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OPTIMAL INVENTORY MANAGEMENT STRATEGY FOR DETERIORATING ITEMS

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ABSTRACT

This paper studied the problem of profit maximization for deteriorating item model where demand function is stock level dependent, deterioration is controllable, vendor can offer trade credit and shortages are completely backlogged. We derived the objective function which is the total average profit function and solved the profit maximization problem by using MatLab. From the numerical illustrations and sensitivity analysis, we found that the optimal total average profit and the optimal ordered quantity are sensitive to the reduced deteriorating rate and initial demand parameters.

1. INTRODUCTION

A common topic been discussed in today's real business world is inventory modelling and management. In inventory management, controlling the rate of deterioration of commodities or items is very important. [1] considered an economic order quantity model. The model investigates the optimal pricing and replenishment policies for instantaneous deteriorating items with backlogging and trade credit under inflation. However, the work did not address control deterioration. The work addressed problem of profit maximization, and obtained the optimal selling price, optimal order quantity and optimal replenishment time. The work by [2] examined a deterministic model for deteriorating items that accounts for biquadratic demand function and shortages. [3] studied an inventory model that account for retention period, deterioration and backlog under two warehouses. They found that two warehouse inventory models can efficiently address various inventory problems, like depreciation, inflation, and shortages. Furthermore, the study considered how overall inventory system can be affected by shortages and cost increase.

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The work by [4] studied deteriorating item inventory model that accounts for displayed stock level and marketing strategy dependent demand. Also, in another work by [5], they obtained the optimal preservation strategies for inventory model that accounts for demand that is stock dependent and holding cost that is time dependent. Furthermore, [6] studied optimal ordering policy for deteriorating items model where demand is power – form stock dependent under two warehouse storage facility. In the work, it is assumed that the vendor satisfies the demands from the rented warehouse first and continues to do so until the goods in the rented warehouse get to zero then demands can be fulfilled from the owned warehouse. So, reduction of goods in the rented warehouse is as a result of deterioration and demands while reduction in goods in the warehouse owned by the vendor is as a result of deterioration at the initial time. However, reduction of goods in the owned warehouse is as a result of deterioration and demands when the goods in rented warehouse get zero. Some of the work that have considered the concept of shortages and backlogging can be found in ([7], [8], [9] and [10]).

Managing inventory of items efficiently may attract a lot of customers and hence increasing demands for such perishable items. Some of such items are fruits, vegetables, yams etc. Thus the work of [11], studied an inventory model that put into consideration stock, price, and time factors to determine the optimal replenishment point and length of inventory cycle. Furthermore, the study addressed different scenarios. Some of them are stock-dependent demand, no stock-dependent demand, and no time effect and also compared and analyzed some special cases. They also presented some numerical examples, sensitivity analysis and comparison studies. They recommended that the study's implications are significant for the processed food, vegetable, fruit, and grocery industries. [12] derived the solution that maximizes the profitability ratio for an inventory model with nonlinear holding cost. However, the minimization problem was not addressed and the comparison between both solutions of maximization and minimization problems was not studied. Other work on this subject are ([13], [14]). The paper by [15] studied instantaneous economic order quantity model that accounts for promotional effort cost, variable cost and units lost due to deterioration. The work determined the optimal order quantity, promotional effort, cycle length and number of units due to deterioration by maximizing the objective function (average profit function). The work also presented some numerical examples to verified the derived results.

In real-world, deterioration of commodities has become very important in certain sector such as food production sectors due to the uncertainties in life spans of food products but this deterioration rate can be controlled to certain level by employing preservation technology to reduce the rate at which such commodities decay. Therefore, our work examined inventory management problem for deteriorating items where demand is stock level dependent, deterioration is controllable under trade credit and complete backlogging. Focusing on how to maximize the average total profit function of the inventory model.

2.0 MODELS FORMULATION

2.1 Constants and Variables

K The ordering cost per order

c the unit purchasing cost

h The holding cost per unit time (excluding interest charges)

s Unit selling price ($s > c$)

c_1 shortage cost per unit per order

c_2 Opportunity cost due to lost sales

I_e Interest earned per $ per unit of time by the retailer

I_p Interest charges per $ in stock per unit of time to the supplier

$I(t)$ The level of inventory at time t

I_m Maximum inventory level for each replenish cycle

I_b Maximum amount of demand backlogged per cycle

M Retailer's trade credit period offered by supplier per unit time

t_1 Time at which the inventory level falls to zero

T Inventory cycle length

Q Retailer's order quantity

ω Maximum capital constraint

ξ Preservation technology cost per unit time for reducing deterioration rate in order to preserve the products $(0 \leq \xi \leq \omega)$

3.1 ASSUMPTIONS AND MODELS

i. The replenishment rate is infinite
ii. Lead time is zero
iii. Planning horizon of the inventory system is infinite
iv. There is no repair or replacement of deteriorated items during the period under consideration
v. The inventory model deals with single item
vi. The reduced deterioration rate $N(\xi)$ is an increasing function of the preservation technology cost ξ where

$$\lim_{\xi \to \infty} N(\xi) = \theta, \qquad \mathrm{N}(\xi) = \theta\left(1 - \mathrm{e}^{-\mathrm{a}\xi}\right), \mathrm{a} > 0$$

vii. The demand rate function $D(t)$ is deterministic and a function of instantaneous stock level $I(t)$. When inventory is positive, $D(t)$ is given by
$D(t) = \alpha + \beta I(t), \qquad 0 \leq t \leq t_1$
And when inventory is negative, $D(t)$ is given by
$D(t) = \alpha, \qquad 0 \leq t \leq T$
$\alpha > 0$ and $0 < \beta < 1$ are positive constants
viii. Shortages are allowed. The unsatisfied demand is backlogged and the fraction of shortages backordered is $b(t) = e^{-\delta t}\ \delta > 0, t$ is the time of waiting for the next replenishment and $0 \leq b(t) \leq 1,\ b(0) = 1.$ Note that if $b(x) = 1$ (or 0) for all t, then the shortages are completely backlogged (or lost). We assumed that the shortages are completely backlogged.

ix. If the trade credit period M is offered, the retailer would settle the account at $t = M$ and pay for the interest charges on the items in stock with rate I_p over the interval $[M, t_1]$ if $t_1 \geq M$ and if the retailer settles the account at $t = M$, the retailer do not need to pay any interest charge on items in stock during the whole cycle if $t_1 \leq M$

x. The retailer can accumulate revenue and earn interest from the beginning of the inventory cycle until the end of the trade credit period offered by the supplier. i.e., the retailer can accumulate revenue and earn interest during the period from $t = 0 \; to \; t = M$ with rate I_e under the trade credit conditions.

Considering the assumptions above, the model for the inventory level at any time is given by

$$\frac{dI(t)}{dt} = \begin{cases} -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) & 0 \leq t \leq t_1 \\ -\alpha b(t) & t_1 \leq t \leq T \end{cases} \tag{1}$$

With boundary condition $I(t_1) = 0$

During the interval $[0, t_1]$, the decrease in inventory level is due to combined effects of deterioration and demand, and the inventory goes to zero during the time interval $[0, t_1]$. But during the interval $[t_1, T]$, shortages can occur and can be completely backlogged. From equation (1), the rate of change of inventory level at any time t can be represented by the following differential equation:

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \leq t \leq t_1 \tag{2}$$

$$\frac{dI_1}{dt} = -\alpha b(t) \qquad t_1 \leq t \leq T \tag{3}$$

With the boundary conditions $I(0) = I_M, I_1(t_1) = 0$

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \leq t \leq t_1$$

$$\frac{dI(t)}{dt} = -\alpha - [\beta + \left(\theta - N(\xi)\right)]I(t)$$

$$= -(\alpha + [\beta + \theta - N(\xi)]I(t))$$

$$\int_t^{t_1} \frac{dI(t)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = \int_t^{t_1} -dt$$

$$\int_t^{t_1} \frac{dI(t)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = -\int_t^{t_1} dt$$

$$\ln\left[\frac{\alpha + [\beta + \theta - N(\xi)]I(t_1)}{\alpha + [\beta + \theta - N(\xi)]I(t)}\right] = -(\beta + \theta - N(\xi))(t_1 - t)$$

$$\frac{\alpha + [\beta + \theta - N(\xi)]I(t_1)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = e^{-(\beta + \theta - N(\xi))(t_1 - t)}$$

Since $I(t_1) = 0$, we Have

$$\frac{\alpha}{\alpha+[\beta+\theta-N(\xi)]I(t)}=e^{-(\beta+\theta-N(\xi))(t_1-t)}$$

$$\alpha+[\beta+\theta-N(\xi)]I(t)=\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}$$

$$[\beta+\theta-N(\xi)]I(t)=\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}-\alpha$$

$$I(t)=\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)}-1]}{\beta+\theta-N(\xi)}$$

$$I_m=I(0)=\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} \tag{4}$$

3.1 Components of the Optimization Problem

Ordering cost per cycle $=K$

Holding Cost $=h\int_0^{t_1}I(t)\,dt$

$$=h\int_0^{t_1}\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)}-1]}{\beta+\theta-N(\xi)}dt$$

$$=\frac{\alpha h}{\beta+\theta-N(\xi)}\int_0^{t_1}[e^{(\beta+\theta-N(\xi))(t_1-t)}-1]\,dt$$

$$=\frac{\alpha h}{\beta+\theta-N(\xi)}\left|\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{-(\beta+\theta-N(\xi))}-t\right|_0^{t_1}$$

$$=\frac{\alpha h}{\beta+\theta-N(\xi)}\left(\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{-(\beta+\theta-N(\xi))}-t_1\right]-\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-0)}}{-(\beta+\theta-N(\xi))}-0\right]\right)$$

$$=\frac{\alpha h}{\beta+\theta-N(\xi)}\left(-\frac{1}{(\beta+\theta-N(\xi))}-t_1+\frac{e^{(\beta+\theta-N(\xi))t_1}}{(\beta+\theta-N(\xi))}\right)$$

$$=\frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-t_1\right) \tag{5}$$

For unsatisfied demands completely backlogged, we have

$$\frac{dI_1}{dt}=-\alpha b(t) \qquad t_1\le t\le T$$

$b(t)=1$, therefore,

$$\frac{dI_1}{dt}=-\alpha$$

$$I_1(t) = -\alpha \int_{t_1}^{t} dt = -\alpha(t - t_1) = \alpha(t_1 - t) \tag{6}$$

Maximum backordered quantity

$I_1(T) = I_{1m} = \alpha(t_1 - T)$

Here, no lost sales or opportunity cost

Purchase cost:

Maximum quantity $= Q = I_m - I_{1m}$

$$Q = I_m - I_{1m} = \frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} + \alpha(T - t_1) \tag{7}$$

$$\text{Purchase cost} = cQ = c\left(\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} + \alpha(T - t_1)\right)$$

$$= \frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} + \alpha c(T - t_1) \tag{8}$$

$$\text{Shortage cost} = -c_1 \int_{t_1}^{T} I_1(t)\, dt = -c_1\alpha \int_{t_1}^{T} (t_1 - t)\, dt$$

$$= -c_1\alpha \left| t_1 t - \frac{t^2}{2} \right|_{t_1}^{T}$$

$$= -c_1\alpha \left[\left(t_1 T - \frac{T^2}{2}\right) - \left(t_1^2 - \frac{t_1^2}{2}\right)\right]$$

$$= -c_1\alpha \left[t_1 T - \frac{T^2}{2} - t_1^2 + \frac{t_1^2}{2}\right]$$

$$= c_1\alpha \left[-t_1 T + \frac{T^2}{2} + t_1^2 - \frac{t_1^2}{2}\right]$$

$$= c_1\alpha \left[\frac{T^2}{2} - \frac{t_1^2}{2} + t_1^2 - t_1 T\right] \tag{9}$$

$$\text{Preservation technology cost} = \xi T \tag{10}$$

$$\text{Sales revenue} = s\left(\int_0^{t_1} D(t)\, dt - I_1(T)\right)$$

$$= s\int_0^{t_1} (\alpha + \beta I(t))\, dt - s\alpha(t_1 - T)$$

$$= s\int_0^{t_1}\left(\alpha+\beta\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)}-1]}{\beta+\theta-N(\xi)}\right)dt+s\alpha(T-t_1)$$

$$= s\int_0^{t_1}\left(\alpha+\beta\alpha\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{\beta+\theta-N(\xi)}-\frac{\beta\alpha}{\beta+\theta-N(\xi)}\right)dt+s\alpha(T-t_1)$$

$$= s\left(\alpha t-\frac{\beta\alpha}{\beta+\theta-N(\xi)}\times\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{\beta+\theta-N(\xi)}-\frac{\beta\alpha t}{\beta+\theta-N(\xi)}\right)\Big|_0^{t_1}+s\alpha(T-t_1)$$

$$= s\left(\alpha t-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t}{\beta+\theta-N(\xi)}\right)\Big|_0^{t_1}+s\alpha(T-t_1)$$

$$= s\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1)$$

$$= s\left[\alpha t_1-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1)$$

$$= s\left[\frac{\alpha t_1[\beta+\theta-N(\xi)]-\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}-\beta\alpha}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1)$$

$$= s\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\alpha(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1)$$

$$= s\alpha\left[\frac{t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1) \tag{11}$$

Deteriorating cost $= d_c\left[I_m-\int_0^{t_1}D(t)\,dt\right]=d_c\left[I_m-\int_0^{t_1}(\alpha+\beta I(t))\,dt\right]$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-\int_0^{t_1}\left(\alpha+\beta\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)}-1]}{\beta+\theta-N(\xi)}\right)dt\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-\left(\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}\right)\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}-\frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-\frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}-\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\right]$$

$$= d_c\left[\frac{\alpha[\beta+\theta-N(\xi)][e^{(\beta+\theta-N(\xi))t_1}-1]-\alpha\beta(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}-\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\right]$$

$$= d_c\left[\frac{\alpha[\theta - N(\xi)][e^{(\beta+\theta-N(\xi))t_1} - 1]}{[\beta+\theta-N(\xi)]^2} - \frac{\alpha t_1[\theta - N(\xi)]}{\beta+\theta-N(\xi)}\right]$$

$$= \frac{d_c\alpha[\theta - N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - t_1\right] \quad (12)$$

Interest payable and earned

If the time the credit period ends is shorter than or equal to the length of period in which the inventory is positive ($M \leq t_1$), the retailer pays for goods and the retailer starts paying the capital opportunity cost for the items in stock with rate I_p. Also, it is assumed that while the account is yet to be settled, the retailer can sell the goods and continue to accumulate sales revenue and earn interest with the rate I_e. Hence, interest earned and payable per cycle for different cases are given below:

Case i: $M \leq t_1$

$$\text{interest payable} = cI_p\int_M^{t_1} I(t)dt$$

$$= cI_p\int_m^{t_1}\left(\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta+\theta-N(\xi)}\right)dt$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\int_M^{t_1}\left(e^{(\beta+\theta-N(\xi))(t_1-t)} - 1\right)dt$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left(\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{-(\beta+\theta-N(\xi))} - t\right)\Bigg|_M^{t_1}$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\left(\frac{e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{-(\beta+\theta-N(\xi))} - t_1\right) - \left(\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}}{-(\beta+\theta-N(\xi))} - M\right)\right]$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}}{(\beta+\theta-N(\xi))} + M - \frac{1}{\beta+\theta-N(\xi)} - t_1\right]$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)} - 1}{(\beta+\theta-N(\xi))} + M - t_1\right] \quad (13)$$

$$\text{interest earned} = sI_e\int_0^m\int_0^t D(s)ds\,dt$$

$$= sI_e\int_0^m\left[\int_0^t D(s)ds\right]dt$$

$$= sI_e\int_0^m\left[\int_0^t (\alpha + \beta I(s))ds\right]dt$$

$$= sI_e \int_0^m \left[\int_0^t \left(\alpha + \frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-s)} - 1]}{\beta+\theta-N(\xi)}\right) ds\right] dt$$

$$= sI_e \int_0^M \left[\int_0^t \left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta+\theta-N(\xi)} - \frac{\beta\alpha}{\beta+\theta-N(\xi)}\right) ds\right] dt$$

$$= sI_e \int_0^M \left[\left(\alpha s - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha s}{\beta+\theta-N(\xi)}\right)\Big|_0^t\right] dt$$

$$= sI_e \int_0^M \left[\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t}{\beta+\theta-N(\xi)}\right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta+\theta-N(\xi)]^2} - 0\right)\right] dt$$

$$= sI_e \int_0^M \left[\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha t}{\beta+\theta-N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right] dt$$

$$= sI_e \left[\left(\frac{\alpha M^2}{2} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right) - \left(0 + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3} - 0 + 0\right)\right]$$

$$= sI_e \left[\frac{\alpha M^2}{2} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2} - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}\right]$$

$$= sI_e \left[\frac{\alpha M^2}{2} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e \left[\frac{\alpha M^2(\beta+\theta-N(\xi)) - \beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}(e^{-(\beta+\theta-N(\xi))M} - 1)}{[\beta+\theta-N(\xi)]^3} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e \left[\frac{\alpha M^2(\theta-N(\xi))}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}(e^{-(\beta+\theta-N(\xi))M} - 1)}{[\beta+\theta-N(\xi)]^3} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right] \quad (14)$$

Case ii: $t_1 \leq M$

Here, the cycle time t, is less than or equal to the credit period M. Thus, no interest will be paid. So, the retailer pays no interest at the end of the inventory cycle. Hence,

$$\text{Interest payable} = 0$$

Meanwhile, from time 0 to t_1, the retailer sells the goods and thus can accumulate sales revenue to earn interest $sI_e \int_0^{t_1} \int_0^{t} D(s)\, ds\, dt$. Furthermore, from time t_1 to M, the retailer can use the sales revenue generated in $[0, t_1]$ to earn interest $sI_e \int_0^{t_1} D(s) ds\, (M - t_1)$. Therefore, the interest earned in this period per cycle can be described by:

$$\text{Interest earned} = sI_e \left[\int_0^{t_1} \int_0^{t} D(s)\, ds\, dt + \int_0^{t_1} D(t) dt\, (M - t_1) \right]$$

$$= sI_e \int_0^{t_1} \int_0^{t} D(s)\, ds\, dt + sI_e \int_0^{t_1} D(t) dt\, (M - t_1)$$

$$= sI_e \int_0^{t_1} \int_0^{t} D(s)\, ds\, dt + sI_e \int_0^{t_1} D(t) dt\, (M - t_1)$$

$$= sI_e \int_0^{t_1} \left[\int_0^{t} D(s) ds \right] dt + sI_e \int_0^{t_1} D(t) dt\, (M - t_1)$$

$$= sI_e \int_0^{t_1} \left[\int_0^{t} (\alpha + \beta I(s)) ds \right] dt + sI_e (M - t_1) \int_0^{t_1} (\alpha + \beta I(t)) dt$$

$$= sI_e \int_0^{t_1} \left[\int_0^{t} \left(\alpha + \frac{\beta\alpha [e^{(\beta+\theta-N(\xi))(t_1 - s)} - 1]}{\beta + \theta - N(\xi)} \right) ds \right] dt$$

$$+ sI_e (M - t_1) \int_0^{t_1} \left(\alpha + \frac{\beta\alpha [e^{(\beta+\theta-N(\xi))(t_1 - t)} - 1]}{\beta + \theta - N(\xi)} \right) dt$$

$$= sI_e \int_0^{t_1} \left[\int_0^{t} \left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1 - s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)} \right) ds \right] dt$$

$$+ sI_e (M - t_1) \int_0^{t_1} \left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1 - s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)} \right) dt$$

$$= sI_e \int_0^{t_1} \left[\left(\alpha s - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1 - s)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha s}{\beta + \theta - N(\xi)} \right) \Big|_0^t \right] dt$$

$$+ sI_e (M - t_1) \left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1 - t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)} \right) \Big|_0^{t_1}$$

$$= sI_e \int_0^{t_1} \left[\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1 - t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)} \right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1 - 0)}}{[\beta + \theta - N(\xi)]^2} - 0 \right) \right] dt$$

$$+sI_e(M-t_1)\left[\left(\alpha t_1-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}\right)-\left(0-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta+\theta-N(\xi)]^2}-0\right)\right]$$

$$=sI_e\int_0^{t_1}\left[\alpha t-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]dt$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e\left(\frac{\alpha t}{2}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right)\Big|_0^{t_1}$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e\left[\left(\frac{\alpha t_1^2}{2}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right)-\left(0+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta+\theta-N(\xi)]^3}-0+0\right)\right]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e\left[\frac{\alpha t_1^2}{2}+\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}\right]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e\left[\frac{\alpha t_1^2}{2}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\frac{\alpha t_1^2\big(\beta+\theta-N(\xi)\big)-\beta\alpha t_1^2}{2\big(\beta+\theta-N(\xi)\big)}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\beta+\theta-N(\xi)]-\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right]$$

$$= sI_e\left[\frac{\alpha t_1^2\big(\theta-N(\xi)\big)}{2\big(\beta+\theta-N(\xi)\big)}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}\right]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right] \qquad (15)$$

4.0 THE PROFIT MAXIMIZATION PROBLEM

The problem now is to determine the optimal value of t which maximizes the objective function $X(t)$. So, the necessary condition for maximizing the total profit function $X(t)$ is:

$$\frac{d}{dt}X(t)=0 \qquad (16)$$

Now, equations (16) can be solved for t to obtain the optimal value of t (say t^*). The sufficient condition for $X(t)$ to be a maximum is:

$$\frac{d^2}{dt^2}X(t)<0 \qquad (17)$$

Thus, if we put the above components into consideration, then the profit function for the retailer can be described by:

$$Y(t)=\frac{1}{T}\{sales\ revenue+interest\ earned-odering\ cost-holding\ cost$$
$$-purchasing\ cost-deteriorating\ cost-shortage\ cost$$
$$-opportunity\ cost-preservation\ cost-interest\ payable\}$$

Case I: $M\le t_1$

Here, opportunity cost $=0$, therefore,

$$Y_1(t_1)=\frac{1}{T}\Bigg\{s\alpha\left[\frac{t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta(e^{(\beta+\theta-N(\xi))t_1}-1)}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1)+sI_e\left[\frac{\alpha M^2\big(\theta-N(\xi)\big)}{2\big(\beta+\theta-N(\xi)\big)}\right.$$

$$\left.+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}\big(e^{-(\beta+\theta-N(\xi))M}-1\big)}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]-K$$

$$-\frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-t_1\right)-\left(\frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}+\alpha c(T-t_1)\right)$$

$$-\frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-t_1\right]-c_1\alpha\left[\frac{T^2}{2}-\frac{t_1^2}{2}+t_1^2-t_1T\right]-\xi T$$

$$\left.-\frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}-1}{(\beta+\theta-N(\xi))}+M-t_1\right]\right\} \quad (18)$$

Case II: $t_1 \leq M$

$$Y(t) = \frac{1}{T}\left\{\begin{matrix} sales\ revenue + interest\ earned - odering\ cost - holding\ cost - purchasing\ cost \\ -deteriorating\ cost - shortage\ cost - opportunity\ cost - preservation\ cost \\ -interest\ payable \end{matrix}\right\}$$

interest payable $= 0$

$$\text{interest earned} = sI_e\left[\frac{\alpha t_1^2(\theta-N(\xi))}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}\right]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right]$$

$$Y_2(t_1) = \frac{1}{T}\left\{s\alpha\left[\frac{t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\left(e^{(\beta+\theta-N(\xi))t_1}-1\right)}{[\beta+\theta-N(\xi)]^2}\right]+s\alpha(T-t_1)\right.$$

$$+sI_e\left[\frac{\alpha t_1^2(\theta-N(\xi))}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}\right]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right]-K$$

$$-\frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-t_1\right)-\left(\frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}+\alpha c(T-t_1)\right)$$

$$\left.-\frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)}-t_1\right]-c_1\alpha\left[\frac{T^2}{2}-\frac{t_1^2}{2}+t_1^2-t_1T\right]-\xi T\right\} \quad (19)$$

5.0 NUMERICAL ANALYSIS AND SENSITIVITY ANALYSIS

Case I: $M \leq t_1$

For numerical illustrations, we solved Equation (18)., an inventory system with the following parameter set. α=1000units, β=0.04, T=40days, M=30days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, c_2=1Naira/unit/order, θ=0.7, K=3000, N=0.02, ξ=0.3, d_c=0.3, I_p=0.14. The optimal time $t_1^* = 38.5320$

Optimal Total average cost $X_1^* = 3873.80$
Optimal order quantity $Q^* = 108960$

Table 1. The effect of the parameter θ on X_1^* and Q^*

N	X_1^*	Q^*
0.02	3873.80	108960
0.04	3842.20	108510
0.06	3811.50	108080
0.08	3781.70	107690
0.10	3752.70	107320
0.12	3724.60	106990
0.14	3697.10	106680

α=40units, β=0.6, T=40days, M=30days, c=1Naira/unit,
h=1Naira/unit time, c_1=1Naira/unit/order, c_2=1Naira/unit/order
K=3000, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, I_p=0.14, $\sigma = 0.03$.

Table 2. The effect of the parameter α on X_1^* and Q^*

α	X_1^*	Q^*
1000	3873.80	111730
2000	7669.70	223460
3000	11466.00	335190
4000	15261.00	446920
5000	19057.00	558650

β=0.7, T=40days, M=30days, c=1Naira/unit, h=1Naira/unit time,
c_1=1Naira/unit/order, c_2=1Naira/unit/order, θ=0.8, K=3000,
$N(\xi)$=0.02, ξ=0.3, d_c=0.3, $\sigma = 0.03$ I_p=0.14.

Case II: $t_1 \leq M$

For numerical illustrations, we solved Equ. (19)., an inventory system with the following parameter set. α=1000units, β=0.04, T=11days, M=10days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, c_2=1Naira/unit/order θ=0.7, K=3000, N=0.02, ξ=0.3, d_c=0.3, I_p=0.14, $\sigma = 0.03$.

The optimal time $t_1^* = 9.4198$
Optimal Total average cost $X_2^* = 4111.40$
Optimal order quantity $Q^* = 27267$

Table 3. The effect of the parameter N on X_1^* and Q^*

N	X_2^*	Q^*
0.02	4111.40	27267
0.04	4118.50	27141
0.06	4128.40	27020
0.08	4141.20	26904
0.10	4156.90	26792
0.12	4175.80	26686
0.14	4198.10	26585

α=40units, β=0.04, T=11days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order,
c_2=1Naira/unit/order, K=3000, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, $\sigma = 0.03$.

Table 4. The effect of the parameter α on X_1^* and Q^*

α	X_2^*	Q^*
1000	4111.40	27267
2000	7947.10	54535
3000	11783.00	81802
4000	15619.00	109070
5000	19454.00	136340

β=0.04, T=11days, c=1Naira/unit, h=1Naira/unit time, c_2=1Naira/unit/order, c_1=1Naira/unit/order,θ=0.7, K=3000, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, $\sigma = 0.03$.

Table (1) and (3), showed that the parameter N (controllable deterioration rate parameter) is sensitive to the optimal total average profit and the optimal ordered quantity. That is, for the case where $M \leq t_1$, as the parameter N increases we can see that the vendor makes less profit and purchased less quantity as the cost of preservation increases. But, for the case where $t_1 \leq M$, the quantity purchased decreases but the profit increases slightly due to the fact that the retailer can generate revenue on the accumulated sales from the commodity throughout the inventory life cycle. Furthermore, Table (2) and (4), we can see that the parameter α (initial demand parameter) is also sensitive to the optimal total average profit and the optimal ordered quantity. We can observe that as the parameter α increases, the optimal ordered quantity and the optimal total average profit also increased. This is consistent with daily practice. If the demand for a commodity increases, the vendor would buy more and sell more and hence profit and quantity ordered would increase.

CONCLUSION

We have examined an inventory profit maximization problem for deteriorating items model that put into consideration demand function that stock level dependent and deterioration that is controllable under trade credit and complete backlogging. We also derived the objective function which is the total average profit function and minimized the objective function to obtained the optimal inventory management strategies for the formulated inventory management problem. We also presented some numerical illustrations and sensitivity analysis to illustrate the derived results.

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