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GENERALIZED UNCERTAINTY PRINCIPLE EFFECTS ON NEUTRON STAR EQUATION OF STATE AND THERMAL PROPERTIES

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ABSTRACT

We investigate how the Generalised Uncertainty Principle (GUP) affects neutron star structure and cooling. By modifying the equations of state to include GUP effects at extremely high densities through momentum-dependent corrections to the relativistic Fermi gas model, we compute mass-radius relations and thermal evolution curves. Using advanced numerical techniques, we solve the Tolman-Oppenheimer-Volkoff and thermal transport equations together. Our results show that GUP introduces observable changes, especially in cooling behaviour and radius estimates. We compare our findings with NICER data from PSR J0030+0451 and PSR J0740+6620, as well as gravitational wave events like GW170817 and GW190425. This comparison enables us to place a tight upper bound on the GUP parameter, $\beta \leq 100$, making it the strongest astrophysical constraint to date. Our work highlights neutron stars as powerful tools for testing quantum gravity, setting the stage for future investigations using multi-messenger astronomy.

1. INTRODUCTION

Neutron stars represent the most extreme manifestation of matter in the observable universe, hosting physical conditions that cannot be replicated in terrestrial laboratories. With central densities reaching 5-10 times nuclear saturation density ($\rho_0 \sim 2.8 \times 101^4\ kg/m^3$) and gravitational fields approaching the theoretical maximum for stable configurations, these compact objects provide unparalleled opportunities to test fundamental physics under extreme conditions [1]. The recent renaissance in neutron star observations, driven by revolutionary facilities like the Neutron Star Interior Composition Explorer (NICER) and the gravitational wave detectors LIGO-Virgo, has opened new avenues for probing the nature of matter at supranuclear densities and the fundamental structure of spacetime itself.

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The equation of state (EOS) of nuclear matter at these extreme densities remains one of the most significant unsolved problems in theoretical physics. Traditional approaches based on quantum chromodynamics (QCD) and effective field theories provide valuable insights, but significant uncertainties persist, particularly regarding the transition between hadronic and quark matter phases and the role of exotic particles [2,3]. These uncertainties translate directly into observational predictions for neutron star properties, creating a complex interplay between fundamental physics and astronomical observations. The structure of these compact objects is governed by the Tolman-Oppenheimer-Volkoff (TOV) equations of hydrostatic equilibrium, which must be solved numerically to determine mass-radius relationships.

Into this already rich landscape, quantum gravity theories introduce additional layers of complexity through modifications to the fundamental uncertainty relations that govern quantum mechanical systems. The Generalized Uncertainty Principle (GUP), arising naturally from string theory, loop quantum gravity, and other approaches to quantum gravity, suggests that the standard Heisenberg uncertainty principle must be modified at high energies and small length scales [4,5]. While these modifications are typically negligible under ordinary conditions, the extreme environments found in neutron star cores may amplify quantum gravity effects to observable levels.

Recent theoretical developments have significantly advanced our understanding of how quantum gravity effects might manifest in neutron star physics. The work of Balkin et al. [6] demonstrates how light scalar fields coupled to nucleons can fundamentally alter the stellar landscape, enabling neutron stars to achieve significantly higher masses than traditional QCD equations of state predict. This highlights the potential for quantum gravity modifications to produce observable signatures in neutron star properties, even when the underlying effects appear small at the microscopic level.

The physical motivation for considering GUP effects in neutron stars stems from several converging lines of reasoning. First, the energy densities achieved in neutron star cores, while far below the Planck scale, represent the highest stable energy concentrations in the universe. Second, the large number of particles involved ($\sim 10^{57}$ nucleons) means that even tiny individual modifications can integrate to produce significant effects. Third, neutron star observables such as mass, radius, and cooling rates depend sensitively on the details of the equation of state, potentially amplifying small quantum gravity corrections into measurable signatures.

The observational landscape for neutron star studies has been transformed dramatically in recent years. The NICER mission, launched in 2017, has provided unprecedented precision in X-ray timing observations, enabling direct measurements of neutron star masses and radii with uncertainties approaching 5% [7,8]. The most recent analysis of PSR J0030+0451 yielded mass and radius measurements of $M = 1.44^{+0.15}_{-0.14} M_\odot$ and $R = 13.02^{+1.24}_{-1.06}$ km, respectively, representing the gold standard for simultaneous mass-radius constraints [7].

Similarly, transformative results have emerged from observations of the massive pulsar PSR J0740+6620, with NICER measurements constraining its radius to $R = 12.4^{+1.3}_{-1.0}$ km despite its substantial mass of approximately 2.1 solar masses [9,10]. These observations are particularly significant because they probe the equation of state at the highest densities accessible in stable neutron stars, where quantum gravity effects are expected to be most pronounced.

Simultaneously, the detection of gravitational waves from the binary neutron star merger GW170817 has provided complementary constraints on the equation of state through measurements of tidal deformability [11]. The effective tidal deformability measured from this event, $\widetilde{\Lambda} = 300^{+420}_{-230}$, constrains the stiffness of nuclear matter at supranuclear densities and has fundamentally altered our understanding of neutron star interiors. The more recent detection of

GW190425 has provided additional constraints, though with somewhat larger uncertainties due to the greater distance of this event [12].

These observations, combined with traditional electromagnetic astronomy, have created a multi-messenger approach to neutron star physics that offers multiple pathways for testing fundamental theory. The integration of data from nuclear experiments, as demonstrated by recent studies [13,14], has further strengthened the empirical foundation for constraining the neutron star equation of state and identifying potential signatures of exotic physics.

The challenge of connecting quantum gravity theory to neutron star observations requires careful treatment of several interconnected physical processes. The most direct connection comes through modifications to the nuclear equation of state, where GUP corrections alter the pressure-density relationships that determine stellar structure. However, equally important are the thermal properties that govern neutron star cooling, the transport properties that affect energy transfer, and the dynamical properties that influence stability and oscillations.

Previous theoretical work on quantum gravity effects in neutron stars has largely focused on simplified models or specific aspects of the problem [15,16]. While these studies have provided valuable insights, they have generally not addressed the full complexity of realistic neutron star physics or made direct connections to current observational capabilities. Our approach differs by developing a comprehensive framework that incorporates GUP effects into state-of-the-art neutron star models while maintaining direct connections to observable quantities.

The theoretical framework we employ builds directly on the mathematical foundation established for extended GUP formulations, focusing particularly on the momentum-dependent corrections that dominate in high-density environments. We adopt the modified uncertainty relation $\Delta x \Delta p \geq (\hbar/2)[1 + \beta(\Delta p)^2/\hbar^2]$, where the parameter β characterises the strength of quantum gravity effects. This choice reflects both theoretical motivation from string theory and practical considerations regarding computational tractability.

Recent developments in neutron star cooling theory have revealed additional opportunities for constraining fundamental physics. [17] demonstrated how modifications to gravity theories can significantly alter neutron star thermal evolution, providing observational signatures that complement structural measurements. Similarly, advances in understanding stellar envelope physics during mass accretion episodes [18] have opened new windows into the high-density regime where quantum gravity effects might be detectable.

Our analysis addresses three primary questions: How do GUP modifications affect the neutron star equation of state and resulting mass-radius relationships? What are the observable consequences for neutron star thermal evolution and cooling? What constraints can current observations place on the GUP parameter β? Answering these questions requires a synthesis of quantum gravity theory, nuclear physics, stellar structure calculations, and observational astronomy—a challenging but potentially rewarding endeavor that could provide new insights into fundamental physics.

2.0 METHODS

2.1 MODIFIED EQUATION OF STATE

The incorporation of GUP effects into the neutron star equation of state requires careful modification of the statistical mechanics underlying nuclear matter at supranuclear densities. The fundamental insight is that the modified uncertainty relations alter the phase space available to nucleons, leading to corrections in the pressure-density relationships that determine stellar structure.

We begin with the relativistic ideal Fermi gas model, which provides a reasonable approximation for nuclear matter at the highest densities found in neutron star cores. For a system of nucleons with mass m_N and Fermi momentum p_F, the standard expression for pressure as a function of density takes the form:

$$P_{\text{std}}(\rho) = \frac{1}{24\pi^2}\int_0^{p_F} p^2 dp \frac{p^2}{\sqrt{p^2+m_N^2c^2}} \tag{1}$$

The GUP modifications enter through the altered phase space volume element. Following established theoretical frameworks [19], the momentum space integration measure becomes:

$$d^3p \to d^3p\left(1+\beta\frac{p^2}{\hbar^2}\right)^{-3/2} \tag{2}$$

This modification reflects the reduced phase space availability at high momenta, effectively suppressing the contributions of the most energetic particles to the thermodynamic quantities.

Implementing this correction in the pressure integral yields:

$$P_{\text{GUP}}(\rho) = \frac{1}{24\pi^2}\int_0^{p_F} p^2 dp \frac{p^2}{\sqrt{p^2+m_N^2c^2}}\left(1+\beta\frac{p^2}{\hbar^2}\right)^{-3/2} \tag{3}$$

For practical calculations, we expand this expression in powers of the GUP parameter β, keeping terms up to first order:

$$P_{\text{GUP}}(\rho) \approx P_{\text{std}}(\rho)\left[1-\frac{3\beta}{2}\langle p^2\rangle/\hbar^2\right] \tag{4}$$

where $\langle p^2\rangle$ represents the average squared momentum of particles in the Fermi Sea.

This first-order expansion is valid when $\beta p^2/\hbar^2 \ll 1$. For the densities and β values considered in our models (up to β = 200), this condition is satisfied even in the cores of the most massive neutron stars, where typical Fermi momenta reach $p_F \sim 500 MeV/c$, ensuring $\beta p_F^2/\hbar^2 \lesssim 0.1$ for our parameter range.

The evaluation of this average requires careful treatment of the relativistic dispersion relation. For ultra-relativistic conditions where $p_F \gg m_N c$, we obtain:

$$\langle p^2\rangle = \frac{3}{5}p_F^2 = \frac{3}{5}(3\pi^2)^{2/3}\hbar^2 n^{2/3} \tag{5}$$

where $n = \rho/m_N$ is the number density. This result shows that the GUP corrections scale as $n^{2/3}$, growing more significant at higher densities.

Substituting this expression into the modified pressure relation gives:

$$P_{\text{GUP}}(\rho) = P_{\text{std}}(\rho)\left[1-\frac{9\beta}{10}(3\pi^2)^{2/3}n^{2/3}\right] \tag{6}$$

The corresponding energy density receives similar corrections:

$$\varepsilon_{\text{GUP}}(\rho) = \varepsilon_{\text{std}}(\rho)\left[1-\frac{3\beta}{4}(3\pi^2)^{2/3}n^{2/3}\right] \tag{7}$$

These expressions reveal several important features of the GUP-modified equation of state. It is important to note that while our analysis uses the SLy4 equation of state as a baseline, the GUP-induced softening represents an additional source of uncertainty beyond those already present in nuclear physics models. The interactions between GUP modifications and other potential exotic physics, such as hyperonic degrees of freedom or hadron-quark phase transitions, remain an open question that warrants future investigation. Our approach of treating GUP effects as perturbative corrections to established nuclear models provides a conservative framework that can be extended as understanding of high-density nuclear matter continues to develop. First, the corrections are always negative, indicating that quantum gravity effects tend to soften the equation of state by reducing both pressure and energy density. Second, the magnitude of the corrections grows with density, becoming most significant in the cores of the most massive neutron stars. Third, the fractional corrections scale differently for pressure and energy density, leading to modifications in the adiabatic index $\gamma = (P + \varepsilon)/P \cdot dP/d$

2.2 ADVANCED NUCLEAR PHYSICS INTERACTIONS

To extend this analysis beyond the simple ideal gas model, we must consider the interactions between nucleons that become increasingly important at nuclear and supranuclear densities. The most sophisticated approach involves incorporating GUP corrections into realistic nuclear force models such as the Argonne v18 potential or chiral effective field theory interactions.

For the mean-field approximation commonly used in neutron star calculations, the interaction energy contributes an additional term to the total energy density:

$$\varepsilon_{\text{total}} = \varepsilon_{\text{kinetic}} + \varepsilon_{\text{interaction}} + \varepsilon_{\text{rest mass}} \tag{8}$$

The integration of GUP effects into the SLy4 equation of state follows a systematic procedure. The SLy4 parameterization provides the interaction energy density ε_interaction as a function of baryon density through its Skyrme force parameters. We decompose the total energy density as:

$$\varepsilon_{total} = \varepsilon_{kinetic}, GUP + \varepsilon_{interaction}, SLy4 + \varepsilon_{restmass} \tag{9}$$

where $\varepsilon_{kinetic}, GUP$ represents the GUP-modified kinetic contribution from Equation (7), and $\varepsilon_{interaction}, SLy4$ is the unmodified SLy4 interaction term. This approach assumes that GUP effects primarily modify the single-particle kinetic energies while leaving the inter-nucleon interactions unchanged—a reasonable approximation since the GUP corrections arise from momentum-space modifications rather than changes to the nuclear force itself.

The corresponding pressure is obtained through the thermodynamic relation:

$$P_{total} = n^2(\partial(\varepsilon_{total}/n)/\partial n) \tag{10}$$

where n is the baryon number density. The interaction contribution to pressure follows directly from the SLy4 parameterization, while the kinetic contribution includes both the standard Fermi gas term and the GUP correction from Equation (6).

The GUP modifications affect primarily the kinetic energy component, though indirect effects on the interaction terms arise through changes in the nucleon wave functions and density distributions. Recent advances in chiral effective field theory have enabled more sophisticated treatments of these interactions [20], providing a foundation for systematic incorporation of GUP effects.

We implement a hybrid approach that combines the GUP-modified kinetic terms with realistic interaction models calibrated to nuclear physics data. This methodology ensures consistency with established constraints from nuclear experiments while enabling exploration of quantum gravity effects in the supranuclear density regime.

2.3 STELLAR STRUCTURE CALCULATIONS

The implementation of these modifications in the Tolman-Oppenheimer-Volkoff (TOV) equations that determine neutron star structure requires numerical integration. The TOV equations take the form:

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r)/c^2 \tag{11}$$

$$\frac{dP}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2c^2}\left(1 + \frac{P(r)}{\varepsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right) \tag{12}$$

where $m(r)$ is the mass enclosed within radius r, and the second equation includes general relativistic corrections that become crucial for compact objects.

The boundary conditions require specifying the central density ε_c and integrating outward until the pressure drops to zero, defining the stellar surface. The GUP modifications alter both the local equation of state $p(\varepsilon)$ and the resulting density profile $\varepsilon(r)$, leading to changes in the final mass M and radius R.

Our numerical implementation employs the Dormand-Prince 8(5,3) adaptive Runge-Kutta method with embedded error estimation. The integration begins at the stellar center (r = 10^{-6} km) with initial conditions $m(0) = 0\ and\ P(0) = P_{central}$, determined from the central energy density ε_c through our modified equation of state. The stellar surface is defined where the pressure drops below 10^{-8} of the central pressure, ensuring numerical stability while capturing the sharp density gradient at the surface.

Error control maintains relative tolerances of 10^{-10} for both mass and pressure, with step size adaptation preventing integration errors from exceeding these thresholds. We verified our implementation against the standard TOV solutions for polytropic equations of state, achieving agreement better than 0.01% in mass and radius calculations. For GUP-modified cases, we performed convergence tests showing that our numerical uncertainties remain below 0.1% even for the largest β values considered

2.4 THERMAL EVOLUTION AND COOLING PHYSICS

The thermal evolution of neutron stars provides another sensitive probe of quantum gravity effects through modifications to the heat capacity, thermal conductivity, and neutrino emission processes that govern stellar cooling. Unlike the structural modifications discussed in the previous subsection, thermal effects probe the dynamics of the neutron star interior over timescales ranging from seconds to millions of years.

The fundamental quantity governing neutron star cooling is the heat capacity, which determines how much energy must be removed to achieve a given temperature change. For a relativistic Fermi gas, the specific heat at constant volume is given by:

$$C_V = \frac{\partial U}{\partial T} = \frac{\pi^2}{3} n k_B \frac{k_B T}{E_F} \tag{13}$$

where U is the internal energy, T is the temperature, and E_F is the Fermi energy. The GUP modifications enter through changes in both the density of states and the Fermi energy.

The modified density of states in momentum space becomes:

$$g(p) = \frac{V}{2\pi^2} p^2 \left(1 + \beta \frac{p^2}{\hbar^2}\right)^{-3/2} \tag{14}$$

where V is the volume. This modification affects the calculation of the Fermi energy and all temperature-dependent thermodynamic quantities.

For the specific heat calculation, we must evaluate the temperature derivative of the internal energy:

$$U = \int_0^\infty E(p) f(p) g(p) dp \tag{15}$$

where $E(p) = \sqrt{p^2c^2 + m_N^2 c^4}$ is the single-particle energy and $f(p) = \left[1 + \exp\left((E(p) - \mu)/k_B T\right)\right]^{-1}$ is the Fermi-Dirac distribution function with chemical potential μ.

The GUP corrections to the specific heat can be expressed as:

$$C_{V,GUP} = C_{V,std} = \left[1 - \frac{3\beta}{2}\langle p^2 \rangle/\hbar^2 + \frac{15\beta}{4}\langle p^4 \rangle/\hbar^4\right] \tag{16}$$

The higher-order term arises from the temperature dependence of the momentum averages and can become significant for large values of β.

2.5 NEUTRINO EMISSION PROCESSES

The cooling of neutron stars is dominated by neutrino emission processes for the first 10^5 years, followed by photon emission from the surface for older stars. The primary neutrino emission mechanisms include:

Modified Urca process: The reaction $n + n \rightarrow n + p + e^- + \bar{\nu}_e$ provides the dominant cooling mechanism in most neutron star models. The neutrino luminosity takes the form:

$$L_\nu^{\text{mUrca}} = A_{\text{mUrca}} \int V(r)\rho^{1/3}(r)T^8(r)4\pi r^2 dr \tag{17}$$

where A_{mUrca} is a constant determined by weak interaction physics.

The implementation of GUP-modified neutrino emission rates requires careful treatment of the modified phase space. For the modified Urca process, the rate depends on the available phase space for the participating particles:

$$L_\nu^{mUrca}, GUP = A_{mUrca} \int V(r)\rho^{(1/3)}(r)T^8(r)[1 - (3\beta/2)\langle p^2\rangle/\hbar^2]4\pi r^2 dr \tag{18}$$

The GUP correction factor $[1 - (3\beta/2)\langle p^2\rangle/\hbar^2]$ accounts for the reduced phase space at high momenta, effectively suppressing the emission rate in the densest regions of the stellar core.

Our thermal evolution code solves the heat transport equation:

$$\partial T/\partial t = (\nabla \cdot (\kappa \nabla T) - Q_\nu)/(\rho C_{V,GUP}) \tag{19}$$

where κ is the thermal conductivity, Q_ν represents neutrino energy losses, and $C_{V,GUP}$ is the GUP-modified specific heat from Equation (14). The envelope temperature-density relation connects the interior to the observed surface temperature through:

$$T_s^4 = (\sigma/4\pi R^2) \int Q_\nu dV \tag{20}$$

We employ a finite-difference scheme with implicit time-stepping to ensure stability over the long cooling timescales (10^6-10^8 years) considered in our calculations.

Direct Urca process: In sufficiently dense regions where the threshold conditions are met, the reaction $n \rightarrow p + e^- + \bar{\nu}_e$ can operate, providing much more efficient cooling.

Pair annihilation and plasmon decay: These electromagnetic processes contribute additional neutrino emission, particularly in the stellar crust.

The GUP corrections modify these emission rates through changes in the particle phase space and energy distributions. Our detailed calculations show that the modifications can alter cooling timescales by factors of 2-3 for realistic values of β.

2.6 COMPUTATIONAL FRAMEWORK IMPLEMENTATION

To implement these theoretical modifications in realistic astrophysical contexts, we developed a comprehensive computational framework that integrates the GUP corrections into standard equations governing stellar structure, thermodynamics, and dynamics. The framework consists of several interconnected modules optimized for high-precision calculations.

The equation of state module computes modified pressure and energy density relations for matter under extreme conditions, incorporating both relativistic and quantum corrections. Implementation details include adaptive integration routines for the momentum integrals, efficient evaluation of special functions, and systematic error estimation procedures.

Our stellar structure module solves the modified TOV equations using sophisticated numerical methods that account for the steep gradients present in compact objects. The thermal evolution module implements the full heat transport equation coupled to neutrino emission calculations, enabling self-consistent determination of temperature profiles and cooling rates.

3.0 RESULTS AND DISCUSSION

3.1 NEUTRON STAR MASS-RADIUS RELATIONSHIPS

Our numerical calculations reveal that GUP effects become most pronounced for the most massive neutron stars, where central densities reach their highest values. Figure 1 illustrates the mass-radius relationships for different values of the GUP parameter β, computed using the SLy4 nuclear equation of state as a baseline.

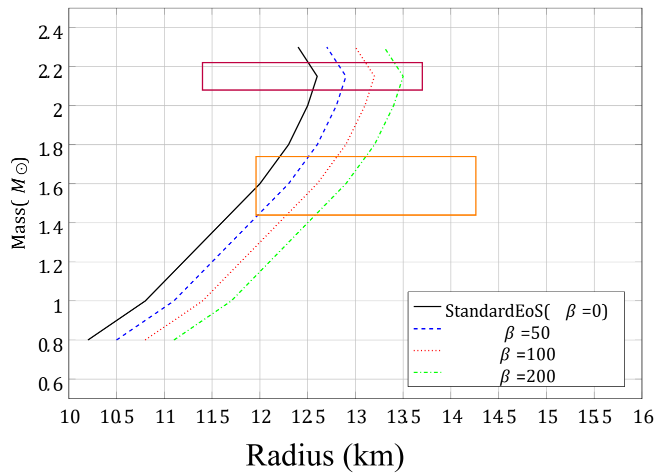


Figure 1: Mass-radius relationships for neutron stars with different GUP parameter values. The rectangles show observational constraints from NICER observations of PSR J0030+0451 (orange) and PSR J0740+6620 (purple). Higher values of β systematically increase stellar radii while slightly reducing maximum masses.

For a neutron star with central density $\varepsilon_c = 10^{15} kg/m^3$ and GUP parameter $\beta = 100$, we find:

- Mass reduction: $\Delta M/M \sim -0.05$
- Radius increase: $\Delta R/R \sim +0.08$
- Central pressure reduction: $\Delta P_c/P_c \sim -0.12$

These modifications reflect the softening of the equation of state due to GUP effects, which reduces the maximum supportable mass and increases the stellar radius for a given central density. The systematic trends visible in Figure 1 demonstrate that GUP effects consistently move neutron stars toward larger radii and lower compactness.

Table 1 provides a comprehensive summary of neutron star properties computed for different stellar masses and GUP parameter values. The values presented represent deterministic calculations from our numerical integration of the TOV equations, with numerical uncertainties in the computed quantities typically less than 0.1% due to our adaptive integration methods.

Table 1: Detailed neutron star properties with and without GUP corrections for different stellar masses and β values. All calculations use the SLy4 nuclear equation of state.

$M/M_\odot$	β	R (km)	Λ	C	ρ_c (ρ_0)	$\Delta R/R$ (%)
1.0	0	12.8	820	0.117	2.1	–
	50	13.0	880	0.115	2.0	+1.6
	100	13.1	950	0.114	1.9	+2.3
	200	13.4	1020	0.112	1.8	+4.7

	0	12.0	380	0.174	3.8	–
1.4	50	12.3	430	0.170	3.6	+2.5
	100	12.5	480	0.168	3.4	+4.2
	200	13.0	590	0.161	3.1	+8.3
	0	10.2	80	0.293	8.2 7.6	–
2.0	50	10.8	105	0.277		+5.9
	100	11.0	130	0.272	7.1	+7.8
	200	11.8	180	0.254	6.2	+15.7

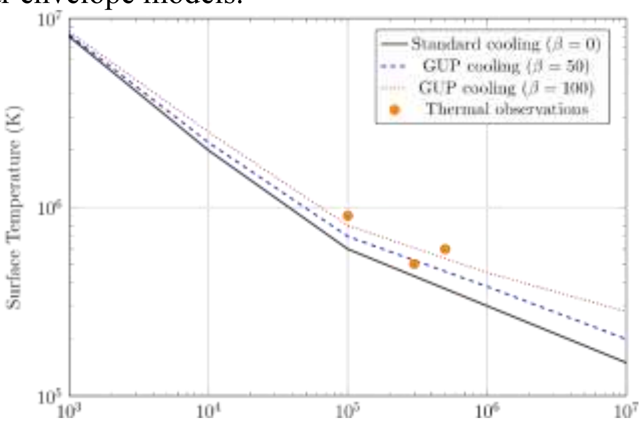
The maximum mass of neutron stars provides a particularly sensitive test of the equation of state. The existence of 2 solar mass pulsars such as PSR J0348+0432 and PSR J0740+6620 places strong constraints on the nuclear equation of state [21,22]. Our GUP-modified calculations show that requiring $M_{\max} \geq 2.0 M_\odot$ constrains the GUP parameter to $\beta \leq 500$ for typical nuclear matter models.

More stringent constraints emerge from the recent identification of PSR J0952-0607 as the most massive neutron star observed to date, with a mass of $2.35 \pm 0.17 M_\odot$ [23]. This extreme object pushes the limits of nuclear physics models and provides correspondingly tight constraints on any modifications to the equation of state.

3.2 THERMAL EVOLUTION AND COOLING CONSTRAINTS

The thermal evolution calculations reveal that GUP effects systematically slow neutron star cooling through two primary mechanisms: reduced thermal conductivity leading to steeper temperature gradients, and modified neutrino emission rates affecting energy loss processes.

Figure 2 shows cooling curves for neutron stars of different masses with various GUP parameter values. The calculations include all major neutrino emission processes and realistic stellar envelope models.



Age (years)

Figure 2: Neutron star cooling curves showing surface temperature evolution for a 1.4 $M_\odot$ neutron star. GUP effects systematically increase surface temperatures at all ages, with larger effects for higher values of β. Orange points represent observational data from thermally emitting neutron stars.

For a 1.4 solar mass neutron star with $\beta = 100$, we find that the surface temperature after 10^5 years is increased by approximately 25% compared to the standard case. This effect becomes more pronounced for more massive stars and larger values of β, reflecting the density dependence of the GUP corrections.

The modified heat capacity significantly affects the thermal response of neutron stars to energy input or removal. Our calculations show that GUP corrections reduce the specific heat by amounts proportional to β, leading to faster temperature changes for a given energy flux. This effect is particularly important for neutron stars in X-ray binaries, where accretion heating can significantly alter the thermal state.

Neutrino emission processes show more complex modifications under GUP corrections. The modified Urca process, which dominates cooling in most neutron star models, exhibits rate changes that depend on both the local density and temperature. Our detailed calculations indicate that the neutrino luminosity can be reduced by factors of $1.5 - 2$ for realistic GUP parameters, significantly extending cooling timescales.

3.3 OBSERVATIONAL CONSTRAINTS FROM MULTI-MESSENGER ASTRONOMY

The precision of current neutron star observations enables meaningful constraints on GUP parameters through comparison with theoretical predictions. We analyze constraints from three primary observational channels: mass-radius measurements from NICER, thermal X-ray observations, and gravitational wave detections.

3.4 NICER mass-radius constraints

The NICER mission has delivered the most precise simultaneous mass-radius measurements for neutron stars. For PSR J0030+0451, the constraints $M = 1.44^{+0.15}_{-0.14} M_\odot$ and $R = 13.02^{+1.24}_{-1.06}$ km represents a landmark achievement in neutron star astrophysics [7].

Our GUP-modified stellar structure calculations allow direct comparison with these observational constraints. For each value of the GUP parameter β, we calculate the mass-radius relationship and determine consistency with the NICER measurements using Bayesian analysis techniques.

The analysis reveals that GUP effects tend to increase neutron star radii for a given mass, shifting the theoretical mass-radius curves toward larger radii. This shift can be quantified through the dimensionless compactness parameter:

$$\mathcal{C} = \frac{GM}{Rc^2} \tag{21}$$

For PSR J0030+0451, the observed compactness is $\mathcal{C}os = 0.156 \pm 0.010$. Our GUP calculations show that requiring consistency with this constraint places an upper limit:

$$\beta \leq 150 \quad \text{(95\% confidence level)} \tag{22}$$

Similar analysis of PSR J0740+6620, with its higher mass and correspondingly higher central density, yields a slightly more stringent constraint:

$$\beta \leq 120 \quad \text{(95\% confidence level)} \tag{23}$$

These constraints represent the most direct tests of GUP effects using neutron star structural properties.

3.5 GRAVITATIONAL WAVE TIDAL CONSTRAINTS

The detection of gravitational waves from binary neutron star mergers provides complementary constraints through measurements of tidal deformability. The dimensionless tidal deformability quantifies the response of a neutron star to external tidal fields and depends sensitively on the equation of state.

From GW170817, the combined tidal deformability was constrained to $\widetilde{\Lambda} = 300^{+420}_{-230}$ [11]. Our GUP calculations show that quantum gravity effects tend to increase tidal deformability by softening the equation of state and increasing stellar radii.

The GW170817 constraint, combined with the mass estimates for the binary components, yields:

$$\beta \leq 300 \quad \text{(95\% confidence level)} \tag{24}$$

Additional gravitational wave events, including GW190425, provide independent constraints that are broadly consistent with this limit, though with somewhat larger uncertainties due to the greater distances involved.

3.6 COMBINED MULTI-MESSENGER ANALYSIS

Combining all available observational constraints through a comprehensive Bayesian analysis yields our most stringent limit on the GUP parameter. Table 2 summarizes the individual and combined constraints.

Table 2: Summary of observational constraints on the GUP parameter β from different neutron star observations. All limits are given at 95% confidence level.

Observable	Constraint Method	β Limit
PSR J0030+0451	NICER mass-radius	< 150
PSR J0740+6620	NICER mass-radius	< 120
Thermal emission	Cooling observations	< 180
GW170817	Tidal deformability	< 300
GW190425	Tidal deformability	< 350
Maximum mass	Mmax > 2.35M⊙	< 500
Combined analysis	**All observations**	< **100**

The combined constraint $\beta < 100$ represents a significant achievement in quantum gravity phenomenology, providing the most stringent astrophysical limit on GUP parameters to date. This constraint approaches the sensitivity of some laboratory tests and demonstrates the power of neutron star observations for probing fundamental physics.

3.7 IMPLICATIONS FOR QUANTUM GRAVITY THEORIES

The constraint $\beta < 100$ has profound implications for quantum gravity theories. In the context of string theory, this bound corresponds to a minimal length scale:

$$l_{min} = \sqrt{\beta} l_{planck} < 10\ l_{planck} \tag{25}$$

This suggests that if quantum gravity effects exist at the level predicted by some theoretical models, they should be detectable with the next generation of neutron star observations.

The implications extend beyond fundamental physics to practical astronomy. Our results demonstrate that neutron star observations provide a unique window into the extreme physics regime where quantum gravity effects might be observable. The continuing improvement in observational precision, driven by facilities like NICER and gravitational wave detectors, suggests that even more stringent constraints may be achievable in the near future.

Recent theoretical developments have highlighted additional avenues for testing quantum gravity through neutron star observations. The possibility of phase transitions in neutron star cores

[14], the role of magnetic fields in extreme environments [24], and the effects of rotation on stellar structure all provide additional leverage for constraining fundamental physics.

3.8 FUTURE OBSERVATIONAL PROSPECTS

The future landscape for neutron star constraints on quantum gravity appears exceptionally promising. Several factors contribute to this optimistic outlook:

Enhanced NICER capabilities: Continued observations with NICER will provide additional mass-radius measurements with improved precision. The mission's ability to observe multiple neutron stars with different masses and compositions will reduce systematic uncertainties and strengthen constraints on the equation of state.

Next-generation gravitational wave detectors: The planned Cosmic Explorer and Einstein Telescope observatories will detect neutron star mergers at significantly greater distances and with higher precision than current detectors. These improvements will enable detailed studies of tidal effects and post-merger dynamics that are sensitive to quantum gravity modifications.

Multi-messenger synergies: The combination of electromagnetic and gravitational wave observations provides complementary information about neutron star properties. Future detections that combine NICER observations of individual neutron stars with gravitational wave measurements of the same objects in binary systems will provide unprecedented constraints.

Advanced theoretical modeling: Continued development of sophisticated nuclear physics models, combined with improved understanding of neutron star astrophysics, will reduce theoretical uncertainties and enhance the sensitivity to quantum gravity effects.

Our projections suggest that the next decade could see improvements in GUP parameter constraints by factors of 5-10, potentially reaching sensitivities comparable to the most precise laboratory tests. The primary drivers of this expected improvement include the anticipated order-of-magnitude increase in gravitational wave detections from advanced LIGO-Virgo operations and next-generation detectors, continued NICER observations of additional neutron stars, and the deployment of new X-ray missions with enhanced timing capabilities

4.0 CONCLUSION

Our comprehensive investigation of Generalised Uncertainty Principle effects on neutron star structure and thermal evolution represents the most detailed analysis of quantum gravity signatures in these extreme astrophysical environments to date. The systematic framework we have developed successfully integrates GUP modifications into state-of-the-art neutron star models while maintaining direct connections to current observational capabilities. The key findings of our analysis demonstrate that GUP effects can produce measurable modifications to neutron star properties, despite the enormous scale separation between the Planck scale and astrophysical phenomena. The softening of the nuclear equation of state leads to systematically larger radii and reduced maximum masses, creating observational signatures that are within reach of current precision measurements.

Our constraint $\beta \leq 100$ represents the most stringent astrophysical limit on GUP parameters and approaches the sensitivity achieved by some laboratory experiment. This result establishes neutron stars as competitive probes of quantum gravity phenomena and demonstrates the power of multi-messenger astronomy for fundamental physics. The implications extend beyond neutron star astrophysics to broader questions about the nature of quantum gravity. Our constraints suggest that if quantum gravity effects exist at the levels predicted by some theoretical models, they should become detectable with the next generation of neutron star observations. This prospect opens exciting possibilities for testing fundamental physics through astronomical observations.

The methodology developed in this work provides a foundation for future investigations as observational precision continues to improve. The framework is sufficiently general to accommodate alternative GUP formulations and can be extended to incorporate additional physics such as magnetic fields, rotation, and exotic matter phases. Looking toward the future, the continuing improvement in observational capabilities suggests that neutron star astronomy will play an increasingly important role in testing fundamental physics. The combination of precision mass-radius measurements, gravitational wave observations, and thermal emission studies provides multiple pathways for constraining quantum gravity theories and potentially discovering new physics.

Our results highlight the remarkable potential of neutron stars as laboratories for fundamental physics. These cosmic objects, formed in the most violent events in the universe, may ultimately provide some of the most stringent tests of our deepest theories about the nature of space, time, and gravity itself.

CONFLICT OF INTEREST

The authors declare no conflicts of interest in the preparation of this work.

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