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REVOLUTIONARY DYNAMICAL INFORMATIVE THEORY OF GRAVITATION FOR A STATIC HOMOGENEOUS SPHERICAL DISTRIBUTION OF MASS

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ABSTRACT

In this article, the revolutionary gravitational scalar potential obtained from our previous research "The Revolutionary Dynamical Theory of Gravitation" is applied to the famous Lagrangian equation of motion. This yields modifications to the Newton's dynamical gravitational equations of motion. The modified dynamical gravitational equations of motion are then applied to the motion of planets in the equatorial plane to obtain a modified planetary equation of motion. The outcomes indicate that, the modified equations of motion and planetary equations of motion are improved with additional expressions of order c^{-2}*, which are not establish in the existing Isaac Newton's gravitational equations of motion and planetary equations of motion. As a consequence of the additional terms, the modified gravitational planetary equations of motion were able to resolve the problems associated with the anomalous orbital precession of the planets and the gravitational deflection of starlight, with similar results obtained by the existing Einstein's general theory of relativity for a static homogeneous spherical distribution of mass.*

1. INTRODUCTION

Sir Isaac Newton, in 1686, made known his dynamical theory of gravitation on the basis of Euclidean geometry. This model was effective in elucidating the gravitational occurrence on Earth and some observations of solar systems and all physical experiments within their respective domains on the order of c^0 [1].

Despite the remarkable accomplishment of the Newtonian dynamical theory of gravitation in explaining the observed gravitational phenomena, the theory remains reasonably inadequate [1]. The Newton's dynamical theory of gravitation was unable to describe the observed gravitational phenomena, such as [1].

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(i) Anomalous orbital precession of planet orbits
(ii) Gravitational shifts caused by the sun
(iii) Gravitational waves
(iv) The geometrical curve of spacetime.
(v) Gravitational redshift
(vi) Change in the period of pulsars

Albert Einstein, in 1915, published his geometrical theory of gravitation on Riemannian geometry, which is commonly known as "general relativity" [2]. General relativity is a good model for the reason that its predictions is what we observe today. General relativity was able to resolve the puzzle associated with the anomalous orbital precession of the planets and the gravitational shifts by the sun [2].

However, this report is inadequate because relating the gravitational field of a black hole in general relativity has a key shortcoming to physical quantities such as spacetime curvature divergence at the center of a blackhole, which is an indication that Einstein's theory of general relativity is a classical theory and does not consider quantum effects [2]. At distances very proximate to the center of a blackhole (closer to Planck's length), spacetime quantum fluctuations are anticipated to play a significant role [2].

When we try to merge general relativity with quantum mechanics, several issues arise, such as [2]:

i. Scales: general relativity describes the behavior of large-scale objects, like planets and stars, while quantum mechanics governs the behavior of tiny particles, like atoms and subatomic particles. The scales at which these two theories operate are vastly different, making it challenging to reconcile them.
ii. Mathematically Frameworks; general relativity relies on smooth, continuous manifolds to describe space-time, whereas quantum mechanics uses wave functions and operators to describe particle behavior. These mathematical frameworks are fundamentally incompatible, making it difficult to merge the two theories.
iii. Time and Space: general relativity describes space-time as a dynamic, flexible entity, while quantum mechanics treats time and space as fixed, background structures. The difference in perspective creates tension when trying to merge the two theories.
iv. Quantization of Space-time: in quantum mechanics, particles are quantized, meaning they come in discrete packets. However, general relativity describes space-time as continuous, smooth fabric. To merge the two theories, space-time itself would need to be quantized, which is highly non-trivial task.
v. Black-hole Singularities: general relativity predicts the existence of singularities, like black-holes, where the curvature of space-time is infinite, even at the smallest scales. Resolving this discrepancy is essential for merging general relativity and quantum mechanics.

In the realm where both quantum effects and gravity are important, such as in the locality of black holes or in the Big Bang, we do not yet have a fully satisfactory theory. This is the realm of quantum gravity, and it is an active area of study [3]. The standard model of particle physics, which explains quantum particles and their interactions, does not embrace gravity, and numerous theories have been recommended to reconcile the apparent inconsistencies between quantum mechanics and general relativity, including string theory, loop quantum gravity and the theory of everything, but none of them have been established by experimental evidence [3]. This signals the breakdown of general relativity and the need for another model that goes beyond general relativity into quantum physics.

In 2013, Howusu publish a research,"Revolution in Mathematics and Theoretical Physics." In this paper, Howusu argued that, if, Newton's noncovariant vector gravitational fields based on Euclidean geometry can describe and explain all physical experiments on gravitational fields correctly to the limit of c^0, then, it is possible to formulate noncovariant laws of physics in line with the phenomenological concept of spacetime, that are capable of extending the existing Newton's noncovariant vector law of gravitational fields from the limit of c^0 to c^{-2} [4]. Howusu proposed the "Riemannian Laplacian Operator," an extended version of the eminent Euclidean Laplacian Operator in spherical polar coordinates as the unique mathematical tool that can enable Newton's noncovariant vector law of gravitational fields to describe and explain all physical experiments on gravitational fields correctly to the limit of c^0 to c^{-2} [4].

In 2024, Nwagbara, O. Obioha, A.O. Igbokwe, S.C. Ogwo, P.O & Nduka, M.N. published an article"Revolutionary Dynamic Theory of Gravitation." In this research, a generalized Newton's gravitational scalar potential was obtained for a static homogeneous spherical massive body by using the Riemannian Laplacian Operator. The results indicate that, the revolutionary dynamical gravitational field equation and gravitational scalar potential exterior to the body contains additional terms that are not found in the famous Newton's gravitational field and scalar potential exterior to the body [5].

This research proposes a revolutionary dynamical theory of gravitation, which challenges traditional understanding of gravity, and is aimed at deriving a unique dynamical gravitational field equation that will be able to explain satisfactorily, the gravitational phenomena from the limit of c^0 to c^{-2}, with results obtained, indicating that, the modified dynamical gravitational equations of motion and planetary equations of motion are improved with additional terms of order c^{-2} which are not established in the famous Newton's gravitational equations of motion and planetary equations of motion.

METHOD

The methods applied to achieve the aim of this research are as follows:

(i) Apply the revolutionary dynamic scalar potential to the renowned Lagrangian equation of motion.

(ii) Solve the equation obtained in (i) by considering the motion of a body in the equatorial plane of the sun, such as a planet, comet or asteroid in the solar system, to obtain the following modified results:
   a. The radial acceleration equation.
   b. The planetary equation of motion.

(iii) The modified planetary equation of motion for the corresponding perihelion advance is solved via the method of perturbation theory.

(iv) The generalized planetary equation of motion for a photon moving in the absence of a gravitational field of the sun is solved to obtain a result that indicates, the gravitational deflection of the starlight.

(v) Compare results obtained in this research with the findings obtained by the famous theory of general relativity.

RESULTS

The revolutionary dynamic scalar potential is given by [5]

$$f(r) = \frac{-Gm}{\mathrm{r}}\left\{1 + \frac{2Gm}{c^2r}\right\} \quad (1)$$

Consider a planet in the gravitational field of the sun; the famous Lagrangian equations of motion is given by [6]

$$L = T - U \quad (2)$$

The potential energy is given by

$$U = \frac{-Gm^2}{\mathrm{r}}\left\{1 + \frac{2Gm}{c^2r}\right\} \quad (3)$$

And the kinetic energy is given by

$$T = \frac{1}{2}mV^2 \quad (4)$$

Equation (4) can be written as

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad (5)$$

Thus equating (4) with (5), we obtain

$$V^2 = \left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad (6)$$

Hence, combining (3) and (5), we obtain the Lagrangian equation and is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{Gm^2}{r}\left\{1 + \frac{2Gm}{c^2r}\right\} \quad (7)$$

Differentiating equation (7) w.r.t $\dot{r}, \dot{\theta}, and\ r$, we obtained set of equations which are given by

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r} \quad (8)$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} \quad (9)$$

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - \frac{Gm^2}{r^2} - \frac{4G^2m^3}{c^2r^3} \quad (10)$$

Simplifying (10), we have

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - \frac{Gm^2}{r^2}\left(1 + \frac{4Gm}{c^2r}\right) \quad (11)$$

The famous Lagrangian equation of motion is given by [6]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \quad (12)$$

Applying equations (8-10) to (12), we obtain

$$m\ddot{r} - mr\dot{\theta}^2 + \frac{Gm^2}{r^2}\left(1 + \frac{4Gm}{c^2r}\right) = 0 \quad (13)$$

Equation (13), further simplifies to

$$\ddot{r} - r\dot{\theta}^2 + \frac{K}{r^2}\left(1 + \frac{4Gm}{c^2 r}\right) = 0 \tag{14}$$

Where $K = Gm$

Consider the motion of a body in the equatorial plane of the sun, such as a comet, asteroid or planet in the solar system, that is, when [7]

$$\theta = \frac{\pi}{2} \tag{15}$$

And

$$\dot{\theta} = \frac{-l}{r^2} \tag{16}$$

Applying equations (16) to equation (14), we obtain

$$\ddot{r} = \frac{-K}{r^2}\left\{1 + \frac{4K}{c^2 r}\right\} + \frac{l^2}{r^3} \tag{17}$$

l is designated to be the constant of motion known as the angular momentum per unit of rest mass.

Equation (17) differs from the famous Newtonian term with the factor $\left\{1 + \frac{4K}{c^2 r}\right\}$.

The exact form of the first integral of the radial motion of equation (17) gives

$$\grave{r}^2 = \frac{2k}{r} + \frac{4k^2}{c^2 r^2} - \frac{l^2}{r^2} \tag{18}$$

Equation (18) contains a correction term that is not found in the renowned Newtonian radial speed equation [7].

Let the transformation of the radial acceleration be of the form [4]

$$\left(1 + \frac{\grave{r}^2}{c^2}\right)\ddot{r} = \frac{-K}{r^2}\left\{1 + \frac{4K}{c^2 r}\right\} + \frac{l^2}{r^3} \tag{19}$$

Hence, we can rewrite equation (19) as

$$\ddot{r} = \frac{-K}{r^2}\left\{1 + \frac{4K}{c^2 r}\right\}\left(1 + \frac{\grave{r}^2}{c^2}\right)^{-1} + \frac{l^2}{r^3}\left(1 + \frac{\grave{r}^2}{c^2}\right)^{-1} \tag{20}$$

By applying (18) to (20), we obtain

$$\ddot{r} = \frac{-K}{r^2} + \frac{2K^2}{c^2 r^3} - \frac{Kl^2}{c^2 r^4} - \frac{4K^2}{c^2 r^3} + \frac{l^2}{r^3} - \frac{2Kl^2}{c^2 r^4} + \frac{l^4}{c^2 r^5} \tag{21}$$

Which simplifies to

$$\ddot{r} = \frac{-K}{r^2}\left\{1 - \frac{2K}{c^2 r}\right\} + \frac{l^2}{r^3} - 3\frac{Kl^2}{c^2 r^4} + \frac{l^4}{c^2 r^5} \tag{22}$$

Equation (22) is the revolutionary dynamical radial acceleration of motion. It contains correction terms not found in the famous Newton's dynamical radial equation of motion.

By the transformation that [4]

$$r(\emptyset) = -\frac{1}{u(\emptyset)} \tag{23}$$

and

$$\grave{r} = -l^2 u^2 \frac{\partial^2 u}{\partial \emptyset^2} \tag{24}$$

Applying (23) and (24) to (22), when $u = \grave{r}$, we obtain

$$\frac{\partial^2 u}{\partial \emptyset^2} = \frac{K}{l^2}\left\{1 - \frac{2K}{c^2 r}\right\} - u + 3\frac{Ku^2}{c^2} + \left(\frac{u^3}{c^2}\right) \tag{25}$$

which, simply reduces to

$$\frac{\partial^2 u}{\partial \emptyset^2} = \frac{K}{l^2}\left\{1 - \frac{2K}{c^2 r}\right\} - u + 3\frac{Ku^2}{c^2} \tag{26}$$

Or

$$\frac{\partial^2 u}{\partial \emptyset^2} = \frac{K}{l^2} - u + 3\frac{Ku^2}{c^2} - 2\frac{K^2 \mathrm{u}}{c^2 l^2} \tag{27}$$

Equation (26) is the revolutionary dynamical planetary equation, and this equation reduces to the pure Newtonian form in the limit of c^0 of speed of light in vacuum, and is given by [1]

$$\frac{\partial^2 u}{\partial \emptyset^2} = \frac{K}{l^2} - u \tag{28}$$

The exact planetary equation of motion for equation (26) reduces to the pure Einstein's form as [3]

$$\frac{\partial^2 u}{\partial \emptyset^2} = \frac{K}{l^2} - u + 3\frac{Ku^2}{c^2} \tag{29}$$

Equation (29) is the famous general relativistic equation of motion [3,4] for the motion of a body in the equatorial plane of the sun, such as asteroid, planet or comet in a solar system at the limit of c^{-2}.

Exact Calculation

Equation (27), resembles the non-exact equation of general theory of relativity for planetary orbits [3], with additional term given by $-2\frac{K^2 \mathrm{u}}{c^2 l^2}$. However, the standard derivation of the anomalous precession formular typically neglects the higher-order term.

Applying the standard orbital equation (29), using the perturbation theory, the unperturbed solution of equation (29) is given by

$$u_0 = \frac{K}{l^2}\{1 + \in cos\emptyset\} \tag{30}$$

The perturbation term is given by: $3\frac{Ku^2}{c^2}$

The first-order correction to the orbit is given by:

$$u_1 = \frac{3K^2}{c^2 l^2}\left(\frac{K}{l^2}(\{1+\in cos\emptyset\})^2\right) \quad (31)$$

This solution corresponds to a conic orbit which is processing in the plane in such a way that its perihelion undergoes a displacement through an angle Δ given by

$$\Delta = \frac{6k^2}{c^2 l^2} \quad (32)$$

We can rewrite equation (31) as [3]

$$\Delta = \frac{6\pi GM}{c^2 a\prime(1-e^2)} \quad (33)$$

where $k = Gm$, $l^2 = GMa\prime(1-e^2)$, e, is known as the eccentricity and where $a\prime$ is also known as the semi-major axis of the orbit.

The results predicted by the revolutionary dynamical informative theory of gravitation (RDITG) are presented in a tabular form:

Table 1. Comparison of the Precession According to Theory and the Observed Values

Planets	Observed Precession Arc-second\Century	Predictions of General Theory of Relativity	Prediction of the Revolutionary Dynamical Informative Theory	Ratio of the Revolutionary Dynamical Informative Theory of Gravitation to Einstein`s General Theory of Relativity	Absolute Difference between RDITG and GTR
Mercury	43.11±0.45	42.945	42.945	1.000000000	0.000000000
Venus	8.4±4.8	8.60	8.60	1.000000000	0.000000000
Earth	5.0±1.2	3.82	3.82	1.000000000	0.000000000
Mars	-	1.40	1.40	1.000000000	0.000000000
Jupiter	-	0.06	0.06	1.000000000	0.000000000
Saturn	-	0.014	0.014	1.000000000	0.000000000
Uranus	-	0.0024	0.0024	1.000000000	0.000000000
Neptune	-	0.00078	0.00078	1.000000000	0.000000000
Icaros	9.8±0.8	10.30	10.30	1.0000000000	0.000000000

Table 1 shows the predictions of the general relativity theory and the modified dynamical informative theory of gravitation for the observed anomalous orbital precession of the planets in our solar system. Column four (4) was obtained when all the necessary data associated with equation (33) were applied. Column five (5) was obtained by comparing the results obtained in column four (4) with the famous results for the observed anomalous orbital precession of the planets in our solar system, obtained by general relativity. The results obtained in column five (5) indicate that, the results of this research fall within the range of acceptability [8].

Equation (33) shows that the precession angle per revolution is inversely proportional to the semi-major axis of the orbit and directly proportional to the eccentricity of the orbit [8].

The results in Table 1, also indicates that, this research is in agreement with Einstein’s general theory of relativity for the observed anomalous orbital precession of the planets in our solar system.

Gravitational Deflection of StarLight

In the gravitational field of the sun, a photon in motion is considered a test particle. The equation used to describe it is given by

$$\frac{\partial^2 u}{\partial \emptyset^2} = \frac{K}{l^2} - u + 3\frac{Ku^2}{c^2}$$

The modified planetary equation of motion (28), can also be applied to the motion of a photon of a gravitational field of the sun. The equation of motion for photon is given by [7]

$$\frac{\partial^2 u}{\partial \emptyset^2} + u = 3\frac{Ku^2}{c^2} \qquad (34)$$

If we neglect the gravitational field, then equation (28) reduces to equation (34).

The solution of equation (34) is a straight line and is given by [7]

$$u = \frac{sin\emptyset}{R} \qquad (35)$$

This is the polar equation of a straight line whose perpendicular distance from the center is R. $r = \frac{1}{u}$ has a minimum value R at$\emptyset = \frac{\pi}{2}$. If we denote $y = rsin\emptyset$,the straight line of equation (35) can be described by [7]

$$y = rsin\emptyset = R = Constant \qquad (36)$$

Using the approximate value of u, we obtain

$$\frac{\partial^2 u}{\partial \emptyset^2} + u = 3\frac{Ku^2}{c^2R^2}sin^2\emptyset \qquad (37)$$

The equation has a special solution as [7]

$$u = \frac{K}{c^2R^2}(1 + cos^2\emptyset) \qquad (38)$$

Hence, the general solution will be of the form

$$u = \frac{1}{R}sin\emptyset + \frac{K}{c^2R^2}(1 + cos^2\emptyset) \qquad (39)$$

or

$$Y = R - \frac{K}{c^2R}(1 + cos^2\emptyset)r \qquad (40)$$

Since in Cartesian coordinates$X = rcos\emptyset \wedge Y = rsin\emptyset$, equation (40) can be written as

$$Y = R - \frac{K}{c^2R}\frac{(2X^2+Y^2)}{(X^2+Y^2)^{\frac{1}{2}}} \qquad (41)$$

For large values of$|X|$, the above solution asymptotically approaches the following expression :

$$Y = R - \frac{2K}{c^2R}|X| \qquad (42)$$

As seen from equation (42), asymptotically, the orbit of the light ray is described by two straight lines in space time. These lines make angles with respect to the X-axis and are given by [8]

$$tan\emptyset = \pm\frac{2K}{c^2R} \quad (43)$$

Far from the central body, the light ray is expected to move in a straight line since spacetime appears flat there. The angle of the deflection $\Delta\emptyset$ between two asymptotes is therefore given by [6]

$$\Delta\emptyset = \frac{4K}{c^2R} \quad (44)$$

Taking c =$2.9979 \times 10^8 ms^{-1}$, G =$6.672 \times 10^{-11} NKg^{-2}m^2, M_{Sun} = 1.989 \times 10^{30}Kg, R_{Sun} = 6.360 \times 10^8 m$ and K = GM.

Applying all these data to equation (44) gives the value $\mathbf{1.75^{II}}$ which is in perfect agreement with the experimentally observed values [3].

DISCUSSION

General relativity is a respectable theory because of its predictions that we observe today. General relativity was able to resolve the puzzle associated with the anomalous orbital precession of the planets as well as the gravitational shifts by the sun [3]. However, its report is inadequate because describing the gravitational field of a blackhole in general relativity has a major limitation to physical quantities such as space-time curvature divergence at the center of a blackhole [8], which is an indication that the general theory of relativity is a classical concept and does not consider quantum effects [9].

In the realm where both quantum effects and gravity are important, such as in the locality of black holes or in the Big Bang, we do not yet have a fully satisfactory theory. This is the realm of quantum gravity, and it is an active area of study [3]. The standard model of particle physics, which explains quantum particles and their interactions, does not embrace gravity, and numerous theories have been recommended to reconcile the apparent inconsistencies between quantum mechanics and general relativity, including string theory, loop quantum gravity and the theory of everything, but none of them have been established by experimental evidence [3]. This signals the breakdown of general relativity and the need for another model that goes beyond general relativity into quantum physics.

This research proposes a revolutionary dynamical informative theory of gravitation (RDITG), which challenges traditional understanding of gravity. The revolutionary dynamical informative theory of gravitation introduced a modified gravitational scalar potential equation, which incorporates the effects of gravitational information on space-time geometry as a result of the correction terms included in the gravitational scalar potential exterior to the body. The correction terms accounts for the results obtained in this research which are in agreement with the results obtained by the existing Einstein's general relativity for a static homogeneous spherical distribution of mass [10].

While the revolutionary dynamical informative theory of gravitation (RDITG) presents a distinct perspective on the nature of gravity, space-time, and the universe, there are indeed connections and shared concepts with general theory of relativity (GTR)

Shared Concepts:

i. Gravitational Field; both RDITG and GTR, recognize the existence of a gravitational field, which is responsible for the curvature of space-time [11].
ii. Space-time Geometry: both theories acknowledge the importance of space-time geometry in describing the behavior of gravity and the motion of objects [11].
iii. Equivalence Principle: the equivalence principle, which states that gravity is equivalent to acceleration, is a fundamental concept in both GTR and RDITG [11].
iv. Resolution of Anomalous Orbital Precession: Both RDITG and GTR resolves the problem of anomalous orbital precession the planets, providing a more accurate description of planetary motion [12 & 13].
v. Gravitational Shift by the Sun: Both RDITG and GTR predicts the gravitational shift by the Sun with similar results, demonstrating their consistency with established gravitational phenomena [10, 12, &14].

Difference and Unique Aspects

i. Introduces a New Scalar Potential: Derived from the research, "the revolutionary dynamical theory of gravitation. The RDITG has a distinctive gravitational scalar potential which differs from the traditional gravitational scalar potential in general relativity [5].
ii. Gravitation is not a fundamental, long-range force, but rather an emergent property of the underlying dynamical information exchange and scalar potential.
iii. Quantum Nature: RDITG framework is compatible with quantum mechanics and field theory, allowing for the description of gravitons as quanta of gravity, providing a more comprehensive understanding of the universe at all scales. GTR, on the other hand, is a classical field theory as it does not accommodate quantum effects.

Relationship between RDITG and GTR

i. Generalization: RDITG can be seen as a generalization of GTR, incorporating quantum mechanics and quantum information theory to provide a more comprehensive understanding of the universe.
ii. Extension: RDITG extends GTR as RDITG is compatible with quantum mechanics, extending to the region of physics where GTR is not applicable.
iii. Alternative Perspective: RDITG offers an alternative perspective on gravity, space-time, and the universe, which complements and expands upon the insights provided by GTR.

In summary, while RDITG and GTR share common concepts and roots, they differ significantly in their scope, methodology, and philosophical underpinnings. RDITG builds upon and extends GTR, providing a more comprehensive and nuanced understanding of the universe.

Implications of RDITG

i. Resolution of Anomalous Orbital Precession: RDITG resolves the problem of anomalous orbital precession the planets, providing a more accurate description of planetary motion.
ii. Gravitational Shift by the Sun: RDITG predicts the gravitational shift by the Sun with similar results obtained by general theory of relativity, demonstrating its consistency with established gravitational phenomena.

CONCLUSION

In this research, the revolutionary dynamical gravitational field intensity obtained from our previous research, "The Revolutionary Dynamical Theory of Gravitation," is applied to the renowned Lagrange's equation of motion to obtain modifications to Newton's dynamical gravitational equations of motion.

The modified dynamical gravitational equations of motion are then applied to the motion of planets in the equatorial plane to obtain a modified planetary equation of motion, and the outcomes of our findings indicate that, the modified equations of motion and planetary equations of motion are improved with additional expressions of order c^{-2}, which are not establish in the existing Isaac Newton's gravitational equations of motion and planetary equations of motion. As a consequence of the additional terms, the modified gravitational planetary equations of motion were able to resolve the problems associated with the anomalous orbital precession of the planets and the gravitational deflection of starlight, with similar results obtained by the existing Einstein's general theory of relativity for a static homogeneous spherical distribution of mass. The results obtained in this research are in agreement with the results obtained by the existing Einstein's general relativity for a static homogeneous spherical distribution of mass.

The publication of this theory marks a turning point and a new era in theoretical physics, pushing the boundaries of what we discern and what we thought we knew about the universe. It invites a new wave of research that could lead to advances not only in cosmology but also in technology, potentially impacting everything from particle physics to space exploration. This research could help us understand the true nature of space, time, and energy, and unlock secrets that have eluded humanity for centuries.

RECOMMENDATIONS
Further research is needed to apply the modified dynamical gravitational equations of motion and planetary equations of motion to other areas of physics, such as cosmology and particle physics.

FUTURE SCOPE
Similar research should be carried out to include spheroidal bodies for future research so as to understand the true nature of bodies under the influence of gravitational fields.

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Nwagbara Obinna conceived the idea, planned the manuscript, prepared the manuscript, and calculated the theoretical analysis of this research. The author discussed the results of this research, and Dr. Ikeri Henry Ifeanyi meticulously reviewed the manuscript.

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The study likely relies on theoretical derivations and comparisons rather than empirical data collection.

COMPETING INTERESTS
The author declares nonfinancial competing interests.

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