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SOLAR PHOTOVOLTAIC ARRAY – STORAGE COMBINATION OPTIMIZATION FOR STAND-ALONE SYSTEMS

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Abstract

This study presents a method of optimizing photovoltaic array and storage in standalone photovoltaic (PV) system. The configuration consists of two components parameters; arrays capacity and storage/(battery) capacity. By adjusting these two parameters, the reliability of the PV system can be determined. The reliability of the PV system is determined based on the concept of Loss of Load Probability (LOLP) from which the optimum storage and array capacities C_S and C_A respectively, may be define with respect to the costs per unit energy generated by the PV array and cost per unit energy stored in the storage. The standalone PV system energy was simulated using data for 21 year for three locations in Nigeria. A numerical method is applied to generate LOLP- array capacity plots for definite storage capacities and iso-reliability plots obtained from these for three locations in Nigeria. The correlation coefficients of iso-reliability lines range from 0.7931 to 0.9967 for the three locations considered. Regression method is applied to the equations for each set of iso-reliability lines to obtain a general equation relating array capacity to storage capacity for each location which is then optimized with respect to cost to obtain an expression for the storage capacity at the lowest system cost.

Keywords: Photovoltaic (PV), Loss of Load Probability (LOLP), State of Charge (SOC), Array Capacity (C_A), Storage Capacity (C_S)

1.0 Introduction

Photovoltaic (or PV) energy generating systems convert the Sun's energy directly into electricity based on the principle of photoelectric effect. PV system configuration known as "stand-alone or off-grid" system are the sole source of power to a load. The load may be appliances in the home, outdoor applications such as water pump, street and traffic lights or other loads. Stand-alone photovoltaic (SAPV) systems are increasingly viable and cost-effective candidates for providing electricity. SAPV system typically consists of a PV array, controller, battery storage and inverter for A.C. loads. The successful operation of the SAPV system is to find the optimum combination of PV array and battery storage to meet load demand. The SAPV system sizing method is essential. The sizing optimization method can help to guarantee reliability of PV system. The merit of a SAPV system should be judged in terms of the reliability of the electricity supply to the load. This is usually quantified by the concept of loss of load probability (LOLP) among others [1]. This concept is defined as the relationship between the energy deficit and the energy demand, as referred to the load, during the total operation time of the installation [2]. A number of research works have been carried out focusing on optimization of PV systems so that the number of PV modules, capacity of storage battery, capacity of inverter and PV array tilt angle can be optimally selected [3]. Khatib [3] developed a sizing method based on monthly-data for LOLP calculation. Optimization was carried out using LOLP optimum array and storage capacities as design criteria. The optimum PV sizing was obtained based on the minimum annualized total cost. Semaoui and other researchers presented an optimization model for optimal sizing of a standalone PV system by using MATLAB–Simulink for a location in Algeria [4]. The optimization methodology was

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implemented in three steps; these are modeling of standalone PV system components, developing load management model and developing the optimization criteria which was done based on LOLP and energy cost. The economical aspect was not included which may increase the cost of the system as well. Khatib and others [5] presented simplified energy flow models for three types of PV power systems using MATLAB. One of these systems is a standalone PV system. The logic of the standalone PV system energy flow was described using a MATLAB line code. The daily output power generated by the PV module/array was calculated using a regression model and the energy produced from the PV array based on a defined period was calculated. Fathi and other researchers [6] described a 7.2kWp PV plant installed in a remote area in Morocco. They studied the effect of the energy management strategy on the system performance based on daily meteorological data and load demand. Their result show that the energy demand and state-of –charge (or SOC) of the battery strongly affected the performance of the system. Dufo-López and others [7] developed a lead-acid battery life time prediction model for optimizing a standalone PV system in Spain. Their focus was to estimate the battery life time in order to reduce the system's capital cost. The standalone PV system components were expressed using a physical modelled while a regression model was used to express the PV array performance. Illanes and others [8] presented a dynamic simulation and modeling of a standalone PV system using the state equation model and numerical integration methods. The initial state of the system and the time evaluation of the inputs were determined, and then a numerical integration method was implemented to model the proposed system. Their work provides an improvement in the sizing results of standalone PV system. Lee, Soto and Modi [9] presented a sizing strategy for isolated standalone micro-grids in Mali. A time series energy balance algorithm based on hourly meteorological and load demand data were utilized to create cost versus availability curve for the system. The method they adopted starts by adjusting the system availability in order to minimize the system's capital cost. The system availability used in this study was the energy shortfall probability (ESP). The analysis starts with quantifying the system availability for each month and is then followed with the determination of the lowest availability which is based on a sub-daily resolution. Bouabdallah and others [10] proposed a techno-economic sizing methodology by using a statistical approach. The methodology begins with calculating the output power of the PV array in order to find the PV array output energy generated based on hourly time series meteorological and load demand data. Accordingly, the difference between the generated energy by PV array and the load demand, the state of charge of the battery was used to determine the required capacity of the battery. Dorji and others [11] used HOMER energy software to analyze the possible options for electrification of a few of remote settlements in the Kingdom of Bhutan, a small neighboring country of India. A harsh mountainous terrain with many settlements scattered all over the place makes a grid connection impossible in most of the cases. In that work, they considered energy needs of households, available renewable resources and current policies and programs on rural electrification. The study revealed that renewable technologies such as wind-battery or PV-battery can be considered as alternatives to grid connections. The study also showed that the most economical approach differs from site to site.

Mandelli along with other researchers [12] proposed a sizing methodology to optimally design an off-grid rural electrification system in Uganda. Daily averages of meteorological and load demand data were utilized. A value of lost load (VOLL) concept was used to determine the lost energy (LE) for all of the configurations. The LE was used as an objective function to find the best PV array size and storage battery capacity. The PV array was modeled using a regression model and the battery was modelled based on a static model which may affect the accuracy of the sizing results. These effects may lead to increase in the cost of generated unit of energy from the proposed model. Nordin and Rahman [13] proposed an optimization sizing method of standalone PV systems based on Malaysia's metrological profile. Hourly load demand data and meteorological data were used. The sizing methodology proposed a search space for the PV array and batteries numbers. The LOLP was calculated for all combinations in the search space. The combinations which had the desired LOLP values were nominated. The best configuration was selected based on the LE. Erusiafe and Chendo [14] presented a life-cycle-cost (LCC) analysis for stand-alone PV system for Ikeja, Lagos, Nigeria. The components of the PV system were all expressed in terms of their present worth value. The economic benefit of operating a PV system may be deduced by comparing the LCC with the cost of buying energy. They obtained an expression for the optimum storage capacity that yielded the lowest LCC. The storage capacity may be expressed as periods of autonomy (dividing the battery capacity by the load energy demand). Sizing methods may be grouped under the following categories which includes Sizing rules based on the designer's experience (intuitive or empirical), Analytical methods, Simulation methods [1] and Artificial Intelligence techniques [15]. This work is focused on the simulation based approach. The objectives are to obtain iso-reliability lines for the three locations considered and a general equation that can be applied to yield an expression for the optimum combination of the two main components of the PV system i.e. the PV array and storage.

2.0 Methodology

2.1 System simulation

The system operation is based on a simplified development of energy generation and energy utilization occurring exclusively. Energy is generated by the array and stored in the storage medium during daytime (from dawn to dusk) and utilized from dusk to dawn. Figure 1 shows the components of a typical stand-alone PV system (with storage).

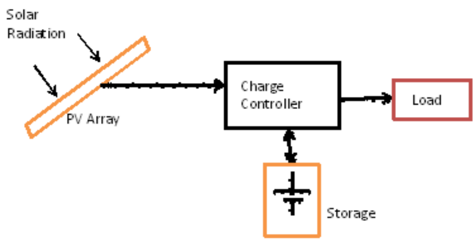


Figure 1: Block diagram of typical stand-alone (off-grid) PV system.

The system shown in figure 1 may be in continuous operation where the function of energy generation, storage and utilization may not be bound to specific periods of the day. The system used for simulation shown in figure 2 however operates under specified condition with respect to energy utilization.

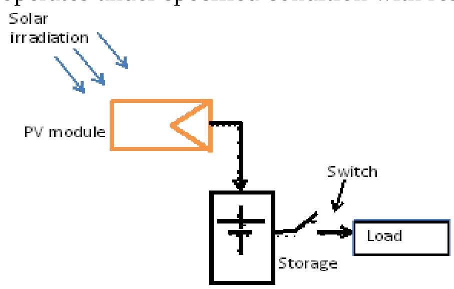


Figure 2: Diagram of PV system used for simulation.

Daily solar irradiation data for a period of 21years (from 2001 to 2021), obtained from NASA Langley Research Center (LaRC) POWER Project funded through the NASA Earth Science/Applied Science Program, was used. The operation of the system in figure 2 is such that energy generated by the array (comprising a number of modules) during the day is stored in the storage unit; stored energy is supplied to the load at night. The state-of-charge (SOC) of the storage (batteries) is updated each day before daytime charging. A situation where the storage is charged enough to meet load demand represents a zero loss of load; a loss of load represents a situation that is to the contrary, where loss of load is greater than zero. The system status is updated each day for the 21year period based on energy generated by the PV array and the charging/discharging of the storage.

2.2 Theory

From figure 2, the daily output or energy produced by a PV module/array is given by

$$C_{PV} = \eta_{PV} A.H \quad (1)$$

where A is the area of the PV module/array, H is daily solar irradiation and η_{PV}, is the efficiency of PV module/array. The status at the front end of a PV system or at the load side at the end of each day of operation is given by

$$\text{Energy Difference} = \sum(C_{PVS} - L) \quad (2)$$

where C_{PVS} is the energy available to the load from the PV system and L is the load energy demand. The result of equation (2) is that it may be either positive ($C_{PVS} > L$) or negative ($C_{PVS} < L$). A negative case implies a deficit and represents the energy that must be sourced elsewhere to run the load. The value of the load is usually adjusted to account for efficiencies associated with system components as well as depth-of-discharge of the storage batteries.

$$Adjusted.Load(L) = \frac{Load}{\eta_A \eta_{CH} \eta_{DOD}}. \quad (3)$$

where η_A and η_{CH} are efficiencies associated with system connection and charging/discharging of storage while η_{DOD}

fraction of full capacity that storage may be discharged. The state-of-charge (SOC) is defined as the ratio of the available capacity Q(t) and the maximum possible charge that can be stored in a battery, i.e., the nominal capacity Q_n [16]:

$$SOC = \frac{Q(t)}{Q_n} \quad (4)$$

A fully charged battery has SOC of 1 or 100% while a fully discharged battery has an SOC of 0 or 0%. The storage state-of-charge for a given day Y is updated at the end of the day to get SOC_Y, as follows:

$$SOC_Y = SOC_{Y-1} + C_{PV} - Adjusted.Load(L) \quad (5)$$

where SOC_{Y-1} is the state-of-charge of the previous day Y-1, C_{PV} is the energy generated on day Y and the energy consumed by the load on day Y. The availability of a PV system – i.e. the ability of the PV system to meet the load demand over a period, say a year - is expressed as a statistical value which is the loss of load probability (LOLP). LOLP is the ratio of annual energy deficits to annual load demand, and it is given by

$$LOLP = \frac{\sum Daily.energy.deficit}{\sum Daily.energy.consumption} \quad (6)$$

The PV array capacity, C_A, and storage capacity, C_S, are defined as follows:

$$C_A = \frac{C_{PV}}{L} \quad (7)$$

$$C_S = \frac{C_B}{L} \quad (8)$$

where C_B, C_{PV}, and L are storage size, PV array size, and load, respectively. They are expressed in Watt-hour (Wh).

Simulation is carried out to obtain variations of loss-of-load probability (LOLP) with array capacity C_A at given storage capacity C_S.

3.0 Result and Discussion

Simulation of the system in figure 2 for Lagos, Abuja and Port Harcourt with solar irradiation for 21 years yielded the following plots.

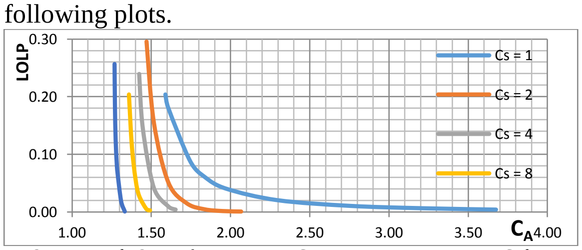


Fig. 3 Variation of LOLP with array capacity C_A at various storage capacities C_S for Lagos.

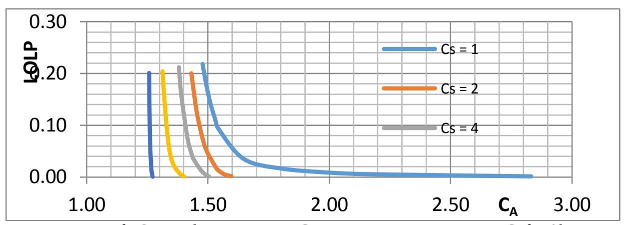


Fig. 4 Variation of LOLP with array capacity C_A at various storage capacities C_S for Abuja.

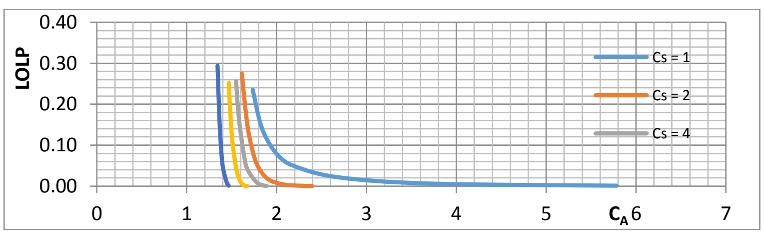
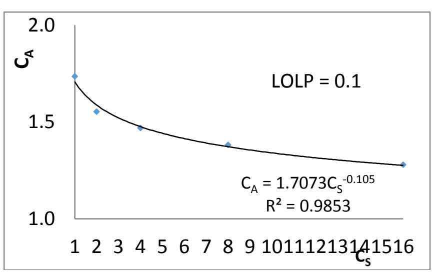
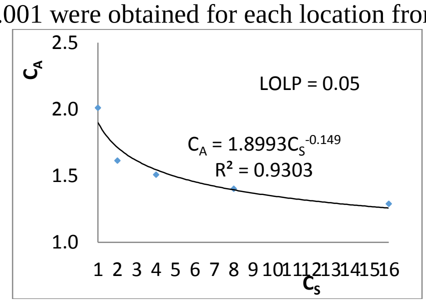


Fig. 5 Variation of LOLP with array capacity C_A at various storage capacities C_S for Port Harcourt.

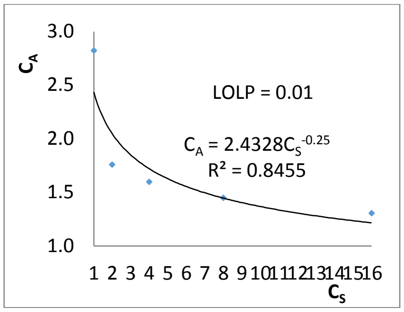
Iso-reliability lines for LOLP = 0.1, 0.05, 0.01 and 0.001 were obtained for each location from figures 3, 4 and 5 as follows:



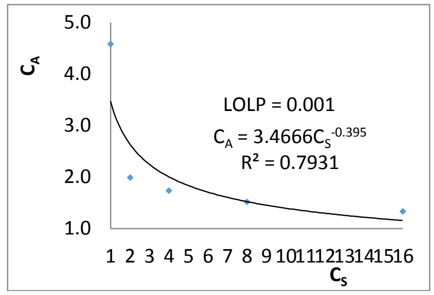
(a) C_A versus C_S for LOLP = 0.1



(b) C_A versus C_S for LOLP = 0.05

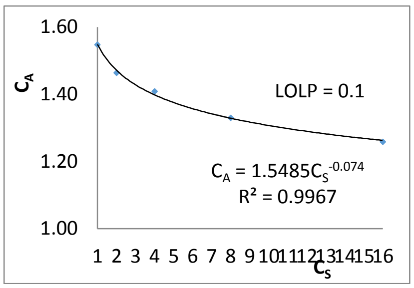


(c) C_A versus C_S for LOLP = 0.01

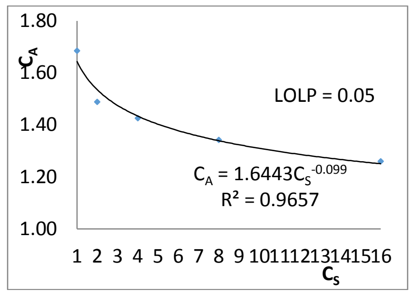


(d) C_A versus C_S for LOLP = 0.001

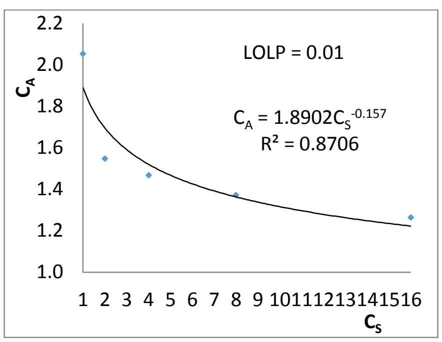
Fig. 6 Iso-reliability lines for various LOLP values for Lagos.



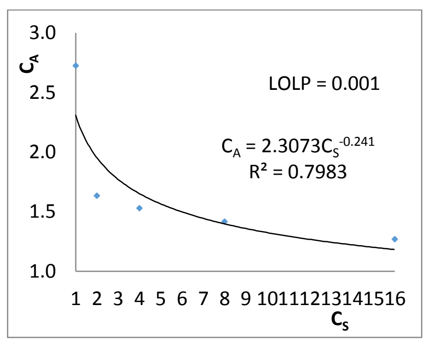
(a) C_A versus C_S for LOLP = 0.1



(b) C_A versus C_S for LOLP = 0.05

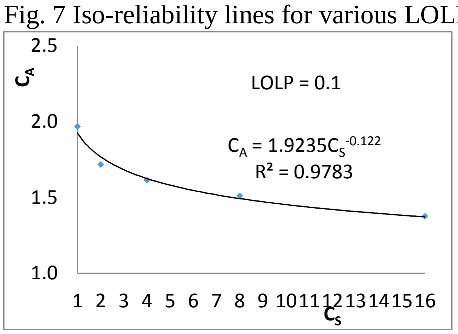


(c) C_A versus C_S for LOLP = 0.01

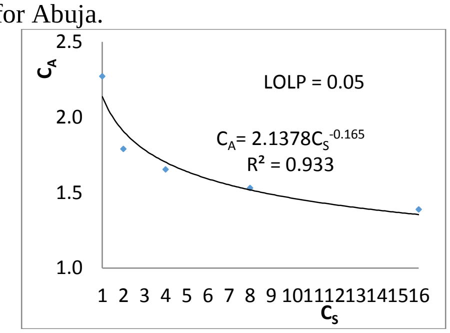


(d) C_A versus C_S for LOLP = 0.001

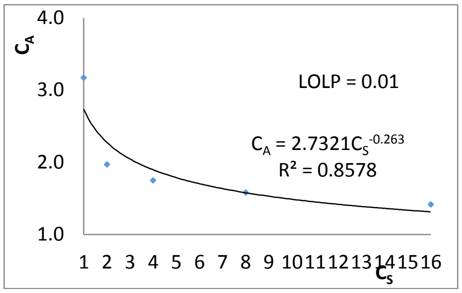
Fig. 7 Iso-reliability lines for various LOLP values for Abuja.



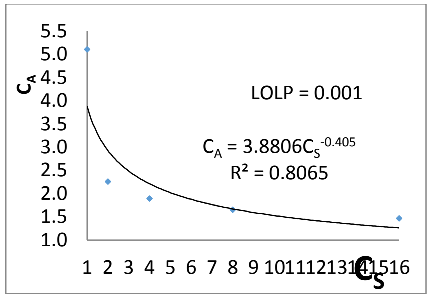
(a) C_A versus C_S for LOLP = 0.1



(b) C_A versus C_S for LOLP = 0.05



(c) C_A versus C_S for LOLP = 0.01



(d) C_A versus C_S for LOLP = 0.001

Fig. 8 Iso-reliability lines for various LOLP values for Port Harcourt.

The equations in figures 6, 7 and 8 are of the form

$$C_A = mC_S^n \quad (9)$$

where m corresponds to the array capacity when C_S = 1 and n is dependent on the persistence of solar irradiation [1]. A regression was carried out to obtain a general equation of the form in Eqn. (8) for each location by correlating m and n each with LOLP. The equations obtained from figures 6, 7 and 8 are;

$$m = -0.384\log_e(LOLP) + 0.7623 \quad (10)$$

$$n = 0.0629\log_e(LOLP) + 0.0398 \quad (11)$$

$$m = -0.165\log_e(LOLP) + 1.1527 \quad (12)$$

$$n = 0.0363\log_e(LOLP) + 0.0096 \quad (13)$$

$$m = -0.427\log_e(LOLP) + 0.8728 \quad (14)$$

$$n = 0.0614\log_e(LOLP) + 0.0193 \quad (15)$$

Equations (10) and (11) were obtained for Lagos with correlation coefficients of 0.9914 and 1.00 respectively; equations (12) and (13) were obtained for Abuja with correlation coefficients of 0.9972 and 1.00 respectively while equations (14) and (15) were obtained for Port Harcourt with correlation coefficients of 0.9915 and 1.00 respectively. Equation (8) provides for different combinations of array size and storage size. Optimization of equation (8) can be carried out with respect to factors that can influence the values of C_A and C_S. One such example is the cost of the system. Given that the cost of the PV system is T_T where

$$T_T = T_A + T_S \quad (16)$$

T_A is the cost of the PV array and all costs associated with installation of the PV array and T_S is the cost of the storage and all costs associated with the installation of the storage. They are given as

$$T_A = C_{WA}\,.C_A\,.L \quad (17)$$

and

$$T_S = C_{WS}\,.C_S\,.L \quad (18)$$

such that T_A becomes

$$T_A = C_{WA}C_A L + C_{WS}C_S L \quad (19)$$

where C_{WA} is the cost per unit energy generated by the PV array and CWS is the cost per unit energy stored in the storage. Equation (9) is substituted into equation (19) to obtain the following:

$$T_A = C_{WA}mC_S^n L + C_{WS}C_S L \quad (20)$$

The lowest value for T_A can be determined by taking the derivative of equation (20) with respect to C_S and setting it to zero as in the following

$$\frac{dT_A}{dC_S} = mnC_S^{n-1}C_{WA}L + C_{WS}L = 0 \quad (21)$$

Evaluating equation (21) gives the storage capacity for optimum PV system size as C_S' where

$$C_S' = \left[-\frac{C_{WS}}{mnC_{WA}}\right]^{1/n-1} \quad (22)$$

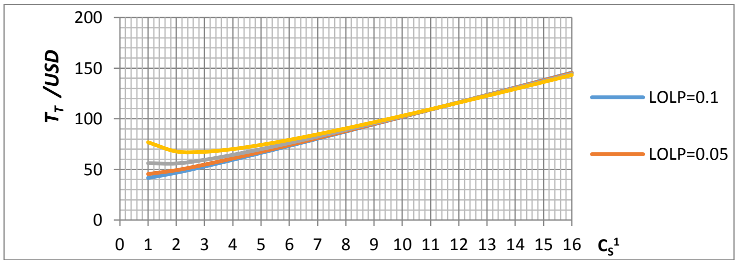


Fig. 9 Illustration of equation (19) showing variation of total system cost with storage capacity.

Figure 9 illustrates the result obtained for Lagos; it shows the curves for the variation of T_T with C_S for various LOLP values. C_{WA} and C_{WS} were computed from current prices for PV modules and batteries as well as mean day solar irradiation for the period considered. The values for these parameters are 0.2USD and 0.074USD respectively. The optimum storage capacity at C_S^1 for various LOLP is shown in table 1.

Table 1. Optimum array and storage capacities for each LOLP.

LOLP	**C_S^1**	**C_A^1**
0.100	0.513	1.831
0.050	0.783	1.970
0.010	1.472	2.209
0.001	2.530	2.402

Note that equation (22) is independent of load *L*. It can be seen that some values of C_S^1 are less than one. The implication is that such LOLP values (LOLP = 0.1 and 0.05) cannot be optimized with respect to system cost since the minimum value of C_S is one.

4.0 CONCLUSION

The simulation of PV system for the purpose of generating iso-reliability lines with respect to Array and storage capacities has been considered. This was carried out for three locations in Nigeria – Lagos, Abuja and Port Harcourt. General equations relating array and storage capacities were obtained through regression method applied to the iso-reliability lines for each location. System optimization for array and storage capacities based on costs associated with array and storage yielded an equation for optimum storage capacity which is independent of load. However, optimization can be considered along with other parameters that can affect energy generation such as module / array inclination angle PV module and battery technology. Although commercial sizing tools are available, they come with various limitations [17]. The method presented in this work is simple, allows for flexibility and geared towards realistic outcomes that can be applied to generate equations for zones and regions that have very similar patterns of solar resource. The shortcomings in some sizing methods which lead to oversizing or under-sizing can be minimized by proper assessment of what the LOLP value should be for any particular project.

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