will reach a state of resonance whenever

$$\beta_1=\beta_0 \quad (32)$$

$$\beta_2=\beta_0 \quad (33)$$

The critical velocity which is the velocity at which resonance takes place can be deduced from the conditions (32) and (33) respectively

$$V_{cr}^{1}=\frac{L}{n\pi}\left[\sqrt{\beta_0}-\sqrt{\alpha_{a5}+\beta_0}\right] \quad (34)$$

and

$$V_{cr}^{2}=\frac{L}{n\pi}\left[\sqrt{\alpha_{a5}+\beta_0}-\sqrt{\beta_0}\right] \quad (35)$$

5. Numerical Simulation and Discussion of Result

The uniform damped shear beam of length $L = 12.98m$, is considered and the load travel with constant velocity $V = 8.128m/s$, moment of inertia $I = 2.87698 \times 10^{-3}$, $\pi = 22/7$, the damping coefficient $C0 = 3000$ and the linear density of the beam $\mu = 2758.291kg/m$. The values of footing stiffness K and shear coefficient G are varied between $0N/m^3$ and $4 \times 10^7 N/m^3$ and the values of axial force N are varied between $0N$ and $4 \times 10^8 N$. The transverse displacement V of the beam is calculated and plotted against time t for various values of axial force N and footing stiffness K, shear coefficient G and load position x. The results are as shown on the various graphs given below.

In Figure 1, the displacement of a simply supported uniform damped shear beam under a uniformly distributed load travelling at constant velocity for various values of axial force N and for fixed values of other parameters is investigated. The graph shows that as the value of axial force N increases, the deflection of the beam decreases noticeably.

The flexure of a simply supported uniform damped shear beam to moving load travelling at constant velocity for various values of footing stiffness K is presented in Figure 2. It is observed that for fixed values of other parameters, higher value of footing stiffness K reduces the transverse displacement of the vibrating beam considerably.

In the same vein, similar graph is plotted against various values of time t in Figure 3 for a simply supported uniform damped shear beam under moving load travelling at constant velocity for varied values of shear coefficient G and for fixed values of other parameters. It is clearly noted that the deflection of the beam decreases significantly with increase in the value shear coefficient G. The response of a simply supported uniform damped shear beam subjected to moving load travelling at constant velocity for varied values of the load position coordinate x and for fixed values of other parameters is displayed in Figure 4. It is evident from the figure that the dynamic deflection at the mid-span of the beam is very large compare to other load positions. Figure 5 shows the comparison of the effects of Winkler foundation and Vlasov foundation on the transverse displacement of a damped shear beam when under the action of moving load travelling at constant velocity. Clearly shown, the deflection displacement of shear beam on Winkler foundation is greater than that of the Vlasov foundation. Likewise, Figure 6 displays the comparison of the effects of Winkler foundation and Vlasov foundation on the angular displacement of a simply supported uniform damped shear beam when under the action of moving load travelling at constant velocity. Interestingly, similar result is also obtained. The angular displacement of shear beam on Winkler foundation is higher than that of the Vlasov foundation.

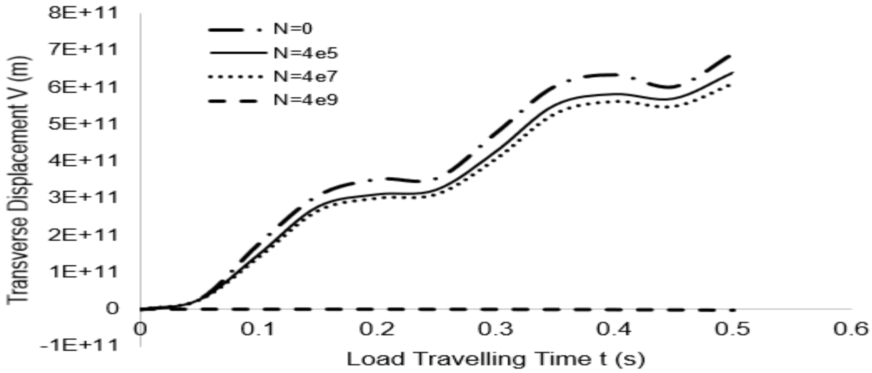

Figure 1: The response of a damped shear beam under the action of moving load for various values of axial force *N and for*
fixed values of other parameters.

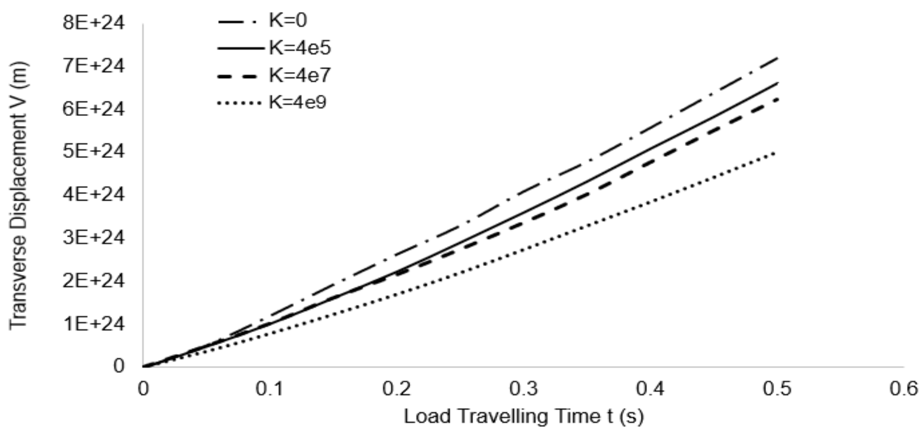

Figure 2: The response of a damped shear beam under the action of moving load for various values of footing stiffness K and for fixed values of other parameters.s.

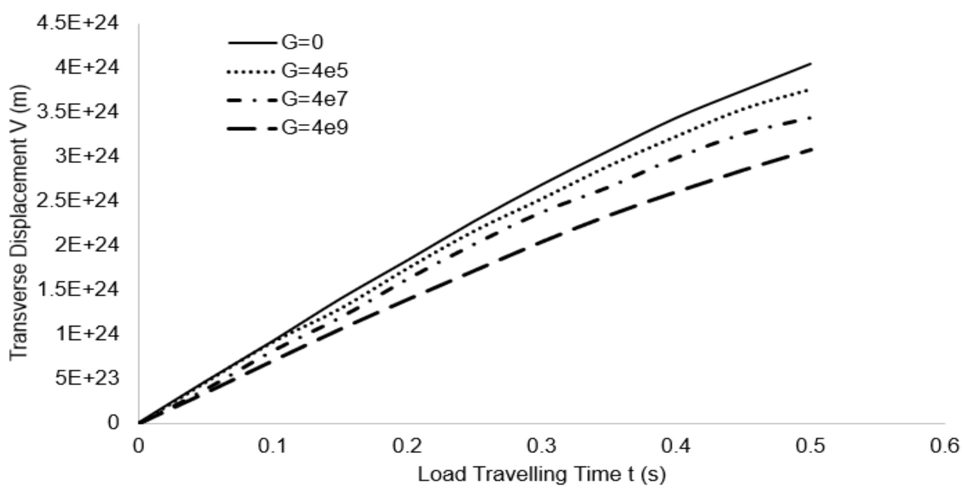

Figure 3: The response of a damped shear beam under the action of moving load for various values of shear modulus G and for fixed values of other parameters.

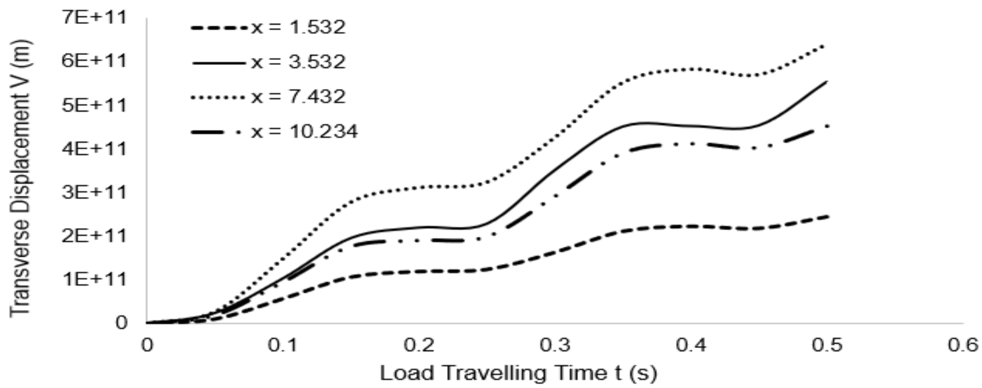

Figure 4: The response of a damped shear beam under the action of moving load for various values of load position x and for fixed values of other parameters.

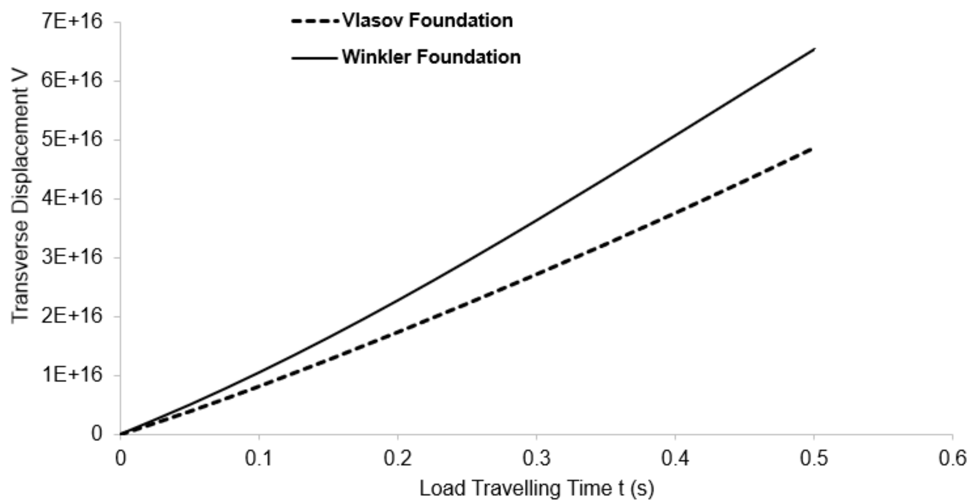

Figure 5: Comparison of the effects of Winkler foundation and Vlasov foundation on the transverse displacement of a damped shear beam subjected to moving load.

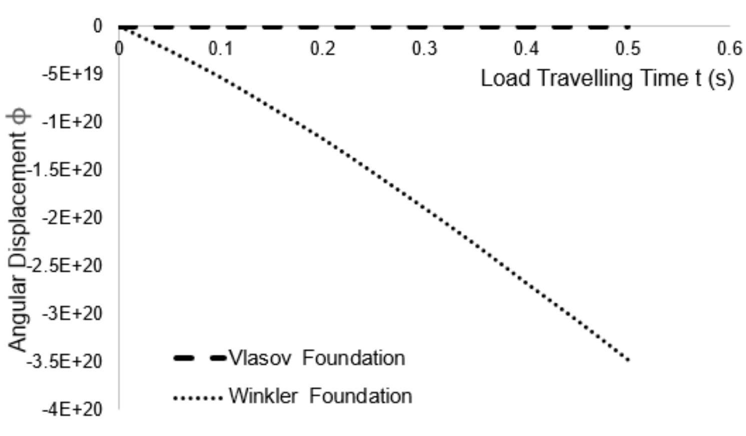

Figure 6: Comparison of the effects of Winkler foundation and Vlasov foundation on the angular displacement of a damped shear beam subjected to moving load

Conclusion

This paper investigates the dynamic behavior of a damped shear beam resting on bi-parametric elastic foundation when under the moving load. The governing equations are coupled second order partial differential equations. Solution procedure, involving finite Fourier transform technique and Laplace transformation in conjunction with convolution theory is used to obtain the solution of the coupled second order partial differential equations describing the motion of the beam-load system. Detailed analyses are performed to investigate the effect of some pertinent structural parameters such as axial force N, footing stiffness K, Shear coefficient G and load position x on dynamic deflection of the beam. It is evident from the plotted curves that the presence of these structural parameters contributes immensely to the stability of the beam when traversed by the travelling load. The study shows that the deflection of the beam reduces significantly with increased axial force, shear coefficient and stiffness of the foundation. Also, it shows that the dynamic deflection at the mid-span of the beam is very large compare to other load positions. The study further compares the effects of Winkler foundation and Vlasov foundation respectively on the transverse displacement and angular displacement of a simply supported uniform damped shear beam when under the action of moving load travelling at constant velocity. Consequently, the study further established the conditions under which the beam-load system will experience resonance phenomenon and the speeds at which this may occur.

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