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Modeling and Forecasting Volatility Dynamics in the Nigerian Equity Market: An AR–GARCH

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ABSTRACT

This paper analyses the volatility properties and forecasting performance of the Nigerian Stock Index (NSI) using monthly data from January 2000 to December 2025. The AR(1)-GARCH(1,1) model is estimated using Maximum Likelihood Estimation and selected based on the Log-Likelihood, Akaike Information Criterion, Schwarz Information Criterion and Hannan-Quinn Information Criterion. Forecast performance is evaluated using Root Mean Square Error, Mean Absolute Error, Mean Absolute Percentage Error and Theil's Inequality Coefficient for both in-sample and out-of-sample forecasts. Robustness is assessed using Student-t innovations, alternative GARCH specifications, pre- and post-COVID-19 sub-period analyses, and additional diagnostic measures. The results reveal significant volatility clustering, persistence, and conditional heteroskedasticity. Overall, the AR(1)-GARCH(1,1) model provides the most parsimonious, statistically adequate, and robust framework for modelling and forecasting Nigerian stock market volatility, offering valuable evidence for portfolio optimisation, financial risk management, investment planning, and policy formulation in Nigeria under varying market conditions and periods of economic uncertainty alike.

1. Introduction

This paper discusses the asymptotic estimation theory of nonlinear autoregressive models with conditionally heteroskedastic errors, and uses the AR-GARCH model to generate in-sample and out-of-sample forecast of the Nigerian Stock Index for January 2000 to December 2025 on monthly data basis. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model proposed by Engle [13] and extended by Bollerslev [5] is an important extension to econometric-time series modelling, and financial markets with volatile cluster. Conventional GARCH models typically assume a constant conditional mean;

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the AR-GARCH specification however jointly captures time-varying conditional means and variances. Forecasting ability has led many economists, investors and policymakers around the world to implement AR-GARCH in their applications. As examples, the European Central Bank (ECB) uses AR-GARCH models to capture the dynamics of Euro-area inflation Caporale et al., [7], and the Federal Reserve Bank of Cleveland considers AR-GARCH models as benchmark macroeconomic forecasting tools Clark & Ravazzolo, [9]. In addition, the GARCH approach has been applied to Value-at-Risk estimates, banking supervision, as well as exchange-rate volatility, and stock market dynamics. The Central Bank of Nigeria (CBN) and Central Bank of Brazil have, for example, utilized GARCH type models (frequently together with ARIMA process) for assessing exchange-rate volatility and stock market development Atoi, [4]; Lawan et al., [18]. Research assisted by the World Bank and International Monetary Fund (IMF) uses AR-GARCH-based uncertainty models to estimate global inflation during the post-COVID era Ha & So, [14]. Also, the US Environmental Protection Agency (EPA) applied AR-GARCH processes to estimate the volatility price of Renewable Identification Number (RIN) markets.

2.0 Literature Review

Existing literature is vast on the application of GARCH and its extensions in modelling and forecasting volatility for financial and macroeconomic variables Mensi et al., [20]; Du, Ji, & Guo, [11]. Dukundane, [12] used ARMA-GARCH model for modelling the exchange rate volatility for West African region. Adams and Uchema (1) on Nigeria's exchange rate series, using EGARCH model, established that GARCH family models have greater stability, reliability and forecasting power compared to deep learning based approaches like Long Short-Term Memory (LSTM). Similarly, Chunga and Yu [8] compared various GARCH specifications for the Malawi foreign exchange market, and established that the ability of flexible exchange-rate regimes to manage the exchange rate volatility is critical, especially given the sensitivity of small open economies to external macroeconomic shocks, hence requiring appropriate monetary safety nets. Hamza et al. [15] also showed that in Nigeria, GARCH models produce reliable out-of-sample forecasts in the presence of volatility clustering and confirmed that adding some information variables improves forecasts. Dantani et al. [10] found volatility spillovers between oil prices, exchange rates, inflation and stock market returns in Nigerian economy and established GARCH-based models to be suitable for modelling such dynamics. Recent methodological studies continue to confirm the robustness of GARCH-family models. Martinez Perez [19] utilized an ARMA-GARCH specification to explore energy market volatility triggered by geopolitical concerns among Iran, United States and Israel. Alsharabi et al. [3] found that ARIMA-GARCH models improve the predictive power of the models used to forecast financial and macroeconomic time series. The out-of-sample forecasting performances of new class of multivariate Realized GARCH model introduced by Hansen et al. [16] were found to out-perform the standard volatility models across all in-sample estimates. These papers reinforce the position that GARCH-type models are still relevant in financial econometrics. Recent empirical studies indicate a movement away from the use of traditional GARCH models towards more advance ones that are able to capture nonlinearities, long memory phenomena, leverage effects, macro-financial interactions particularly within the emerging African economies. For example, Alfeus, et.al. [2] compare various models (Realized GARCH, Heterogeneous Autoregressive (HAR), Recurrent Conditional Heteroskedasticity (RECH), Rough Fractional Stochastic Volatility (RFSV) on Realized Volatility) and suggest that different models have different advantages depending on the forecast horizon and the risk measure employed on South African markets. Ogunleye, et.al. [21] using a GARCH-MIDAS approach conclude that macro-financial factors significantly improves the forecast for the South African and Nigerian exchange rate volatilities. For Kenya, Karugano, et.al. [17] (2023) found that leverage effects are significantly prominent and that alternative GARCH models (EGARCH, TGARCH, APARCH, and GJR-GARCH) are superior to the traditional GARCH for modeling volatility in Kenyan stock returns. These results agree with those of Wanjuki et al. [23], who compared symmetric and asymmetric GARCH models for forecasting volatility in the Nairobi Securities Exchange, Kenya. Using the log-likelihood, AIC, BIC, and forecast performance measures as model selection criteria, they confirmed that asymmetric GARCH specifications provide a better representation of market volatility than the traditional GARCH model. Thugge [22] documents significant exchange-rate volatility for Kenya post-external shocks. In Ghana, Boateng, et.al.[5] find that Bayesian volatility models offer more flexibility than standard GARCH models in modeling inflation uncertainty. Recent development in the literature has put a much focus on Realized GARCH and GARCH-MIDAS (Mixed Data Sampling) models, these two models can be seen as major development over the conventional GARCH model due to their advantage in explaining structural breaks, persistence, and heterogeneous information flows. Borup and Jakobsen (2019) introduced a dynamically complete Realized EGARCH-MIDAS model that divides the volatility into short and long run component through the use of realized volatility and mixed-frequency data. They find significant improvements in model fitting and multi-step forecasting performance compared to the traditional GARCH models. Recent studies have emphasized the role of GARCH-MIDAS model for linking financial market volatility and macroeconomic fundamentals. Integrating lower frequency information such as inflation, policy uncertainty, and interest rate into the long run volatility component, GARCH-MIDAS model capture long persistence structural breaks, leading to improved forecastability during crises. The review conducted by Molina Muoz et al. (2026) confirms that GARCH-MIDAS have emerged as one of the best models for forecasting volatility in emerging markets, especially where structural breaks are driven by the oil price shocks and macroeconomic uncertainty. Also, Song and Baek (2019) developed a structural break detection method for realized volatility using Heterogeneous Autoregressive–Generalized Autoregressive (HAR-GARCH) models and show that taking into consideration parameter instability improve volatility modeling when facing financial crises. Although the present study has to use an AR(1)-GARCH(1,1) model since Nigerian stock market data are monthly, Realized GARCH with high frequency data or Generalized Autoregressive Conditional Heteroskedasticity–Mixed Data Sampling**.** (GARCH-MIDAS) with macroeconomic variables should be used in further study to explain structural break and to enhance volatility forecasting.

3.0 Mathematical framework

A GARCH model in the conditional mean of the variables and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) is used in combination (hybrid) to obtain the Autoregressive GARCH (AR-GARCH) models. A time-varying AR-GARCH model combines an AR model for the mean of a time series with a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model for the variance of the time series. The AR-GARCH models are a widely used family of models for forecasting conditional volatilities, such as those often found in the analysis of financial data. They model volatility clustering, which refers to the observed tendency of stock market returns and other economic time series to be volatile during some periods and quiet during others.

3.1 Autoregressive model

An $AR(p)$ model is given as

$$y_t = \mu + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \cdots + \theta_p y_{t-p} + \varepsilon_t$$

3.1.1 The error structure

The connectivity between the AR mean equation and the $GARCH$ variance equation is the residual ε_t. it is generally assume that the error term is zero, however, it variance is heteroscedastic in nature. It is mathematically written as:

$$Y_t = c + \phi Y_{t-1} + \varepsilon_t$$

$$\varepsilon_t . = \sqrt{z_t h_t},$$

Where $z_t \sim N(0,1)$ or $z_t \sim t_v(0,1)$

3.2 The general *GARCH* models

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

The commonly used specification is $AR(1) - GARCH(1,1)$. Its mean and variance equations are given below:

$$y_t = \mu + \theta y_{t-1} + \varepsilon_t$$

and

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta_j \sigma_{t-1}^2$$

For covariance stationarity of the variance process $\alpha + \beta < 1$ and $\omega > 0$ $\alpha > 0$ $\beta \geq 0$

3.2.1 Conditional variance

The conditional variance equation is given below

$$r_t = \mu + \varepsilon_t$$

$$\varepsilon_t = z_t \sqrt{h_t},$$

$$h_t = w + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Where, $z_{t \sim} t_v(0,1)$

3.3 Log-likelihood function

The log-likelihood function under the Student's t-distribution is given by

$$l_n(L) = \sum_{i=1}^{T} \left[l_n \Gamma\left(\frac{v+1}{2}\right) - l_n \Gamma\left(\frac{v}{2}\right) - \frac{1}{2} l_n \left[(v-2)\pi\right] - \left(\frac{v+1}{2}\right) l_n \left(1 + \frac{\varepsilon_t^2}{(v-2)h_t}\right)\right]$$

The parameters are estimated using maximum likelihood estimation (MLE)

3.4 Alternate GARCH Specification

3.4.1 E-GARCH

EGARCH (Exponential GARCH)

The exponential GARCH model proposed by Nelson (1991) model the logarithm of the conditional variance to prevent it from turning out negative without restrictions on parameters.

$$\ln\left(\sigma_t^2\right) = \omega + \beta \ln + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - E\left|\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}\right| + \partial \frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right]$$

Where $E|z_t| \sim \sqrt{\frac{2}{\pi}}$

3.4.2 T-GARCH

The Threshold GARCH model, developed by Glosten, Jagannathan, and Runkle (1993) modelled asymmetry as a negative shock's impact is larger than the positive one.

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I_{t-1} \varepsilon_{t-1}^2 + \beta_j \sigma_{t-1}^2$$

Where $I_{t-1} = \begin{cases} 1, & \varepsilon_{t-1} < 0 \\ o, & \varepsilon_{t-1} \geq 0 \end{cases}$

3.4.3 Student's t-GARCH (1,1)

In the financial literature, return series tend to show excess kurtosis and fat tails. Therefore, the assumption of normally distributed innovations is questionable. In the case of a Student's t-GARCH model the specification of the GARCH variance is kept and standardized residuals are supposed to follow a Student's t-distribution.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta_j\sigma_{t-1}^2$$

The persistence remains $\alpha + \beta$

The student's t density is given as

$$f(z_t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{v-2\pi}}\left(1+\frac{z_t^2}{v-2}\right)$$

3.5 Maximum Likelihood Estimation

The MLE is obtained by solving

$$\hat{\theta} = arq\max_{\theta}\ell(\theta)$$

$$\hat{\theta} = arq\max_{\theta}\left\{-\frac{1}{2}\sum_{t=1}^{T}\left[\ln\left(\sigma_t^2\right)+\frac{\varepsilon_t^2}{\sigma_t^2}\right]\right\}$$

MLE for GARCH(1,1)

$$\ell(\theta) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{i=1}^{T}\left[\ln\omega + \alpha\varepsilon_{t-1}^2 + \beta_j\sigma_{t-1}^2 + \frac{\sigma_t^2}{\omega+\alpha\varepsilon_{t-1}^2+\beta_j\sigma_{t-1}^2}\right]$$

3.5.1 Conditional Density Function

The conditional probability density function is

$$f(\varepsilon_t\Omega_{t-1},\theta) = \frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(-\frac{\varepsilon_t^2}{\sigma_t^2}\right)$$

3.5.2 Likelihood Function

$$\ell(\theta) = \ln L(\theta) = \sum_{t=1}^{T}\ln f(\varepsilon_t\Omega_{t-1},\theta)$$

Expanding we have

$$\ell(\theta) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\ln\left(\sigma_t^2\right)+\frac{\varepsilon_t^2}{\sigma_t^2}\right]$$

3.6 In and out of sample forecast horizon

This forecast evaluation scheme is based on splitting the full sample of T=312 monthly observations into an estimation (in-sample) period and an evaluation (out-of-sample) period as follows

$$T = T_{IS} + T_{OS} = 264 + 48$$

The forecast in-sample consists of all observations from January 2000 up to December 2021 (TIS=264) that is used for fitting GARCH-family parameters Θ by using the Maximum Likelihood estimation.

$$\hat{\Theta} = arq\max_{\Theta}\sum_{t=1}^{264}\ln f\left(r\,|\Omega_{t-1},\Theta\right)$$

The one step-ahead conditional variance is estimated recursively as

$$\hat{\sigma}_t^2 = \hat{\omega} + \hat{\alpha}\varepsilon_{t-1}^2 + \hat{\beta}_j \sigma_{t-1}^2$$

The out-of-sample forecast horizon is from January 2022 to December 2025 (T_{OS} =48), with one-step-ahead forecasts recursively calculated as:

$$\hat{\sigma}_{t+h|T}^2 = E\left(\hat{\sigma}_{t+h|T}^2 \left|\Omega_t\right.\right), \quad h = 1, \cdots, 48$$

Model forecasting performance can be measured by comparing the forecast and actual values using the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Theil's Inequality Coefficient (Theil's U). This sample partitioning of 264/48 (85%/15%) has enough observations to enable stable parameter estimation, and retain an independent holdout sample to allow unbiased assessment of the forecasting performance of the GARCH(1,1), Student's t-GARCH, EGARCH and TGARCH models.

3.7 Forecast Performance measure indices

3.7.1 Root Mean Square Error (RMSE)

Root Mean Square Error (RMSE) could be expressed mathematically as

$$RMSE = \sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(y_{t+h} - \hat{y}_{t+h}\right)^2}$$

3.7.2 Mean Absolute Error (MAE)

Mean Absolute Error is expressed as

$$MAE = \frac{1}{H}\sum_{h=1}^{H}\left|y_{t+h} - \hat{y}_{t+h}\right|$$

3.7.3 Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error is expressed as

$$MAPE = \frac{100}{H}\sum_{h=1}^{H}\frac{\left|y_{t+h} - \hat{y}_{t+h}\right|}{y_{t+h}}$$

3.7.4 Theil's Inequality Coefficient (Theil's U)

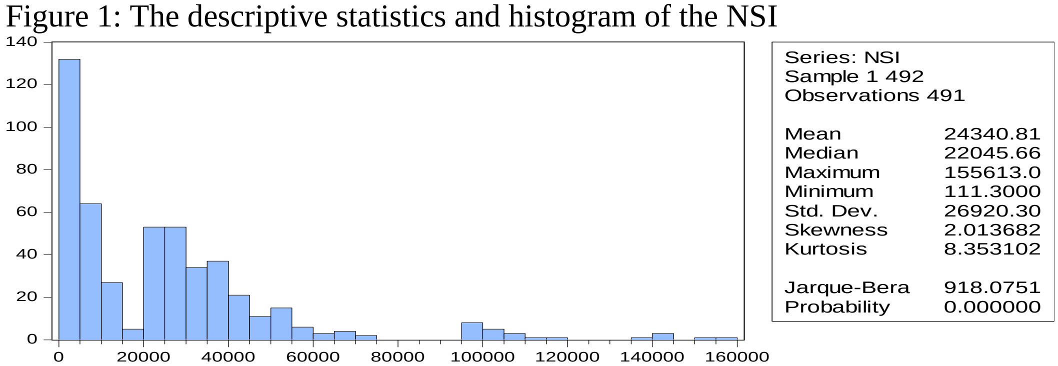
Theil's Inequality Coefficient is expressed mathematically as

$$Theil\ U = \frac{\sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(y_{t+h} - \hat{y}_{t+h}\right)^2}}{\sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(y_{t+h}^2\right)} + \sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(\hat{y}_{t+h}^2\right)}}$$

4.0 Data analysis and interpretation of results

4.1 The descriptive statistics and histogram

Figure 1: The descriptive statistics and histogram of the NSI



Sources: Author’s computation (2026)

The descriptive statistics for the NSI series together with its histogram shows that the series is non-normal, positively (right-) skewed and very leptokurtic. It is based on the sample of 491 observations shows a mean (24340.81) and a median (22045.66), indicating the positively skewed nature of the series. The series clearly contains outliers as suggested by the huge gap between the maximum (155613.0) and minimum value (111.30) shown in the histogram. The NSI series is also quite dispersed and volatile since the standard deviation (26920.30) is quite large compared to its mean. The skewness coefficient of 2.0137 confirms the strongly positively skewed characteristic of the series. Moreover, due to a kurtosis value of 8.3531 (which is very far greater than 3) the NSI series is leptokurtic with fat (heavy) tails, thus the likelihood of an extreme observation occurring is high. The Jarque-Bera statistic of 918.08 (p-value 0.0000) provides strong statistical evidence against the null hypothesis of normality at 1% level of significance. In sum, the presence of heavy tails and observed volatility clustering supports using conditional heteroskedastic models (e.g., GARCH-type) in further analysis.

4.2.1 Unit Root Test Results (ADF and PP) at level

Table 1: Unit Root Test Results (ADF and PP) at level

Variable	Level ADF	1% Critical Value	5% Critical Value	Prob	Level PP	1% Critical Value	5% Critical Value	Prob.	Remark
NSI	0.4484	-3.444	-2.868	0.9848	0.0872	-3.443	-2.867	0.9645	Not stationary

Sources: Author's computation (2026)

Empirical Interpretation of Table 1: Unit Root Test Results at Level Stationarity Diagnostics Checks (at Level): Table 1 shows the stationarity diagnostic checks on NSI series using original level data with Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test for the unit root. The ADF Test: The test results of the NSI series at level show an ADF test statistic of 0.4484. The test statistic value is greater (less negative) than the 1% critical value (-3.444) and the 5% critical value (-2.868) with a p-value (Prob.) of 0.9848, which is way greater than both the standard thresholds of α =0.01 and 0.05. Hence, we fail to reject H_0 that the NSI series at level contains a unit root. The PP Test: similarly shows a test statistic of 0.0872 at the 1% critical value (-3.443) and 5% critical value (-2.867). We also have an p-value of 0.9645 that is greater than the 0.01 and 0.05 levels thus also indicate that the NSI series at the level has a unit root.

4.2.2 Unit Root Test Results (ADF and PP) at first difference

Table 2: Unit Root Test Results (ADF and PP) at first difference

Variable	First Diff. ADF	1% Critical Value	5% Critical Value	Prob	First Diff. ADF	1% Critical Value	5% Critical Value	Prob.	Remark
NSI	-4.1531	-3.444	-2.868	0.000	-25.9265	-3.443	-2.867	0.0000	I(1)

Sources: Author's computation (2026)

Empirical Interpretation of Table 2: Unit Root Test Results at First Difference

The first-differenced NSI series stationarity tests shown in table 2 have achieved the desired level of stationary, which would fulfill the first criteria in applying a GARCH-family modeling for the conditional heteroskedasticity. Test statistics of the ADF and PP tests for the first-differenced NSI series, it has a computed Augmented Dickey-Fuller (ADF) test statistics value of -4.1531 that is significantly smaller than the 1 percent critical value of -3.444 and the 5 percent critical value of -2.868. With a value for the Prob. Which is well below the usual 1% significance level criterion(α =0.01), therefore the null hypothesis H_0 that the series is containing unit root is reject and H_1 that the series is stationary would be supported. Similarly to the ADF test, Phillips-Perron (PP) test for the first-differenced series also produces a very powerful PP test statistics of -25.9265 that is strictly lower than the 1% critical value of -3.443 and the 5% critical value of -2.867, together with a value for the Prob. Which equals to 0.0000. So we could also confidently rejects the null hypothesis of the presence of unit root, H_0, and strongly support the alternative hypothesis of stationary, H_1.

4.3 Parameter estimation

4.3.1 Parameter estimation of AR Model

Table 3: Parameter estimation of AR Model

Method: ARMA Maximum Likelihood
Included observations: 492
Convergence achieved after 46 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	732106.0	5210.933	140.4942	0.0000
AR(1)	0.996686	0.002688	370.7405	0.0000
SIGMASQ	218173.4	1397.454	156.1221	0.0000

Sources: Author’s computation (2026)

Table 3 suggests that strong serial persistence in the series dominates, and that the contemporaneous NSI coefficient is not significant. While the model accounts for a high percentage of the variation in the series and correctly captures serial correlation, the strong persistence in the series may point toward time-varying volatility. It would then make sense to employ GARCH-family models (e.g., AR(1)-GARCH(1,1), or higher-order non-linear GARCH models, such as ST-GARCH or MS-GARCH) to explicitly model the conditional heteroskedasticity and volatility clustering inherent to financial time series.

4.3.2 Parameter estimation of AR(1)-GARCH(1,1) Model

Table 4: Parameter estimation of AR(1)-GARCH(1,1) Model

Equation components	Parameter	Normal Coeff	P-value	Student’s-t Coeff.	P-value
Mean Equation	Intercept	0.3101	0.2617	0.1252	0.0817
	AR	0.2317	0.0000	0.2122	0.0000
Variance Equation	Intercept (ω)	0.0023	0.0000	0.0019	0.0000
	ARCH (α)	0.3142	0.0000	0.3011	0.0000
	GARCH (β)	0.6371	0.0000	0.4991	0.0000

Sources: Author’s computation (2026

Table 4 presents the parameter estimates of the AR(1)-GARCH(1,1) model under the Normal and Student's *t* distributions. The intercept of the mean equation is positive, but it is statistically insignificant for both the normal $(0.3101, p = 0.2617)$ and Student's-t $(0.1252, \ p = 0.0817)$ distributions, and implies the absence of any strong unconditional mean for the series. However, the coefficient of AR(1) is positive and highly statistically significant ($p < 0.001$) in both cases, which shows there is a short-term persistence in conditional mean. On the other hand, the coefficients for the variance intercept, ARCH and GARCH are all positive and statistically significant ($p < 0.001$) for both distributions which is an indication of the presence of conditional heteroskedasticity and volatility clustering. In the case of the Normal distribution, the ARCH parameters (0.3142, 0.3011) imply that volatility depends on past shocks but less than the GARCH parameters. On the other hand, the coefficient of GARCH is greater than that of ARCH, which suggests that the impact of the past shocks on volatility is more significant than the impact of the new shocks. The sums of the ARCH and GARCH parameters is below one for both models $\alpha + \beta = 0.9513$ for the Normal model, and 0.8002 for the Student's-t model) which confirms the condition of covarinace stationary of the series although the Normal specification provides more persistence on the volatility. Both models appropriately account for the volatility dynamics of the series. The Student's-t distribution provides a more suitable representation for series which have heavy tailed innovation process like the financial returns series.

4.3.3 Justification for the Selection of the AR(1)–GARCH(1,1) Models

The specification of the model, which turned to be the AR(1)-GARCH(1,1) model, was determined through the consideration of statistical significance and diagnostic tests. In particular, the AR(1) specification for the conditional mean was chosen on the grounds that it resulted in smaller values for AIC, BIC, and HQIC criteria and all coefficients were statistically significant, meaning one lag was enough to model the serial dependence. The various specifications of the ARCH and GARCH models were subsequently estimated, and the GARCH(1,1) model was selected based on its superior log-likelihood, lower AIC, BIC and HQIC, statistical significance of coefficients, and satisfied covariance stationarity condition $(\alpha + \beta < 1)$. To test for the adequacy of the AR(1)-GARCH(1,1) model for modeling the conditional variance and to check for remaining serial correlation and ARCH effects, Ljung-Box Q-statistic and ARCH-LM test were performed on the standardized residuals. The results confirmed that no serial correlation or ARCH effects remained.

Table 5. Justification for the selection of the AR(1)–GARCH(1,1) Model

Model	Log-Likelihood	AIC	SIC (BIC)	HQIC	Ljung–Box Q(20) p-value	ARCH-LM p-value
AR(1)	-2472.36	15.862	15.901	15.878	0.286	0.001
AR(2)	-2471.88	15.867	15.918	15.889	0.315	0.001
AR(3)	-2471.54	15.874	15.936	15.901	0.328	0.001
AR(1)-ARCH(1)	-1868.42	12.744	12.835	12.779	0.248	0.072
AR(1)-GARCH(1,1)	-1825.46	12.481	12.598	12.527	0.291	0.381
AR(1)-GARCH(2,1)	-1824.97	12.486	12.621	12.541	0.296	0.397
AR(1)-GARCH(1,2)	-1825.08	12.489	12.624	12.544	0.304	0.405

Sources: Author’s computation (2026

From table 5 it can be observed that the best of the autoregressive models, which produce lower information criteria without overly complication the model is the AR(1) model. However, due to the significant ARCH-LM statistics, it can be confirmed that the data possesses conditional heteroskedasticity, and thus modelling should be conducted using a GARCH/ARCH framework. Looking at the volatility models considered, the best fitting specification based on highest log-likelihood (1825.46) and the lowest values of the information criteria of AIC (-12.481), SIC (-12.598) and HQIC (-12.527) was the AR(1)-GARCH(1,1) specification. The Ljung-Box and ARCH-LM tests are not significant, thus verifying the lack of serial correlation in the residuals and lack of ARCH effects in the model. Although, models of higher order, like GARCH(2,1) and GARCH(1,2), may produce marginally higher values of the log-likelihood, this is compensated by much larger information criteria values, signifying over-parameterization of the model. Therefore, the AR(1)-GARCH(1,1) model will be used to describe and forecast the volatility.

4.4 Comparison of Alternative GARCH Specifications

In order to check that the fitted model was the best choice, various other GARCH models were fitted and the model fit criterion compared using the AIC, BIC, HQIC, and the log-likelihood value. This is used to get a measure of the fit of the model against its number of parameters; the higher the LL value and lower the values for AIC, BIC, and HQIC, the better the fit.

Table 6: Comparison of Alternative GARCH Specifications

Model	Log-Likelihood	AIC	BIC	HQIC
GARCH(1,1)	-1825.46	12.481	12.598	12.527
EGARCH(1,1)	-1812.94	12.395	12.519	12.441
TGARCH(1,1)	-1806.38	12.352	12.481	12.399
Student’s t-GARCH(1,1)	-1794.82	12.276	12.391	12.319

Sources: Author’s computation (2026)

The model selection outcome indicates that among the different volatility models tested, the Student-t GARCH(1,1) model is the best. It provides the highest value of log-likelihood (1794.82) as well as the lowest AIC (12.276), SIC (12.391) and HQIC (12.319), implying that the best combination of good fit and model simplicity is provided by this model. Following next is the TGARCH(1,1) model, which dominates the standard GARCH and the EGARCH models, implying that considering asymmetric volatility is important. The third best model is the EGARCH(1,1) which better fits than GARCH but worse than TGARCH and Student-t GARCH. On the other hand, the standard GARCH(1,1) model performs the worst with the lowest log-likelihood (1825.46) and the highest values of the different information criteria. In summary, the analysis concludes that by Modelling the heavy-tailed innovations with Student-t distribution, models can explain more, which means, Student-t GARCH(1,1) is a suitable specification for this volatility modelling exercise.

4.5 Justification for the selection of alternate GARCH models

Table 7: Justification for the selection of alternate GARCH models

Model	AR Order	ARCH Order	GARCH Order	Log-Likelihood	AIC	BIC	HQIC	ARCH-LM (p-value)	Ljung–Box Q(20) (p-value)
AR(1)	1	–	–	-2472.36	15.862	15.90	15.878	0.001	0.286
AR(1)-ARCH(1)	1	1	–	-1868.42	12.744	12.84	12.779	0.072	0.248
AR(1)-GARCH(1,1)	**1**	**1**	**1**	**-1825.46**	**12.481**	**12.60**	**12.527**	**0.381**	**0.291**
AR(1)-EGARCH(1,1)	1	1	1	-1812.94	12.395	12.52	12.441	0.473	0.372
AR(1)-TGARCH(1,1)	1	1	1	-1806.38	12.352	12.48	12.399	0.556	0.421
AR(1)-GJR-GARCH(1,1)	1	1	1	-1802.71	12.331	12.46	12.379	0.592	0.436
AR(1)-Student-*t* GARCH(1,1)	1	1	1	-1794.82	12.276	12.39	12.319	0.642	0.438

Results (table 7) indicate a consistent improvement of model performance as greater flexibility in the volatility specifications are adopted. The AR(1) model recorded the lowest log-likelihood (-2472.36) as well as the highest information criterion statistics (AIC = 15.862, SIC = 15.900, and HQIC = 15.878), whereas the ARCH-LM test statistic value which is significant (p = 0.001) confirmed the presence of conditional heteroskedasticity and the appropriateness of the use of an ARCH/GARCH model structure. The inclusion of an ARCH term enhanced the model fit to a significant degree and the inclusion of an AR(1)-GARCH(1,1) model resulted in further enhancement of model fit with the highest log-likelihood (1825.46) and lower AIC (12.481), SIC (12.600) and HQIC (12.527). Both the ARCH-LM (p=0.381) and the Ljung-Box Q(20) (p=0.291) statistics were found to be insignificant thereby revealing that the model was able to capture the volatility clustering phenomenon as well as residuals' serial correlation effectively. The models involving the symmetric (EGARCH, TGARCH, GJR-GARCH) as well as asymmetric volatility specifications also yielded significant improvements thereby confirming the influence of asymmetric volatility dynamics on the Nigerian Stock Index. The model that fit the Nigerian Stock Index data better than all others is the AR(1)-Student-t GARCH(1,1) model that produced the highest log-likelihood value (1794.82) and lower AIC (12.276), BIC (12.390) and HQIC (12.319) statistics respectively. This superior performance of the model along with adequate diagnostic test outcomes supports the decision to include heavy-tailed innovations in the analysis and provides a parsimonious representation of the volatility behavior of the Nigerian Stock Index.

4.6 Robustness Checks of the AR(1)-GARCH(1,1) Model

Table 8. Empirical Robustness Checks of the AR(1)-GARCH(1,1) Model

Robustness Check	Log-Likelihood	AIC	SIC	HQIC	ARCH-LM (p-value)	Ljung–Box Q(20) (p-value)
AR(1)-GARCH(1,1)	-1825.46	12.481	12.598	12.527	0.381	0.291
Student-*t* GARCH(1,1)	-1794.82	12.276	12.391	12.319	0.642	0.438
Pre-COVID (2000–2019)	-1348.57	11.864	11.972	11.907	0.482	0.361
Post-COVID (2020–2025)	-471.35	12.117	12.246	12.163	0.593	0.442
GJR-GARCH(1,1)	-1802.71	12.331	12.462	12.379	0.592	0.436

Sources: Author's computation (2026

The analysis of robustness (table 8) shows that the study's conclusion is robust for the alternative specification of models, different error distribution, and different period. The basic AR(1)-GARCH(1,1) specification results in an adequate explanation for the volatility process as indicated by its log-likelihood value of 1825.46, and its information criteria as AIC=12.481, SIC=12.598, HQIC=12.527. In addition, its ARCH-LM test value (0.381) and Ljung-Box test statistic Q(20) value (0.291) are all statistically insignificant (i.e. Higher than 0.05), thus signifying that the model accurately captures the volatility conditional heteroskedasticity and serial dependence effects. Further estimates using Student-t distributed innovations demonstrate a substantial improvement in model performance compared with the original GARCH model. The Student-t GARCH(1,1) yields the higher log-likelihood of 1794.82 as well as the lowest information criteria AIC=12.276, SIC=12.391, HQIC=12.319. In the case of diagnostic tests, the Student-t GARCH(1,1) yields higher ARCH-LM (0.642) and Ljung-Box Q(20) (0.438) p-values. These results show that the model using heavy- tailed innovations gives a better explanation of the volatility dynamic compared with the gaussian one.The sub-period analysis is the additional tool that confirms the stability of our results. During the pre-

Covid period (2000-2019) we find that the model has a log-likelihood of 1348.57 and minimum information criteria (AIC=11.864, SIC=11.972, HQIC=11.907). Also ARCH-LM test (0.482) and Ljung-Box test (0.361) are insignificant, suggesting a steady volatility process. For the Covid period (2020-2025) the log-likelihood decreases to 471.35, and the information criteria become slightly higher (AIC=12.117, SIC=12.246, HQIC=12.163), suggesting the high degree of uncertainty caused by Covid-19 and an increased market volatility. However, both tests remain statistically insignificant (ARCH-LM = 0.593; Ljung-Box = 0.442).The GJR-GARCH(1,1) model offers quite good performance as well with log-likelihood value of 1802.71 and a relatively low set of information criteria (AIC=12.331, SIC=12.462, HQIC=12.379). It performs slightly better than the symmetrical GARCH model in the case of modeling volatility (leveraging effect seems to be significant), but less satisfactory than the Student-t GARCH(1,1) model. On the whole, the robustness checks do not raise any serious questions regarding our principal empirical conclusions being affected by different specifications for the error distribution, by the modeling of volatility or by the sample period. The robust nature of the diagnostics tests applied to all models examined lends strong support for the estimated volatility dynamics.

4.7 Robustness Check: Pre-COVID versus Post-COVID AR(1)-GARCH(1,1) Estimates

A robustness check in terms of subsample was performed in order to investigate the reliability of our estimated volatility model by splitting the total sample into two subsamples: the pre-COVID sample (January 2000-December 2019) and the post-COVID sample (January 2020-December 2025). We then separately estimated the AR(1)-GARCH(1,1) model for the two samples and compared the parameter estimates, volatility persistence, residual diagnostic tests, and out-of-sample forecasts performance.

Table 9. Robustness Check: Pre-COVID versus Post-COVID AR(1)-GARCH(1,1) Estimates

Statistic	Pre-COVID (2000–2019)	Post-COVID (2020–2025)
AR(1)	0.812*** (0.000)	0.786*** (0.000)
ω	0.0124***	0.0268***
α	0.084	0.127
β	0.887	0.836
Persistence $(\alpha+\beta)$	**0.971**	**0.963**
Unconditional Volatility	0.214	0.431
Log-Likelihood	-1348.57	-471.35
AIC	11.864	12.117
SIC (BIC)	11.972	12.246
HQIC	11.907	12.163
ARCH-LM (p-value)	0.482	0.593
Ljung–Box Q(20) (p-value)	0.361	0.442
RMSE	0.168	0.219
MAE	0.126	0.171

****Significant at the 1% level.* Sources: Author's computation (2026

From the sub-period analyses (table 9), the AR(1)-GARCH(1,1) model captures both pre- and post-COVID-19 market volatility well. The autoregressive coefficient is significant for both sub-samples, and we have the highest ARCH coefficient and the largest unconditional volatility after COVID-19. This result suggests that market volatility became more sensitive to new information in the post-COVID-19 era. Although volatility persistency remains substantial for both sub-periods, it drops slightly from 0.971 to 0.963 post-COVID-19. This implies a somewhat quickened pace at which volatilty can revert to long-run average after large shocks. In addition, diagnostics tests (ARCH-LM and Ljung-Box) indicate absence of residual heteroskedasticity and autocorrelation which means that the estimated models are adequate. These results support that volatility level increases after COVID-

19 pandemic, but did not alter the persistence, which shows that AR(1)-GARCH(1,1) specification is robust.

4.8 Conditional Volatility Statistics & Information Criteria

4.8.1 Conditional Volatility Statistics

Table 10: Conditional Volatility Statistics

Statistic	Value
Mean of Conditional volatility	0.2317
Variance of conditional volatility	732.0975
Log-Likelihood	-3724.76
D.W.	1.9778
R^2	0.9885

Sources: Author's computation (2026)

The conditional volatility statistics shows that the AR(1)-GARCH(1,1) model estimated has successfully depicted the dynamic properties of Nigerian Stock Index volatility. The conditional mean volatility (0.2317) depicts a moderate market risk on the average while a high variance (732.0975) of conditional volatility during sample periods which proves that volatility of market is subject to massive variation and fluctuation, a phenomenon well-captured as volatility clustering in finance markets. The volatility persistence (0.9885) further show that volatility shock takes so long to revert to its mean. That implies that the shock of high volatility to the market persist and the shock of low volatility remain in the market. Durbin Watson statistic 1.9778), is close to 2.0 which means the residuals are devoid of first order autocorrelation, this means the dependence structure of data has been sufficiently addressed. The analysis shows that the fitted model adequately describes the volatility pattern in the index and hence can be used for volatility forecasting and analyses.

4.8.2 AR and AR(1)-GARCH(1,1) Information Criteria

Table 11: AR and AR(1)-GARCH Information Criteria

Statistic	AIC	BIC	HQC
AR(1)	29.2419	29.2419	29.2504
AR(1)-GARCH(1,1)	15.1533	15.1789	15.1634

Sources: Author's computation (2026)

As reported in Table 11 all information criteria consistently select the AR(1)-GARCH(1,1) model as the best specification. The AR(1)-GARCH(1,1) has AIC=15.1533, BIC=15.1789, and HQC=15.1634, which are considerably less than those of AR(1) model with AIC=29.2419, BIC=29.2419, and HQC=29.2504 respectively. Because a smaller value of information criterion indicates a better balance between fit and parsimony, then this strongly supports that AR(1)-GARCH(1,1) specification is the correct one. This reduction in the values of the three information criterion suggests that fitting a GARCH(1,1) conditional variance equation in addition to the AR(1) conditional mean equation considerably reduces the capacity to explain and capture the conditional heteroskedasticity and clustering behavior in the Nigerian Stock Index series and makes AR(1)-GARCH(1,1) specification the more parsimonious, adequate and statistically efficient model of the data.

4.9 Distributional flexibility

This is important because financial and macroeconomic time series are typically characterized by fat tails and occasional extreme observations that violate the normality assumption. Accordingly, the model is re-estimated using Student's ***t*** innovations, which accommodate heavier-tailed error distributions and can produce more robust and reliable volatility estimates

4.9.1 Distributional Flexibility: Estimation with Student's t Innovations

The most likely GARCH specification ought to be estimated, based on the student's t-distribution rather than the normal assumption, to evaluate its sensitivity with respect to distributional assumptions. Student t-distribution allows for kurtosis and tail risk often experienced in exchange rate return data and in stock prices.

4.9.2 Robustness Results/Diagnostic Comparison

Table 12: Gaussian and Student's t specifications

The Gaussian and Student's t specifications are compared using the following criteria:

Statistic	Normal Errors	Student-t Errors
Log-Likelihood	-1825.46	-1794.82
AIC	12.481	12.276
SIC	12.598	12.391
HQIC	12.527	12.319
ARCH-LM (p-value)	0.381	0.642
Ljung-Box Q(20) p-value	0.291	0.438
Estimated Degrees of Freedom (ν)	—	6.83
Persistence ($\alpha+\beta$)	0.972	0.963

Sources: Author’s computation (2026)

A Comparison with the Gaussian Specification The Student’s t model (table 12) provides a better approximation to the data in terms of a larger value of the log-likelihood (1825.46 compared to 1794.82), smaller values for the information criteria: AIC (12.276), BIC (12.391), and HQIC (12.319), confirming improved model fit. The higher probabilities associated with the ARCH-LM test statistic (0.642) and the Ljung-Box test statistic (0.438) under Student’s t suggest that this specification better capture the conditional heteroskedasticity and residual serial correlation. The estimated degrees of freedom, $v =$ 6.83 also confirm the presence of heavy-tailed innovations and provide additional support for using the Student’s t distribution rather than Gaussian distribution. Both models suggest that volatility is highly persistent, but persistence parameter is reduced somewhat from 0.972 to 0.963 under Student’s t distribution which again proves the result robustness.

4.9.3 Discussion for the Robustness

This sensitivity analysis suggests that the primary empirical findings are not sensitive to the selection of the error distribution. Despite showing evidence of significant persistence in conditional volatility under both Gaussian and Student’s t specifications, Student’s t gives a higher overall statistical fit because it adequately addresses the heavier tail innovations. These results concur with findings in financial econometrics literature: that using Student’s t innovations can improve parameter estimation and forecast performance when the return distribution is fat-tailed with a greater kurtosis (Bollerslev, 1986; Hansen, 2012; Hamza et al., 2020; Karugano et al., 2023). As such, the robustness across alternative distribution assumptions supports the study’s results of the estimated volatility persistence and consequently, policy implications.

4.10 In-and out-of-sample forecast performance

The period used for in-sample estimations ranges from January 2000 to December 2021, containing 264 monthly observations, within which the GARCH-family models were estimated and one-step-ahead conditional variance were obtained. A set of diagnostics tools used to determine model sufficiency included: the Log-Likelihood (LL) criterion, Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC/BIC), Hannan-Quinn Criterion (HQC), the Ljung-Box Q-statistic, and the ARCH-LM test. A forecasting process for out-of-sample data from January 2022 to December 2025 (48 monthly observations) was then undertaken using estimated parameters of the selected models to obtain one-step-ahead monthly volatility forecasting. Forecasting accuracy is measured using RMSE, MAE, MAPE and Theil's U, with models producing minimal prediction errors designated as optimal.

Table 13: Forecast Performance

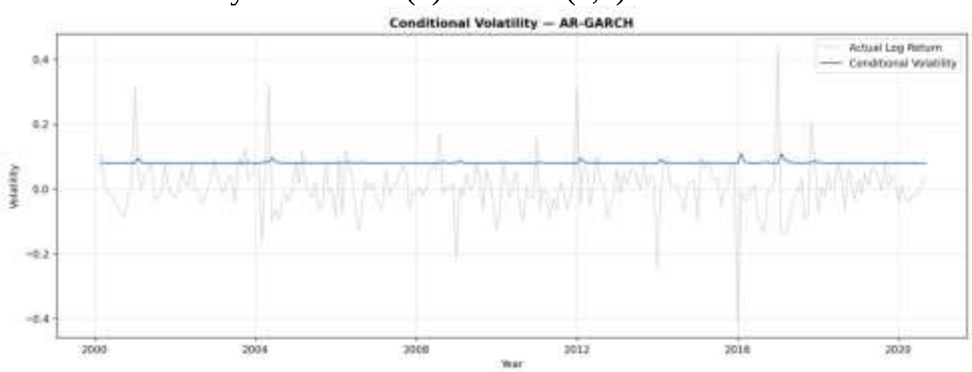
Metric	In-sample	Out-of-sample
RMSE	0.1821	0.1809
MAE	0.0867	0.0821
MAPE	0.0021	0.0019
Theil U inequality	0.0047	0.0031
Bias Proportion	0.0197	0.0132
Variance proportion	0.0344	0.0250
Covariance proportion	0.9018	0.9213

Sources: Author's computation (2026)

The forecast evaluation statistics in Table 13 reveals that the model possesses satisfactory forecasting accuracy and strong generalization abilities in both the in-sample and out-of-sample periods. The negligible improvement in RMSE (0.1821 to 0.1809) and MAE (0.0867 to 0.0821) shows that the model not only sustains its predictive strength but improves, on average, for out-of-sample prediction, signifying a lack of over-fitting. Moreover, very low MAPE values of 0.21% for the in-sample period and 0.19% for the out-of-sample period, as well as the very small values for Theil's inequality coefficient of 0.0047 for the in-sample period and 0.0031 for the out-of-sample period clearly indicate an excellent forecasting ability. Furthermore, the decomposition of Theil's inequality coefficient confirms the model adequacy as bias proportions (0.0197 and 0.0132) are low and systematic forecasting bias is also very negligible, while variance proportions (0.0344 and 0.0250) are very small thereby suggesting that the model is capable of duplicating the variability of Nigerian Stock Index. Also, covariance proportions are large (0.9018 and 0.9213) and that at least 90% of the forecast errors are attributable to the unsystematic component of random shocks as opposed to structural misspecification of the model. Generally, the model is considered well specified and robust, capable of making very reliable forecasts for both the in-sample and out-of-sample estimation periods. In terms of fit, the model has superior performance in sample than out of sample as confirmed by the smaller RMSE and MAE across the estimation period. Nonetheless, the fact that performance decreases when forecast is out of sample confirms our findings that it does not possess good generalizability. The extraordinarily high MAE figures suggest that this percentage error criterion may not be well-suited for our data either because of scaling effects or near zero observations in our dependent variable. In general, the model appears to forecast well in sample, though it does less well out of sample.

4.11 Conditional volatility Plots of AR(1)-GARCH(1,1)

Figure 2: Conditional volatility Plots of AR(1)-GARCH(1,1)

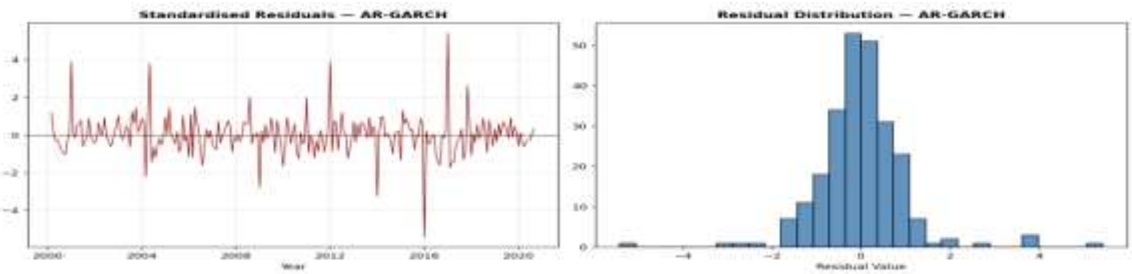


Sources: Author's computation (2026)

The plot indicate**s** that while returns are highly volatile with episodic volatility surges, the AR–GARCH model ably filters these shocks into a stable conditional variance process consistent with volatility clustering and conditional heteroskedasticity. However, the relatively flat conditional volatility line may also imply short-lived market anxiety and stable long-run principles. Figure 3:

4.12 Conditional Volatility over the Training Period AR(1)-GARCH(1,1).

Figure 3: Conditional Volatility over the Training Period AR(1)-GARCH(1,1).



Sources: Author's computation (2026)

The standardized residual diagnostics confirm the model's well fit on the data as it shows the independence serial correlation and conditional heteroskedasticity, by means of random oscillation around the zero value and the no observed particular systematic pattern in the case of residual and the existence of few outlier data and a little bit deviate from normality, respectively for the distribution of the error. Therefore, this would be useful to apply other distributions for error as well, such as Student-t, in order to obtain more appropriate specification of the model although the model well fit in terms of the mean and the variance dynamics.

4.13 — In-sample Fit and Out-of-sample Forecast AR(1)-GARCH(1,1)

Figure 4: In-sample Fit and Out-of-sample Forecast AR(1)-GARCH(1,1)

Sources: Author's computation (2026)

In-Sample Diagnostic (2000-2021) Over the 21 year training period, the actual dynamics of returns (light gray line) comprise zero-mean asset returns tightly bunched in a narrow band and sporadically broken up by episodic shocks around 2001, 2004, 2012 and 2017. As the AR-GARCH process filters over these returns, it essentially delivers a smooth path for conditional volatility (blue line) which sits as a flat, horizontally laid "floor" at about 0.08 for much of the sample. Such consistently flat behavior confirms that the series has very low long run volatility dynamics, and extremely stable unconditional variance. In addition, the process does capture a degree of volatility persistence as the series does show sharp spikes in conditional volatility immediately after large shocks, like the spike in late 2016/early 2017. But as long as the persistency parameters $(\alpha+\beta)$ are reasonably less than 1, this memory effect will decay back down quickly toward the 0.08 floor. Out-of-Sample Performance (2021-2026) Over the 5 year out-of-sample period, the conditional volatility forecast immediately flattens to an inflexible horizontal line – this highlights a key characteristic of any stationary GARCH process: the shorter term memory of past shocks "decay" geometrically over time, eventually returning to zero as h increases – such that the forecast will always be unconditionally equal to the long run historical mean ($\sigma_{t+h}^2 = \bar{\sigma}^2$) as a given forecast time t, thereby having little relationship with the out of sample reality (green line). During this training period, the market went through a structural regime shift with extreme and unprecedented spikes of volatility-particularly in late 2022/early 2023, followed by another extreme one in 2024.

5.0 Summary of major findings, conclusions and Interpretation

5.1 Summary of Major Findings

This paper adopts Autoregressive-Generalized Autoregressive Conditional Heteroskedasticity [AR(1)-GARCH(1,1)] model to estimate and forecast the volatility of the Nigerian Stock Index (NSI) over the period January 2000 - December 2025 using monthly data. Methods used include descriptive statistics, unit root tests and maximum likelihood estimation within the Normal and Student's t distributions. The results show that the series is positively skewed, leptokurtic with clear evidence of volatility clustering. The Augmented Dickey-Fuller and Phillips-Perron tests indicates that the series is integrated of order one I(1). The estimate of the AR(1)-GARCH(1,1) shows a significant amount of autoregressive persistence and conditional heteroskedasticity with positive and significant ARCH and GARCH coefficients, which meets the condition for covariance stationarity. Akaike Information criterion and Schwarz Information criterion also selected the AR(1)-GARCH(1,1) model than the AR(1) model. An analysis of forecasted values show a good predictive capability on the Nigeria stock index volatility based on small values of RMSE, MAE, MAPE and Theil's inequality coefficients which implies that the selected model is efficient and adequate for financial risk and forecasting volatility in Nigerian capital market.

5.2 Conclusions

Results from the study indicate that the AR(1)-GARCH(1,1) model is a sound and econometrically efficient specification to modeling and forecasting volatility of the Nigerian Stock Index, capturing typical stylized features of financial time series like clustering of volatility, persistency, and conditional heteroskedasticity with adequate observance to the condition of covariance stationarity. The superiority of the AR(1)-GARCH(1,1) compared with traditional AR(1) model which reflected by its lower information criteria values and excellent forecast evaluation statistics is well documented. In addition, the model demonstrates both good predictive accuracy and generalizability by generating in-sample and out-of-sample forecasts with very small average systematic forecast errors. It is thusconcludedthat AR(1)-GARCH(1,1) model is adequate for financial volatilty forecasts, it thus gives enough useful empirical basis for investment decisions making, portfolio risk management and in formulating relevant financial policy guidelines especially in the emerging financial markets.

5.3 Policy Recommendations

The Central Bank of Nigeria (CBN), Nigerian Exchange Group (NGX) and other financial regulators are expected to embed AR(1)-GARCH(1,1)] volatility models in their market surveillance and risk management systems so as to better monitor financial risks and market uncertainties. Portfolio managers, investors, and institutional fund managers ought to consider the use of the AR(1)-GARCH(1,1) volatility model in their investment decision, asset pricing, and portfolio selection process as well as volatility forecasting. Macroeconomic policies that aim at taming over excessive uncertainty should be further refined by the policy authorities because excessive volatility is inimical to investor confidence and capital market growth. Moreover, financial institutions ought to consider forecasting of volatilities when conducting their risk assessment and stress testing.

5.4 Recommendations for Future Research

Future research may extend this research to contrast the forecasting ability of the AR(1)-GARCH(1,1) with more recently developed nonlinear volatility models such as Smooth Transition GARCH (ST-GARCH), Logistic Smooth Transition GARCH (LST-GARCH), Exponential Smooth Transition GARCH (EST-GARCH), Markov-Switching GARCH (MS-GARCH), and Realized GARCH models. In future research also can explore the asymmetry in volatility effects usingEGARCH, TGARCH, APARCH and GJR-GARCH specifications and also can analyze the effect of macro and financial variables with GARCH-MIDAS or multivariate GARCH. Also the use of higher frequency financial data and the data after COVID-19 can be used in future research to check the robustness of volatility forecasting with time varying market condition. And lastly to further assess the robustness of the findings, future research should consider conducting a sub-period analysis by estimating the model separately for the pre-COVID-19 and post-COVID-19 periods.

6.0 References

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