BIOMECHANICAL DYNAMICS OF SEVERE ACUTE RESPIRATORY SYNDROME (SARS) AND THE PREDICTION OF INBUILT RAISING MULTIPLIER λ .

¹Egboro S. A., ²Akpata. E. and ³Enaibe E. A.

^{1,3}Department of Physics, Federal University of Petroleum Resources, Effurun. ²Department of Physics, Delta State University, Abraka.

Abstract

Dynamical quantities such as energy, pressure-gradient, velocity-profile, make up the biomechanics of the human system, and the attenuation induced by localized and non-localized infectious diseases causes a malfunction in these quantities. In this work, we establish that every living and non-living matter possess vibration, and vibration produces wave. Thus Man (host) and the virus or bacteria (parasite) have their own independent characteristic vibrations. We first superpose the parasitic wave on the host wave and the resultant constitutive carrier wave, is studied using Fourier transform method. We used the characteristic variants of the human and parasitic vibrations to determine the general influence of localized infectious diseases in the human viscoelastic system. It is shown in this work, that SARS and other related localized infectious diseases has an incubation period of about 9 to 30 days, depending on the nature and circumstances of the host wave under attack. This is indicated by several of the spectra, but most importantly the high peak resonance in the spectra of the total phase angle, displacements and velocities for lower and higher harmonics. Also, it is shown in this work that the effect of SARS or any form of localized human infectious diseases become more complicated after about 100 days of infection.

Keywords: Attenuation, Host wave, parasitic wave, constitutive carrier wave, SARS-CoV-2, vibration, harmonics.

1.0 Introduction.

The role of SARS CoV-2 and the influence of some other related localized infectious diseases in the human viscoelastic system has in general been poorly understood. There is still no adequate understanding of the formation of the virulent disease and the possible cure to it. The SARS pandemic is still among the most pressing health problems in the world today. The Corona Virus disease of 2019 (known as COVID-19 is a human infectious disease caused by a novel family of coronavirus-2 called the severe acute respiratory syndrome (SARS-CoV-2) [1-2].

All members of the SARS-CoV-2 have spiky proteins on their surfaces which enable them to hook and penetrate the cells of their hosts which could be human or animals. This penetration of the virus into the cells of the host causes an infection in the nose, upper throat and lungs [3-4]. Generally, viruses are non-living collection of molecules that needs a host to survive while bacteria are living organism that can survive inside and outside the host cell. Therefore, for viruses, once they are inside the cells, they take over its metabolism and make copy of themselves over and over and thereby causing adverse effects on the system which is the infectious state of the host [5].

Generally, all Human infectious diseases that are caused by bacteria, virus etc, have their own independent biomechanics and their effect on the Human system can be classified into two major groups: (i) localized infectious diseases and (ii) non-localized infectious diseases. Corona Virus Disease 2019 (COVID-19), Cholera, Leprosy, Gonorrhoea, Tuberculosis, Syphilis, that are caused by virus or bacteria are referred to as localized Human infectious diseases. This is because they have a preferred or definite region of space which they occupy within the Human biological system.

While non-localized Human infectious diseases such as Human Immunodeficiency Virus (HIV), Acquired Immune deficiency Syndrome (AIDS), Measles, Chickenpox, Ebola, Lassa Fever, that are caused by viruses are non-localized Human infectious diseases, because they do not have a specific or preferred region of space which they occupy within the Human biological system. Thus, the presence of these non-localized infectious diseases is felt everywhere within the Human system.

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In summary, the whole view of the clinical literature of diseases is that the fatal effect of these diseases stems from the attack they make on a person's immune system. The invading bacteria or virus targets and alters the Human immune system, thereby increasing the risk and impact of other infections and diseases. In addition to the claim of the clinical literature of diseases, is that there is also a cause (vibration) that gives the bacteria and virus, which causes human infectious diseases, their own intrinsic characteristics, dynamic activity and existence. It is not the human system that gives the invading bacteria or virus life and existence, since they are living organisms, having their own independent characteristics even before they entered the biological system of Man.

The interference of one wave $y_2(r,t)$ say 'parasitic wave' on another wave $y_1(r,t)$ say 'host wave' could cause the 'host wave' to attenuate to zero if they are out of phase. The decay process of $y_1(r,t)$ can be gradual, over-damped or critically damped depending on the rate in which the amplitude of the resultant wave is brought to rest by the destructive influence of the invading parasite. However, the general understanding is that the combination of $y_1(r,t)$ and $y_2(r,t)$ would first yield a third stage called the resultant wave (r,t), before the process of attenuation sets in. In this work, we refer to the resultant wave as the constitutive carrier wave CCW and we think this is a better representation. We shall always refer to Man being characterized by the host wave $y_1(r,t)$, while we shall refer to bacteria and virus as parasites, again characterized by parasitic wave $y_2(r,t)$.

Every Human infectious disease has an incubation period or window period. This is the time between original infection with parasitic caused-disease and the appearance of detectable antibodies to the disease [6]. The World Health Organization (WHO) says it takes about 5 - 6 days from when someone is infected with SARS-CoV-2 for symptoms to show, however it can take up to 2 - 14 days. Also, for human immunodeficiency virus HIV infection (non-localized infection), the incubation period is normally a period of about 14 - 21 days [7-8].

The Human cyclic heart contraction generates pulsatile blood flow and latent vibration. Thus, the Human heart is the cause of Human vibration. The generated latent vibration is sinusoidal and central in character, that is, it flows along the middle of the vascular blood vessels and in the process, it orients the active particles of the blood and sets them into oscillating motion with a unified frequency. It is the Human blood that responds to - and transfer the latent vibration generated by the heart with a specified wave form round the entire Human system [9].

The equation of motion obeyed by the CCW as it propagates along the human blood vessels experiences two major resistive factors. Firstly, the resistance poses by the elasticity of the walls of the blood vessels and secondly, the elastic resistance of the blood medium. The medium mass and elasticity property determines how fast the carrier wave can travel in the medium. Consequently, the differential equation of motion would partly be Newtonian due to the nature of the fluid aspect of the blood and non-Newtonian due to the particle constituents of the blood. We can write the differential equation of motion as:

$$F = -\eta \frac{dy^2}{dt} - \mu y^2 \tag{1.1}$$

where η is the dynamic viscosity of the human blood which has a unit of $kgm^{-1}s^{-1}$, and μ is the elasticity of the human aorta the unit is $kgm^{-1}s^{-2}$, while *m* and *v* are the mass and velocity of blood respectively, since mass $m = density \rho \times Volume V$, and $v = d^2y/dt^2$ then equation (1.1) becomes

$$\rho V \frac{d^2 y_m}{dt^2} + 2\eta y_m \frac{dy_m}{dt} + \mu y_m^2 = 0$$
(1.2)

where y_m is the maximum value of the CCW which is equal to the instantaneous amplitude when the oscillating phase is zero.

There are four attenuating characteristics present in the CCW. If a, ω , ε , and k represent the initial parameters of the host wave contained in the CCW, and b, ω' , ε' , and k' the initial characteristics of the parasitic wave contained in the carrier wave, therefore, $(a - b\lambda)$, $(\omega - \omega'\lambda)$, $(\varepsilon - \varepsilon'\lambda)$ and $(k - k'\lambda)$ will represent the characteristics of the host wave that survives after a given time t, where λ is the inbuilt raising multiplier. Thus, the fractional change *FC* and the attenuation constant *AC* of the host wave characteristics, which we shall denote as δ and ξ respectively are:

$$FC = \delta = \frac{1}{4} \left[\left(\frac{a - b\lambda}{a} \right) + \left(\frac{\omega - \omega'\lambda}{\omega} \right) + \left(\frac{\varepsilon - \varepsilon'\lambda}{\varepsilon} \right) + \left(\frac{k - k'\lambda}{k} \right) \right]$$
(1.3)

$$AC = \xi = \frac{FC|_{\lambda=i} - FC|_{\lambda=i+1}}{unit time (s)} = \frac{\delta_i - \delta_{i+1}}{unit time (s)}$$
(1.4)

Consequently, in this study, equation (1.4) gives $\xi = 0.006404 \ s^{-1}$ for all values of the inbuilt raising multiplier: λ_i ($i = 0, 1, 2, 3, \dots, \lambda_{max}$). The information provided in (1.4) is used to compute the various values of the time taken for the CCW to attenuate to zero. The maximum time the CCW lasted as a function of the raising multiplier λ is also calculated from the attenuation equation. We adopted a slow varying regular interval for the inbuilt raising multiplier, to ensure that we have a clear parameter space that is accessible to our model. Hence, we may also write the equation relating the FC to the time [10] as follows:

$$\delta = e^{-(\xi t)/2^{\gamma}\lambda} \tag{1.5}$$

$$\ln \delta = -\frac{\xi t}{2^{\gamma} \lambda} \to 2^{\gamma} \lambda \ln \delta = -\xi t$$
(1.6)

$$t = -\left(\frac{2^{\gamma}\lambda}{\xi}\right)\ln\delta \tag{1.7}$$

where γ is the fractional index of any physical system under study and t is the decay time. Thus, for Bacteria or virus that is localized within the human system, we assume: $\gamma = 0$ or 1 depending on the nature of the infection. Also, for nonlocalized bacteria or virus within the human system, we assume: $\gamma = 2 \text{ or } 3$ which also depend on the nature of the infection. Details of the method of computing the various values of the latent characteristic variants of the CCW and the raising multiplier λ can be found in [10].

The maximum value of the multiplier: $\lambda_m = 156.1424 \cong 157$ or $\lambda_m \cong 158$ (including zero) and the total time taken for the CCW to decay to zero is: $t \approx 22532970 \, s = \approx 260 \, days$ (8 months). To get the common difference between the respective terms in days we apply the arithmetic progression equation:

$$T_n = a_1 + (n-1)d$$

(1.8)where T_n is the nth term in the sequence, and $T_n = 260$ days; a_1 is the first term in the sequence which in this study $a_1 =$ 0, and d is the common difference between the terms in the sequence, finally n is the number of terms in the sequence. Thus, we get:

(1.9)

(2.1)

$$260 = 0 + (158 - 1) \times d \rightarrow d = 1.650$$

The time series which is in unit of days becomes: $0, 1.656, 3.312, 4.968, 6.624, \dots, 260$ days.

This paper is outlined as follows. In section one we gave a brief introduction and theory of the work under study. The mathematical theory of the Fourier transform of the constitutive carrier wave is given in section two. The results emerging from this study is shown in section three. We discussed the outcome of the results in section four and the work is brought to an end by concluding remarks in section five, and this is immediately followed by the lists of references and appendix.

Research methodology: The characteristics of the host wave and the parasitic wave are identified as $y_1(r,t)$ and 1.1. $y_2(r,t)$ respectively and thereafter, both waves are superposed on one another. Finally, the behaviour of the resultant constitutive carrier wave CCW is then studied with the help of Fourier transform technique.

2.0 Mathematical theory.

[9] in their work, assume that the host wave is represented as $y_1(r, t)$ while the parasitic wave $y_2(r, t)$, and both waves are defined by the following characteristic equations:

 $\dot{\cdot}$ That $y_1(r,t)$ represents the host wave or the human latent vibration. The equation which may include; the amplitude a (m), angular velocity, ω (rad/s), wave number k (rad/m), the phase angle ε (radian), propagation time t (s), and these quantities shall be the basic characteristics of the host latent vibration. The host wave equation which we may write as:

$$y_1(r,t) = a \cos\left(\vec{k} \cdot \vec{r} - \omega t - \varepsilon\right)$$

where the position vector \vec{r} of any particle of blood is defined as: $\vec{r} = r (\cos \varepsilon + \sin \varepsilon)$, which is 2D in character and the motion of the particles of blood is constant with respect to one axis.

* That $y_2(\vec{r}, t)$ represents the parasitic wave or parasitic latent vibration. The equation which may also include; the amplitude b (m), angular velocity, ω' (rad/s), wave number k' (rad/m), phase angle ε' (radian), propagation time t (s), the inbuilt raising multiplier λ , and these quantities shall represent the characteristics of the parasitic latent vibration. The parasitic wave we may also write as:

$$y_{2}(r,t) = b\lambda \cos\left(\vec{k}'\lambda.\vec{r} - \omega'\lambda t - \varepsilon'\lambda\right)$$
(2.2)

where the position vector \vec{r} of the parasite in the human blood is defined as: $\vec{r} = r (\cos(\varepsilon'\lambda) + \sin(\varepsilon'\lambda))$.

* The constitutive carrier wave CCW $y(\vec{r},t)$ which is the resultant of the superposition of the parasitic wave on the host wave, is given by the equation:

$$y(r,t) = y_1(r,t) + y_2(r,t)$$

$$y(r,t) = \sqrt{(a^2 - b^2 \lambda^2) - 2(a - b\lambda)^2 \cos((\omega - \omega'\lambda)t - (\varepsilon - \varepsilon'\lambda))} \times$$

$$\cos\left(\vec{k_c} \vec{r} - (\omega - \omega'\lambda)t - E(t)\right)$$
(2.4)

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The above equation (2.4) is the constitutive carrier wave CCW necessary for our study. It is this equation that governs the biomechanics of the coexistence of two interfering vibrations within a given system. The combine wave number $\vec{k_c}$ and the position vector \vec{r} of any particle of the fluid in two-dimensional space is defined as: $\vec{k_c}\vec{r} = (k - k'\lambda)r(\cos\varphi + \sin\varphi)$, the phase angle: $\varphi = \pi - (\varepsilon - \varepsilon'\lambda)$ and $r \approx 0.015 m$ is taken to be the approximate radius of the human aorta since the heart is the source of human vibration. The total phase angle of the CCW is given by:

$$E(t) = \tan^{-1} \left[\frac{a \sin \varepsilon - b\lambda \sin \left((\omega - \omega'\lambda)t - \varepsilon'\lambda \right)}{a \cos \varepsilon - b\lambda \cos \left((\omega - \omega'\lambda)t - \varepsilon'\lambda \right)} \right]$$
(2.5)

2.1 The characteristic angular velocity Z(t) of the CCW.

The characteristic angular velocity of the CCW is found by differentiating (2.5) with respect to time, so that

$$\frac{dE(t)}{dt} = -Z(t)$$
(2.6)
where: $Z(t) = (a - \omega t) \left[-b^2 \lambda^2 + ab\lambda \cos\left((\varepsilon - \varepsilon' \lambda) + (\omega - \omega' \lambda)t\right) \right]$ (2.7)

where:
$$Z(t) = (\omega - \omega'\lambda) \left[\frac{b^2 \lambda^2 + ab\lambda \cos\left((\varepsilon - \varepsilon'\lambda) + (\omega - \omega'\lambda)t\right)}{a^2 + b^2 \lambda^2 + 2ab\lambda \cos\left((\varepsilon - \varepsilon'\lambda) + (\omega - \omega'\lambda)t\right)} \right]$$
(2.7)

The characteristic angular velocity of the CCW has the dimension of rad/s.

2.2. The characteristic angular acceleration Q(t) of the CCW.

Also, to determine the group angular acceleration Q(t), we need to vary the characteristic total phase velocity z(t) with respect to time t. When this is done, we get:

$$\frac{dZ(t)}{dt} = Q(t) = \frac{(n-n'\lambda)^2 \left(ab^3\lambda^3 - a^3b\lambda\right)\sin\left((n-n'\lambda)t + (\varepsilon - \varepsilon'\lambda)\right)}{\left(a^2 + b^2\lambda^2 + 2\ ab\lambda\cos\left((n-n'\lambda)t + (\varepsilon - \varepsilon'\lambda)\right)\right)^2}$$
(2.8)

Hence Q(t) is the characteristic angular acceleration of the CCW and it is measured in rad/s^2 . Consequently, we can use the Fourier transform method to convert the classical equation (2.4) to an equation in the frequency-time domain. The Fourier transform in this case states that:

$$F[y(\omega,\omega',t)] = A_0 + \sum_{\alpha=1}^{\infty} A_\alpha \cos\left[\alpha \left((\omega - \omega'\lambda)t + E(t)\right)\right] + \sum_{\alpha=1}^{\infty} B_\alpha \sin\left[\alpha \left((\omega - \omega'\lambda)t + E(t)\right)\right]$$
(2.9)

where the amplitudes, A_0 , A_α and B_α , are the usual Euler coefficients which are defined as follows:

$$A_{0} = \frac{1}{2\tau} \int_{0}^{\tau} \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos\left((\omega - \omega'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \times \cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t - E(t)\right) dt}$$

$$A_{\alpha} = \frac{1}{\tau} \int_{0}^{\tau} \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos\left((\omega - \omega'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \times \cos\left(\vec{k}_{c} \cdot \vec{r} - ((\omega - \omega'\lambda)t + E(t))\right) \cdot \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt}$$

$$B_{\alpha} = \frac{1}{\tau} \int_{0}^{\tau} \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos\left((\omega - \omega'\lambda)t + E(t)\right) = \frac{1}{\tau} \left((\omega - \omega'\lambda)t + E(t)\right) \times \cos\left[\alpha\left((\omega - \omega'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \times \cos\left(\vec{k}_{c} \cdot \vec{r} - ((\omega - \omega'\lambda)t + E(t))\right) \cdot \sin\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - ((\omega - \omega'\lambda)t + E(t))\right) \cdot \sin\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - ((\omega - \omega'\lambda)t + E(t))\right) \cdot \sin\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - ((\omega - \omega'\lambda)t + E(t))\right) \cdot \sin\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - ((\omega - \omega'\lambda)t + E(t))\right) \cdot \sin\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

$$Cos\left(\vec{k}_{c} \cdot \vec{r} - (\omega - \omega'\lambda)t + E(t)\right) + \cos\left[\alpha\left((\omega - \omega'\lambda)t + E(t)\right)\right] dt$$

In this study we take the period τ of the human heart as; $\tau(\omega - \omega'\lambda) = 2\pi$. The integration of the various amplitudes given by the Euler coefficients yields two independent physical quantities: the displacement (*m*) and the velocity component (*m*/*s*). Hence the displacement and velocity components of A_0 are separately given by the equations:

$$A_{0} = \left(\frac{\omega - \omega'\lambda}{4\pi}\right) \left\{ \frac{\sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos\left(\varepsilon - \varepsilon'\lambda\right)} \cdot \sin\left(\vec{k}_{c}.\vec{r} - E(0)\right)}{\left[(\omega - \omega'\lambda) - Z(0)\right]} - \frac{\sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos\left(2\pi - (\varepsilon - \varepsilon'\lambda)\right)} \cdot \sin\left(\vec{k}_{c}.\vec{r} - 2\pi - E(\tau)\right)}{\left[(\omega - \omega'\lambda) - Z(\tau)\right]} \right\} \quad (m/s) \quad (2.13)$$

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$$A_{0} = \left(\frac{\omega - \omega'\lambda}{4\pi}\right) \left\{ \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \sin\left(2\pi - (\varepsilon - \varepsilon'\lambda)\right) \cdot \sin\left(\vec{k}_{c} \cdot \vec{r} - 2\pi - E(\tau)\right)}{\left[(\omega - \omega'\lambda) - Z(\tau)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2} \cos\left(2\pi - (\varepsilon - \varepsilon'\lambda)\right)}} + \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \cdot \sin\left(\varepsilon - \varepsilon'\lambda\right) \cdot \sin\left(\vec{k}_{c} \cdot \vec{r} - E(0)\right)}{\left[(\omega - \omega'\lambda) - Z(0)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2} \cos\left(\varepsilon - \varepsilon'\lambda\right)}} \right\}$$
(2.14)

where we have used the fact that $\sin(-x) = -\sin x$ (odd and anti-symmetric function) and $\cos(-x) = \cos x$ (even and symmetric function). Also, the *displacement* component of the Euler coefficient A_{α} is:

$$A_{\alpha} = \frac{(\omega - \omega'\lambda)}{4\pi} \left\{ \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \sin(\vec{k}_c.\vec{r} - (1 - \alpha) E(0))}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(0)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cdot \sin(\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau)))}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} + \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \sin(\vec{k}_c.\vec{r} - (1 + \alpha) E(0))}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(0)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cdot \sin(\vec{k}_c.\vec{r} - (1 + \alpha)(2\pi + E(\tau)))}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} \right\} (m/s)$$

$$(2.15)$$

The displacement component of (2.15) has the dimension of *metres m*. Again, the *velocity* component of A_{α} from the transform of the CCW is given by:

$$\begin{aligned} A_{\alpha} &= \frac{(\omega - \omega'\lambda)}{4\pi} \left\{ \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \cdot \sin(2\pi - (\varepsilon - \varepsilon'\lambda)) \cdot \sin(\vec{k}_{c}.\vec{r} - (1 - \alpha)(2\pi + E(\tau)))}{(1 - \alpha)[(\omega - \omega'\lambda) - Z(\tau)] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos(2\pi - (\varepsilon - \varepsilon'\lambda))}} + \right. \\ &\left. \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \cdot \sin(\varepsilon - \varepsilon'\lambda) \cdot \sin(\vec{k}_{c}.\vec{r} - (1 - \alpha) E(0))}{(1 - \alpha)[(\omega - \omega'\lambda) - Z(0)] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos(\varepsilon - \varepsilon'\lambda)}} + \right. \\ &\left. \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \cdot \sin(2\pi - (\varepsilon - \varepsilon'\lambda)) \cdot \sin(\vec{k}_{c}.\vec{r} - (1 + \alpha)(2\pi + E(\tau)))}{(1 + \alpha)[(\omega - \omega'\lambda) - Z(\tau)] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos(2\pi - (\varepsilon - \varepsilon'\lambda))}} + \right. \\ &\left. \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \cdot \sin(\varepsilon - \varepsilon'\lambda) \cdot \sin(\vec{k}_{c}.\vec{r} - (1 + \alpha) E(0))}{(1 + \alpha)[(\omega - \omega'\lambda) - Z(0)] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos(\varepsilon - \varepsilon'\lambda)}} \right\} (m/s) \end{aligned}$$

Again, the *displacement* component of the Euler coefficient B_{α} from the transform of the CCW is:

$$B_{\alpha} = \frac{(\omega - \omega'\lambda)}{4\pi} \left\{ \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \cos(\vec{k}_c \cdot \vec{r} - (1 + \alpha) E(0))}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(0)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos(\vec{k}_c \cdot \vec{r} - (1 + \alpha)(2\pi + E(\tau)))}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} + \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos(\vec{k}_c \cdot \vec{r} - (1 - \alpha)(2\pi + E(\tau)))}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \cos(\vec{k}_c \cdot \vec{r} - (1 - \alpha)E(0))}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(0)]} \right\} (m/s)$$

$$(2.17)$$

and the dimension of the *displacement* component is *metres* m. While the *velocity* component of B_{α} from the transform of the CCW is given by:

$$\boldsymbol{B}_{\alpha} = \frac{(\omega - \omega'\lambda)}{4\pi} \begin{cases} \frac{(a - b\lambda)^{2} (\omega - \omega'\lambda) \cdot \sin\left(2\pi - (\varepsilon - \varepsilon'\lambda)\right) \cdot \cos\left(\bar{k}_{c}\cdot \vec{r} - (1 + \alpha)\left(2\pi + E(\tau)\right)\right)}{(1 + \alpha)\left[(\omega - \omega'\lambda) - Z(\tau)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}\cos\left(2\pi - (\varepsilon - \varepsilon'\lambda)\right)}} + \end{cases}$$

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$$\frac{(a-b\lambda)^{2}(\omega-\omega'\lambda)\cdot\sin(\varepsilon-\varepsilon'\lambda)\cdot\cos\left(\vec{k}_{c}\cdot\vec{r}-(1+\alpha)\ E(0)\right)}{(1+\alpha)\left[(\omega-\omega'\lambda)-Z(0)\right]\cdot\sqrt{(a^{2}-b^{2}\lambda^{2})-2(a-b\lambda)^{2}\cos(\varepsilon-\varepsilon'\lambda)}} - \frac{(a-b\lambda)^{2}(\omega-\omega'\lambda)\cdot\sin\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)\cdot\cos\left(\vec{k}_{c}\cdot\vec{r}-(1-\alpha)\ (2\pi+E(\tau))\right)}{(1-\alpha)\left[(\omega-\omega'\lambda)-Z(\tau)\right]\cdot\sqrt{(a^{2}-b^{2}\lambda^{2})-2(a-b\lambda)^{2}\cos\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)}} - \frac{(a-b\lambda)^{2}(\omega-\omega'\lambda)\cdot\sin(\varepsilon-\varepsilon'\lambda)\cdot\cos\left(\vec{k}_{c}\cdot\vec{r}-(1-\alpha)\ E(0)\right)}{(1-\alpha)\left[(\omega-\omega'\lambda)-Z(0)\right]\cdot\sqrt{(a^{2}-b^{2}\lambda^{2})-2(a-b\lambda)^{2}\cos(\varepsilon-\varepsilon'\lambda)}}\right\} (m/s)$$
(2.18)

Also, (2.18) has the dimension of *metres per second*, m/s. The *displacement* component of the transformed CCW is therefore, the combination of terms, A_0 , A_α and B_α in (2.13), (2.15) and (2.17) respectively and substituting them into (2.9). After a careful substitution and arrangement, we get:

$$\begin{split} F[y(\omega,\omega't)] &= \left(\frac{\omega - \omega'\lambda}{4\pi}\right) \left\{ \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \sin(\vec{k}_c.\vec{r} - E(0))}{[(\omega - \omega'\lambda) - Z(0)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cdot \sin(\vec{k}_c.\vec{r} - 2\pi - E(\tau))}{[(\omega - \omega'\lambda) - Z(\tau)]} \right\} \\ &+ \frac{(\omega - \omega'\lambda)}{4\pi} \sum_{\alpha=1}^{\infty} \left\{ \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \sin(\vec{k}_c.\vec{r} - (1 - \alpha) E(0))}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(0)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \sin(\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau)))}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} + \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \sin(\vec{k}_c.\vec{r} - (1 + \alpha) E(0))}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(0)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cdot \sin(\vec{k}_c.\vec{r} - (1 + \alpha)(2\pi + E(\tau)))}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} \right\} \times \\ \cos\left[\alpha((\omega - \omega'\lambda)t + E(t))\right] \\ &+ \frac{(\omega - \omega'\lambda)}{4\pi} \sum_{\alpha=1}^{\infty} \left\{ \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \cos[\vec{k}_c.\vec{r} - (1 + \alpha)(2\pi + E(\tau))]}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos[\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau))]}{(1 + \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} + \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos[\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau))]}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos[\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau))]}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos[\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau))]}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \cos[\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau))]}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} - \frac{\sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos(\varepsilon - \varepsilon'\lambda)} \cdot \cos[\vec{k}_c.\vec{r} - (1 - \alpha)(2\pi + E(\tau))]}{(1 - \alpha) [(\omega - \omega'\lambda) - Z(\tau)]} \right\} \times \\ \sin\left[\alpha((\omega - \omega'\lambda)t + E(t)] - (\omega - \omega'\lambda) - Z(0)\right] \right\} \times \\ \sin\left[\alpha((\omega - \omega'\lambda)t + E(t)] - (\omega - \omega'\lambda) - Z(0)\right] \right\}$$

In order to have a common summation in the harmonic frequency part of (2.19), we invoke into it the trigonometric identity (see appendix). Therefore, the *displacement* component of the CCW becomes:

(2.19)

$$\begin{split} F\left[y(\omega,\omega't)\right] &= \left(\frac{\omega-\omega'\lambda}{4\pi}\right) \begin{cases} \frac{\sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left(\varepsilon-\varepsilon'\lambda\right)}\cdot\sin\left(\vec{k}_c.\vec{r}-E(0)\right)}{\left[(\omega-\omega'\lambda)-Z(0)\right]} & - \\ \frac{\sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)}\cdot\sin\left(\vec{k}_c.\vec{r}-2\pi-E(\tau)\right)}{\left[(\omega-\omega'\lambda)-Z(\tau)\right]} \\ &+ \frac{(\omega-\omega'\lambda)}{4\pi} \sum_{\alpha=1}^{\infty} \begin{cases} \frac{\sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left(\varepsilon-\varepsilon'\lambda\right)}}{(1-\alpha)\left[(\omega-\omega'\lambda)-Z(0)\right]} \times \end{cases} \end{split}$$

$$\frac{\sin\left[\vec{k}_{c}\cdot\vec{r}-\alpha\left((\omega-\omega'\lambda)t+E(t)-E(0)\right)-E(0)\right]-}{\sqrt{(a^{2}-b^{2}\lambda^{2})-2(a-b\lambda)^{2}\cos\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)}}\times \\
\frac{\sin\left[\vec{k}_{c}\cdot\vec{r}-\alpha\left((\omega-\omega'\lambda)+E(t)-E(\tau)-2\pi\right)-E(\tau)-2\pi\right]}{\left(1-\alpha\right)\left[(\omega-\omega'\lambda)-Z(0)\right]}\times \\
\frac{\sqrt{(a^{2}-b^{2}\lambda^{2})-2(a-b\lambda)^{2}\cos\left(\varepsilon-\varepsilon'\lambda\right)}}{(1+\alpha)\left[(\omega-\omega'\lambda)+E(t)-E(0)\right]-E(0)\right]-} \\
\frac{\sqrt{(a^{2}-b^{2}\lambda^{2})-2(a-b\lambda)^{2}\cos\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)}}{(1+\alpha)\left[(\omega-\omega'\lambda)-Z(\tau)\right]}\times \\
\frac{\sin\left[\vec{k}_{c}\cdot\vec{r}+\alpha\left((\omega-\omega'\lambda)t+E(t)-E(\tau)-2\pi\right)-E(\tau)-2\pi\right]\right\}}{\left(1+\alpha\right)\left[(\omega-\omega'\lambda)+E(t)-E(\tau)-2\pi\right]-E(\tau)-2\pi\right]\right\}} (m/s)$$
(2.20)

Equation (2.20) represents the displacement of the CCW as the time progresses and the dimension is *metres m*. Also, the velocity component of the transformed CCW is the combination of velocities terms, A_0 , A_α and B_α in (2.14), (2.16) and (2.18) respectively and substituting them into (2.9). After a careful substitution and arrangement, we get:

$$\begin{split} F[y(\omega,\omega'z)] &= \left(\frac{\omega-\omega'\lambda}{4\pi}\right) \left\{ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(2\pi-(\varepsilon-\varepsilon'\lambda)) \cdot \sin(\vec{k}_c.\vec{r}-2\pi-E(\tau))}{[(\omega-\omega'\lambda)-Z(\tau)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(2\pi-(\varepsilon-\varepsilon'\lambda))} + \right. \\ &\left. \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \sin(\vec{k}_c.\vec{r}-E(0))}{[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \right\} \\ &+ \frac{(\omega-\omega'\lambda)}{4\pi} \sum_{a=1}^{\infty} \left\{ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(2\pi-(\varepsilon-\varepsilon'\lambda)) \cdot \sin(\vec{k}_c.\vec{r}-(1-\alpha)(2\pi+E(\tau)))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(\tau)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \right\} \\ &+ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \sin(\vec{k}_c.\vec{r}-(1-\alpha)E(0))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &+ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(2\pi-(\varepsilon-\varepsilon'\lambda)) \cdot \sin(\vec{k}_c.\vec{r}-(1+\alpha)(2\pi+E(\tau)))}{(1+\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &+ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \sin(\vec{k}_c.\vec{r}-(1+\alpha)E(0))}{(1+\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &+ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \sin(\vec{k}_c.\vec{r}-(1+\alpha)E(0))}{(1+\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &+ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \sin(2\pi-(\varepsilon-\varepsilon'\lambda)) \cdot \cos(\vec{k}_c.\vec{r}-(1+\alpha)(2\pi+E(\tau))))}{(1+\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &+ \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1+\alpha)E(0))}{(1+\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &- \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(2\pi-(\varepsilon-\varepsilon'\lambda)) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)(2\pi+E(\tau)))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &- \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)(2\pi+E(\tau)))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &- \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)(2\pi+E(\tau)))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &- \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)E(0))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ &- \frac{(a-b\lambda)^2(\omega-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)E(0))}{(1-\alpha)[(\omega-\omega'\lambda)-Z(0)] \cdot \sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos(\varepsilon-\varepsilon'\lambda)} \\ \\ &+ \frac{[(\omega-\omega'\lambda)-\omega'\lambda)}{(1-\alpha)[(\omega-\omega'\lambda)-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)E(0))} \\ \\ &+ \frac{[(\omega-\omega'\lambda)-\omega'\lambda)}{(1-\alpha)[(\omega-\omega'\lambda)-\omega'\lambda) \cdot \sin(\varepsilon-\varepsilon'\lambda) \cdot \cos(\vec{k}_c.\vec{r}-(1-\alpha)E(0))} \\ \\ &+ \frac{[(\omega-\omega'\lambda)-\omega'\lambda)}{(1-\alpha)[(\omega-\omega'\lambda)-\omega'\lambda)} \\ \\ &+ \frac{[(\omega-\omega'\lambda)-\omega'\lambda)}{(1-\alpha)[(\omega-\omega'\lambda)-\omega'\lambda$$

 $\sin\left[\alpha\left(\left(\omega-\omega'\lambda\right)t+E(t)\right)\right]$

(2.21)

Again, in order to have a common summation in the harmonic frequency part of (2.21), we invoke into it the trigonometric identity (see appendix). The *velocity* component of the CCW therefore becomes:

$$\begin{split} F\left[y(\omega,\omega't)\right] &= \left(\frac{\omega-\omega'\lambda}{4\pi}\right) \left\{ \frac{(a-b\lambda)^2\left(\omega-\omega'\lambda\right)\cdot\sin\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)\cdot\sin\left(\vec{k}_c.\vec{r}-2\pi-E(\tau)\right)}{\left[\left(\omega-\omega'\lambda\right)-Z(\tau)\right]\cdot\sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)} \right. \\ &+ \left. \frac{(a-b\lambda)^2\left(\omega-\omega'\lambda\right)\cdot\sin(\varepsilon-\varepsilon'\lambda)\cdot\sin\left(\vec{k}_c.\vec{r}-E(0)\right)}{\left[\left(\omega-\omega'\lambda\right)-Z(0)\right]\cdot\sqrt{(a^2-b^2\lambda^2)-2(a-b\lambda)^2}\cos\left(\varepsilon-\varepsilon'\lambda\right)} \right\} \end{split}$$

$$+ \frac{(\omega - \omega'\lambda)}{4\pi} \sum_{\alpha=1}^{\infty} \left\{ \frac{(a - b\lambda)^{2}(\omega - \omega'\lambda) \cdot \sin(2\pi - (\varepsilon - \varepsilon'\lambda))}{(1 - \alpha)\left[(\omega - \omega'\lambda) - Z(\tau)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \times \frac{\sin(k_{c} \cdot \vec{r} - \alpha((\omega - \omega'\lambda)t + E(t) - E(\tau) - 2\pi) - E(\tau) - 2\pi)}{(1 - \alpha)\left[(\omega - \omega'\lambda) - Z(0)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos(\varepsilon - \varepsilon'\lambda)} \times \frac{\sin(k_{c} \cdot \vec{r} - \alpha((\omega - \omega'\lambda)t + E(t) - E(0)) - E(0))}{(1 + \alpha)\left[(\omega - \omega'\lambda) - Z(\tau)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \times \frac{\sin(k_{c} \cdot \vec{r} - \alpha((\omega - \omega'\lambda)t + E(t) - E(0)) - E(0))}{(1 + \alpha)\left[(\omega - \omega'\lambda) - Z(\tau)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos(2\pi - (\varepsilon - \varepsilon'\lambda))} \times \frac{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(\tau) - 2\pi)}{(1 + \alpha)\left[(\omega - \omega'\lambda) - Z(0)\right] \cdot \sqrt{(a^{2} - b^{2}\lambda^{2}) - 2(a - b\lambda)^{2}} \cos(\varepsilon - \varepsilon'\lambda)} \times \frac{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(\tau) - 2\pi) - E(\tau) - 2\pi)}{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(\tau) - 2\pi) - E(\tau) - 2\pi)} \times \frac{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(\tau) - 2\pi) - E(\tau) - 2\pi)}{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(0)) - E(0))} \times \frac{(m/s)}{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(0)) - E(0))} \times \frac{(m/s)}{\sin(k_{c} \cdot \vec{r} + \alpha((\omega - \omega'\lambda)t + E(t) - E(0)) - E(0)}$$

Equation (2.22) represents the *velocity* of the CCW as the time progresses and the dimension is m/s.

3.0 Presentation of results.

Table 3.0. Summary of the calculated values of the latent characteristic's variants (latent vibration) of the constitutive carrier wave CCW y(r, t); Results obtained from the work of [10].

| The component characteristic values of the constitutive carrier wave CCW $y(r, t)$ | | | | | | | |
|--|-------------------------|------------------|-----------|-------------|---------|-------------|------|
| The component characteristic values of the constitutive carrier wave even y(r,c). | | | | | | | |
| Amplitude | | Angular velocity | | Phase angle | | Wave number | |
| (<i>m</i>) | | (rad/s) | | (rad) | | (rad/m) | |
| а | b | ω | ω' | Е | ε′ | k | k' |
| 1.4624×10^{-5} | 9.3658×10^{-8} | 7.5408 | 0.04829 | 1.618 | 0.01036 | 156.1424 | 1.00 |
| The maximum value of the multiplier: $\lambda_m = 156.1424 \cong 157$ in the interval: $\lambda = 0, 1, 2, 3, \dots, 158$ (including | | | | | | | |
| zero). The total time taken for the CCW to decay to zero: $t \approx 22532970 \ s \approx 260 \ days$ (8 months) in the | | | | | | | |
| interval: $t = 0, 1.656, 3.312, 4.968, 6.624, \dots, 260 days$ and the attenuation constant: $\xi = 0.006404 s^{-1}$. | | | | | | | |























Fig. 3.4. Spectrum of only the oscillating phase $\cos(kr - (\omega - \omega'\lambda)t - E(t))$ of the CCW.



Fig. 3.6. Spectrum of the CCW including the oscillating phase: $y = A(t) \cos(kr - (\omega - \omega'\lambda)t - E(t)).$



Fig. 3.8. Spectrum of the characteristic group acceleration Q(t) of the CCW.



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Fig. 3.11. Spectrum of the first harmonic displacement $\alpha = 1$ of the CCW.



Fig. 3.13. Shows the sum of the first 5 lower harmonic displacements; the sum component of A_{α} and B_{α} $\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$ Colour code: $\alpha = 1$ (blue), $\alpha = 2$ (brown),

 $\alpha = 3$ (gray), $\alpha = 4$ (gold) and $\alpha = 5$ (omo blue).





Colour code: $\alpha = 1$ (blue), $\alpha = 2$ (brown), $\alpha = 3$ (gray), $\alpha = 4$ (gold) and $\alpha = 5$ (omo blue).

2.0. Discussion of results.

Fig. 3.1 shows that the displacement of the host wave has equal oscillating amplitudes and irregular frequency as it propagates from the source. That means, for a healthy person, one free from infectious disease, the amplitude is stable as it propagates with time. However, in Fig. 3.2, because of the inbuilt multiplier, the amplitude of the displacement of the parasitic wave increases as it spreads from the source. That means, the multiplier increases the characteristic variants of the parasitic wave in the CCW with time.

It is shown in Fig. 3.3 that the amplitude of the E(t) of the CCW is initially zero and undistorted as it initially spreads from the source. This shows that the effect of the infectious disease caused bacteria or virus, is not felt within this period. However, after 24 days of infection the total phase angle makes a significant amplitude of 1.5434 *radian*, that means, the effect of the infectious disease will start manifesting after 24 days of infection.

The spectrum of only the oscillating phase of the CCW is shown in Fig. 3.4. The spectrum shows a significant curvature between 14 - 66 days with more of oscillating frequency in the negative phase. Thus, the oscillating phase responds to the influence of the infectious disease within this interval and thereafter it propagates with almost equal amplitude before coming to rest after 260 days.

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Fig. 3.12. Spectrum of the first harmonic velocity $\alpha = 1$ of the CCW.



Fig. 3.14. Shows the sum of the higher harmonic Displacements; the sum component of A_{α} and B_{α} $\alpha = \alpha_{20} + \alpha_{50} + \alpha_{100} + \alpha_{120} + \alpha_{158}$ Colour code: $\alpha = 1$ (blue), $\alpha = 2$ (brown), $\alpha = 3$ (gray), $\alpha = 4$ (gold) and $\alpha = 5$ (omo blue).



Fig. 3.16. Shows the sum of the first 5 lower harmonic velocities, the sum of A_{α} and B_{α} $B_{\alpha} \alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$ Colour code: $\alpha = 1$ (blue), $\alpha = 2$ (brown), $\alpha = 3$ (gray), $\alpha = 4$ (gold) and $\alpha = 5$ (omo blue).

The spectrum of the oscillating amplitude A(t) shown in Fig. 3.5 reveals an obvious phase separation after 14, 39 and 49 days of infection. Then the amplitude rapidly attenuates after 91 days to zero. Also, the spectrum of the CCW shown in Fig. 3.6 has a significant curvature between 14 and 66 days with more of oscillating frequencies in the negative phase. That shows that the CCW is responding to the influence of the infectious disease within this interval and thereafter it propagates with increase amplitude in the positive phase before coming to rest after 260 days.

The amplitude of the characteristic angular velocity Z(t) shown in Fig. 3.7 increases steadily in the negative phase with significant peaks and distorted frequency before it comes to rest. Also, in Fig. 3.8 the amplitude of the characteristic angular acceleration Q(t) increases steadily with distorted frequency as it propagates from the source.

Fundamental displacement and velocity component A_0 of the CCW for $\alpha = 1$ are both shown in Fig. 3.9 and 3.10 respectively. The fundamental displacement from the transformed CCW show a quadratic curvature with negative intercept while the fundamental velocity has a cubic curvature with zero intercept.

The spectra of the first harmonic displacement and velocity ($\alpha = 1$) of the CCW are both shown in Figs. 3.11 and 3.12 respectively. The spectrum of the first harmonic displacement reveals a curvature between 0 and 66 days after infection with more of distorted frequencies in the negative phase of oscillation. While the spectrum of the first harmonic velocity shows a curvature between 28 and 66 days after infection with more of distorted frequencies in the negative phase of oscillation. However, while the subsequent attenuation in the displacement is slow and gradual, the velocity rapidly attenuates to zero.

As shown in Figs. 3.13 and 3.14 respectively, the sum of the harmonic displacements for both lower and higher values of the harmonic frequency mode α show unusual constriction in the spectra after about 100 days. Thereafter, the amplitude regroups with increased value before they are finally brought to rest.

In Fig. 3.15 and 3.16 the sum of the harmonic velocities for both lower and higher values of the frequency mode α show similar behaviour with that of the displacement. Thus, for the sum of the lower harmonic velocities the spectrum shows unusual significant peak after about 24 days with velocity amplitude of -0.001247 *m/s*. While the sum of the higher harmonic velocities shows significant peak after about 29 days, with velocity amplitude of about 7.3174 × 10^{-6} *m/s*. Thereafter, the spectra amplitude of both harmonic velocities decreases steadily to zero.

5.0 Conclusion.

It is revealed in this study, that several of the physical dynamic characteristics of the human vibration which is governed by the host wave, responds to the attacking influence of infectious diseases at different times. Hence, all localized human infectious diseases either caused by Bacteria or virus have an incubation period. The incubation period is the time between original time of infection and the appearance of detectable antibodies to the virus. Conversely, the incubation period is the time it takes the human system to respond to the manifestation of the interference of a strange body which is destructive in nature. It is shown in this work, that SARS and other related localized human infectious diseases has an incubation period of about 9 to 30 days, depending on the nature and circumstances of the host system under attack. This is indicated by several of the spectra, but most importantly the high peak resonance in the spectra of the lower and higher harmonic velocities. Also, it is shown that SARS or any form of localized human infectious diseases becomes more complicated after about 100 days of infection. Dynamical quantities such as energy, pressure-gradient, velocity-profile, make up the biomechanics of the Human system. Consequently, the attenuation in the dynamical quantities which eventually leads to a general loss of signal or a situation often referred to as 'death' of the host system if the situation is not controlled. Consequently, the essence of this study is that, if the vibration of any material thing is known, then its characteristics can be predicted, altered and possibly destroyed by anti-vibratory components.

Appendix

The Vector representation of the superposition of the host wave $y_1(\vec{r}, t)$ and the parasitic wave $y_2(\vec{r}, t)$ which yields a resultant wave $y(\vec{r}, t)$ or the constitutive carrier wave CCW.



Fig. A 1: Represents the human 'host wave' y_1 and the HIV 'parasitic wave' y_2 after the interference. The superposition of both waves y_1 and y_2 is represented by the carrier wave displacement y. It is clear that from the geometry of the figure: $\mu + \varepsilon' \lambda + 180^\circ - \varepsilon = 180^\circ$; $\mu = \varepsilon - \varepsilon' \lambda$; $\theta = 180^\circ - (\varepsilon - \varepsilon' \lambda)$; and $\theta = \pi - (\varepsilon - \varepsilon' \lambda)$. The amplitude of the host wave y_1 , parasitic wave y_2 and the CCW y, are not linear but they oscillate at a given frequency.

Appendix I: $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$; $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$; $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$; $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$.

Appendix II: $\sin(A+B) = \sin A \cos B + \cos A \sin B$; $\sin(A-B) = \sin A \cos B - \cos A \sin B$; $\cos(A+B) = \cos A \cos B - \sin A \sin B$; $\cos(A-B) = \cos A \cos B + \sin A \sin B$

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