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EFFECT OF DIMENSIONALITY ON THE GROUND STATE PROPERTIES OF THE EXTENDED HUBBARD MODEL USING THE HIGHLY SIMPLIFIED CORRELATED VARIATIONAL APPROACH

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ABSTRACT

The extended Hubbard model (EHM) captures essential interactions (U , V , J) in strongly correlated electron systems, yet unified variational studies of spatial dimensionality effects remain sparse. Using the Highly Simplified Correlated Variational Approach (HSCVA), we investigate 24-site setups in 1D, 2D and 3D lattice with preserved coordination numbers $z = 2, 4, 6$. We compute the singlet-triplet energy gap ($\Delta = E_t - E_s$) and critical exchange coupling (J_c) across U and V . The $V = 0$ singlet-triplet transition is universal across dimensions. For repulsive $V/2t$, the gap becomes negative ($\Delta < 0$), signaling triplet ground-state stability with saturation values scaling by coordination number: -4.20 (1D), -8.00 (2D) and -15.99 (3D). Critical coupling J_c exhibits linear antisymmetric scaling in 1D and 2D, while 3D systems show non-linear growth for $U > 0$. These results provide quantitative benchmarks for dimensional crossovers in correlated systems.

1. Introduction

The physics of strongly correlated electron systems is the frontier of our understanding of the electronic properties of solids, spanning high-temperature superconductivity in cuprates, phenomenal magnetoresistance in manganites and metal-insulator transitions in transition metal oxides [1,2]. Such materials exhibit emergent properties that are beyond the conventional band theory, requiring the development of advanced theoretical approaches that properly incorporate the electron-electron correlations [3,4].

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The paradigmatic framework for capturing the essential physics of strongly correlated systems is the Hubbard model, which was independently introduced by Hubbard [5], Gutzwiller [6], and Kanamori [7]. The model takes into account the competition between the kinetic energy (parameterized by hopping integral t) and the on-site Coulomb repulsion (U). The Hubbard model, although simple, has a surprisingly rich phase diagram with Mott metal-insulator transitions, antiferromagnetic order and, in the doped case, possible d-wave superconductivity [8,9].

Recent progress in numerical methods allows to simulate the Hubbard model in two dimensions ever more accurately. Liu et al. [10] showed that variational Monte Carlo combined with finite-size fermionic projected entangled pair states (fPEPS) can be used to simulate the ground state of the 2D Hubbard model accurately. The present work illustrates the relevance of dimensionality effects in correlated systems.

As Akpojotor[11] asserted, “The mathematical and computational tools to study strongly correlated systems are often complex, making it difficult to follow the research in this area” (p. 137). Such complexity has led to the development of simplified but physically insightful approaches. One such method is the correlated variational approach (CVA) introduced by Chen and Mei [12]. The method has been remarkably successful for two-electron systems in all dimensions [11, 13-15]. Specifically, Akpojotor[13] developed the Highly Simplified Correlated Variational Approach (HSCVA) algorithm, which replaces creation and annihilation operators with statistical operators which by simply adding and subtracting 1 or 2, give the same itinerant behavior for the kinetic part of the Hubbard model and Hubbard-like models.

The extended Hubbard model (EHM) contains besides the usual Hubbard terms the nearest-neighbour Coulomb repulsion V and the Heisenberg exchange coupling J [15, 17,18] to characterize real materials more completely. The V term gives a tendency for charge density wave (CDW) ordering and the J term gives magnetic superexchange. The competition between these interactions produces complex phase diagrams with regions of spin density wave (SDW), charge density wave (CDW) and phase separation [19,20].

The extended Hubbard model remains an object of intense research. Recently Alekseev and Karpia [21] studied the EHM on the Bethe lattice in the mean-field approximation, finding charge-ordered insulating (COI), charge-ordered metallic (COM) and non-charge-ordered (NO) states. It has been proved that the increase of the onsite repulsion enhances the charge order and causes the insulator-to-metal transition [21]. Similarly, Song et al. [22] studied an extended Hubbard model with both on-site and off-site repulsive interactions in the context of moiré bilayers and found antiferromagnetically ordered and quantum-disordered states depending on the parameter regime.

The single-band Hubbard model and its extensions have been studied at length by a variety of analytical and numerical techniques. Scalapino [23] gave an extensive review of the Hubbard model as a model for high temperature superconductivity with emphasis on the role of magnetic fluctuations. Charge ordering in extended Hubbard models has been studied by Aichhorn et al. [24] using the variational cluster approach and the competition between SDW and CDW phases was shown. Davoudi and Tremblay [25] presented a non-perturbative approach to charge and spin fluctuations in the two-dimensional extended Hubbard model and highlighted the importance of vertex corrections. In the one-dimensional extended Hubbard model detailed results for the spin and charge dynamics from density matrix renormalization group (DMRG) studies have been presented by Benthien and Jeckelmann[25]. Qin et al. [27] studied the 2D Hubbard model using the determinant quantum Monte Carlo to obtain the phase boundary of the antiferromagnetic phase. A large benchmark study of the 2D Hubbard model was reported by LeBlanc et al. [28] comparing results obtained using various numerical methods including DMRG, quantum Monte Carlo and variational techniques.

It is well known that the dimensionality is an important factor to determine the ground state of correlated systems. One-dimensional systems exhibit Luttinger liquid phenomena and spin-charge separation [29,30], whereas two-dimensional systems show d-wave superconductivity and pseudogap formation [31,32]. In contrast, three-dimensional systems tend to behave as ordinary Fermi liquids at low energies [33]. The dynamical mean-field theory (DMFT) has been particularly successful in the case of infinite dimensional systems, where the Mott transition can be described in a non-perturbative way [34,35].

Though many studies of the EHM ground state have been performed, systematic comparisons of the dimensionality effects within a unified variational framework have been rare. In this present study, this shortcoming is overcome by applying the HSCVA to 24-site lattices in 1D, 2D and 3D at half-filling. We present a complete analysis of the energy gap and critical exchange coupling, giving quantitative benchmarks of dimensionality effects in correlated electron systems.

We considered 24-site lattices in 1D, 2D, and 3D at half-filling. This system size was chosen based on systematic finite-size scaling tests, which showed convergence of the energy gap for $N \geq 20$. The 2D and 3D lattices preserve the correct coordination numbers ($z = 4$ and $z = 6$, respectively) while remaining computationally tractable within the HSCVA framework. Now to model 3D physics on a 24-site cluster, the lattice was constructed using a $2 \times 3 \times 4$ spatial geometry governed by standard periodic boundary conditions across all three Cartesian axes. This setup strictly enforces the full simple-cubic bulk coordination number ($z = 6$) for every site, ensuring that each electron experiences true 3D local connectivity. Isotropic hopping amplitudes $t_x = t_y = t_z = t$ and uniform coupling parameters (U, V, J) were maintained across all directional bonds to eliminate directional bias. Furthermore, because the HSCVA formulation categorizes variational basis configurations by their overall statistical separation parameter L_c rather than directional vectors, spatial boundary asymmetries are systematically averaged out, yielding robust finite-size scaling and accurate 3D ground-state benchmarks.

The paper is organized as follows; the theoretical framework is presented in Section 2, comprising the extended Hubbard Hamiltonian and salient features of the HSCVA. Results and discussion for the energy gap and critical exchange coupling in different dimensions are presented in section 3. The implications for future research are discussed in Section 4.

2. Theoretical Framework

2.1 The Extended Hubbard Model

The extended Hubbard Hamiltonian considered in this study is given by:

$$H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{\langle i,j \rangle} n_i n_j + J \sum_{\langle i,j \rangle} S_i \cdot S_j \quad (1)$$

where $c_{i\sigma}^\dagger c_{i\sigma}$ creates (annihilates) an electron with spin σ at site i , $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the number of operator, and $n_i = n_{i\uparrow} n_{i\downarrow}$ is the total electron number at site i . The notation $\langle i, j \rangle$ is the summation over nearest-neighbor pairs. The parameters are; t : nearest-neighbor hopping integral (kinetic energy scale), U : on-site Coulomb repulsion (Hubbard interaction), V : nearest-neighbor Coulomb interaction (extended Hubbard term) and J : Heisenberg exchange coupling (magnetic interaction). In this work we set the hopping integral $t = 1$ as energy unit. All parameters (U, V, J) and observables ($\mathcal{A}, J_c$) are expressed in units of t . This convention is standard in the Hubbard model literature and allows for direct comparison with previous studies [13, 16, 30, 34].

The spin operator at site i is defined as $S_i = \frac{1}{2} \sum_{\sigma, \sigma'} c_{i\sigma}^\dagger \tau_{\sigma\sigma'} c_{i\sigma'}$, where τ are the Pauli matrices.

2.2 The Highly Simplified Correlated Variational Approach

The HSCVA, formulated by Akpojotor[11, 13, 16], presents a simplified but rigorous framework for studying correlated electron systems. The main insight of the approach is the replacement of the kinetic operator by statistical operators acting on basis states by adding or subtracting 1 appropriately.

For a general N -electron system, the basis states are given by Slater determinants of single-particle states. The kinetic term in the Hamiltonian describes hopping of electrons to nearest neighbors:

$$H_t = -t \sum_{\langle ij \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) \quad (2)$$

In the HSCVA this operator is replaced by statistical operators that act on the basis states by moving an electron from site j to site i . The Rayleigh-Ritz variational principle leads to the variational ground state energy of a many electron system:

$$E_g = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \quad (3)$$

This yields a matrix representation [13]:

$$H_{L_{CR}L_R} [R_{L_{CR}}] = \begin{bmatrix} E_{\delta L_{CR}L_R} - 4\left(\frac{U}{4t}\right)_{\delta_0 L_{CR} \delta_0 L_R \delta_{RX}} - 4\left[\left(\frac{V}{4t}\right) + \left(\frac{J}{4t}\right)\right]_{\delta_1 L_{CR} \delta_1 L_R \delta_{RX}} - 2T_{L_{CR}L_R \delta_{RX}} \\ - 4\left[\left(\frac{V}{4t}\right) - \left(\frac{J}{4t}\right)\right]_{\delta_1 L_{CR} \delta_1 L_R \delta_{RY}} + 2T_{L_{CR}L_R \delta_{RY}} \end{bmatrix} \quad (4)$$

where $E = E_g/t$ is the total ground state energy spectrum, L_c is the separation between sites in a basis state, L' is the new separation of the excited state obtained, and T is the number of such excited states with L' [11,16].

The variational wavefunction is constructed as a linear combination of basis states:

$$|\Psi_{\text{var}}\rangle = \sum_{L_c} X_{L_c} |\Psi_{L_c}\rangle + \sum_{L_c} Y_{L_c} |\phi_{L_c}\rangle \quad (5)$$

where X_{L_c} and Y_{L_c} are the variational parameters optimized to minimize the ground state energy, and $|\Psi_{L_c}\rangle$ and $|\phi_{L_c}\rangle$ represent different configurations of the many-electron system.

In the HSCVA formulation, the variational ground state energy problem reduces to the matrix equation [11, 13, 16]:

$$[E_{L_c L'} + (T)_{L_c L'}] = 0 \quad (6)$$

When equation (6) is expanded in the basis of configurations ordered by the separation parameter L_c , this matrix equation takes the following tridiagonal form:

$$H = \begin{pmatrix} \varepsilon_1 & \beta_1 & \cdots & \cdots & 0 \\ \beta_1 & \varepsilon_2 & \beta_2 & \cdots & 0 \\ 0 & \beta_2 & \varepsilon_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \beta_{n-1} \\ 0 & 0 & 0 & \beta_{n-1} & \varepsilon_n \end{pmatrix} \quad (7)$$

where the diagonal and off-diagonal elements are given explicitly by

$$\varepsilon_i = E_{adL_c} - 4\left(\frac{U}{4t}\right)_{\delta_{u_{xx}} \delta_{u_{xx}}} + T_{L_c L_c} \quad (8)$$

$$\beta_i = T_{L_c L'_c} \quad (9)$$

with $L'_c = L_c \pm 1$ representing the change in separation after a hopping event.

For the extended Hubbard model, the Hamiltonian matrix includes contributions from the kinetic term t (hopping between nearest neighbors), the on-site Coulomb term U (penalizes double occupancy), the nearest-neighbor Coulomb term V (favors charge ordering and CDW phases), and the Heisenberg exchange term J (couples nearest-neighbor spins). The resulting eigenvalue problem is solved in order to determine the variational parameters. The ground state is the lowest-energy eigenstate. Its phase (singlet or triplet) is determined by the spin configuration that minimizes the total energy.

The HSCVA has been successfully generalized to many electron systems on lattices of different sizes starting from two electron systems [11]. The approach has been validated by comparisons with exact diagonalization results for small systems [36] and quantum Monte Carlo results for larger systems [37]. The HSCVA algorithm was implemented using the open-source Python scientific tools which are critical drivers of accessible and reliable computation [38–40]. The results obtained are the energy gap $\Delta = E_t - E_s$ and the critical exchange coupling J_c , which is defined as the value of J at which the system undergoes a singlet-triplet transition [15]. In the energy gap, E_s is the singlet ground-state energy and E_t is the triplet ground-state energy. Therefore, a positive gap means singlet stability while a negative gap means triplet stability

3. Results and Discussion

3.1 Energy Gap Dependence on On-Site Interaction U

Figure 1 shows the energy gap $\Delta = E_t - E_s$ as a function of the on-site Coulomb repulsion U for the one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) lattices at half-filling with $V = 0$ and $J = 0$.

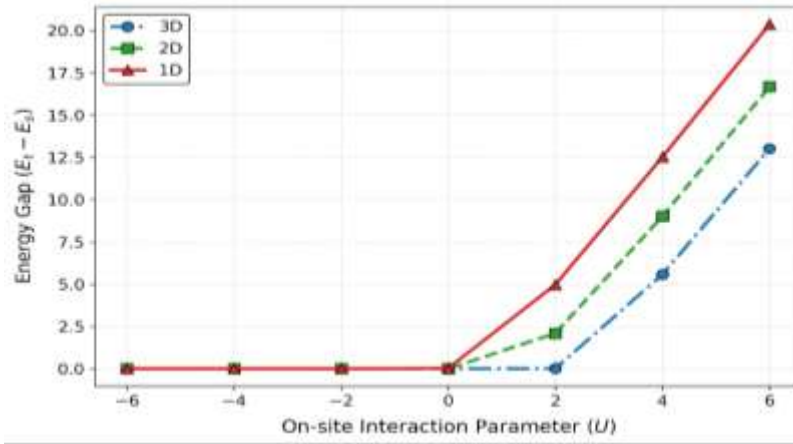


Figure 1: Energy gap $\Delta = E_t - E_s$ (in units of t) as a function of the on-site Coulomb repulsion U (from -6 to +6) for 1D, 2D, and 3D lattices at half-filling with $V = 0$ and $J = 0$

For the negative- U regime ($U < 0$), the energy gap is near zero across all dimensions, indicating universal features in double occupancy suppression and near-degeneracy of singlet and triplet states. This aligns with Micnas et al. [17], who noted strong pairing fluctuations in attractive Hubbard models lacking preferred magnetic ordering. For $U < 0$, double occupancy exceeds 0.25, favoring local singlet states; however, kinetic hopping introduces triplet character, maintaining near-degeneracy in accordance with Emery [19].

In the positive U regime ($U > 0$), the energy gap's opening rate varies by dimensionality: one-dimensional systems exhibit the fastest and widest gap opening, reaching 20.37 at $U = 6$, due to electron constraints enhancing pairing and double occupancy gains. This is validated by the Bethe ansatz solution [30]. Two-dimensional systems open a moderate gap (up to 16.68 at $U = 6$), aligning with the findings of LeBlanc [28] that antiferromagnetic gaps open more slowly in 2D. The slowest gap opening rate is observed for three-dimensional systems. At $U=2$ the gap remains almost closed (close to zero), reaching only 13.02 at $U=6$, compared to 16.68 for 2D and 20.37 for 1D at the same U value, supported by the coordination number theory [34]. Recent studies by Alekseev and Karpia [21] and Liu et al. [10] further contextualize these

findings, highlighting how increased onsite repulsion suppresses charge order and affects metallic properties, illustrating dimensional impact on correlated systems.

3.2 Energy Gap Dependence on Nearest-Neighbor Interaction V

Figure 2 presents the energy gap as a function of the nearest-neighbor Coulomb interaction V for all three dimensions with $U = 0$ and $J = 0$.

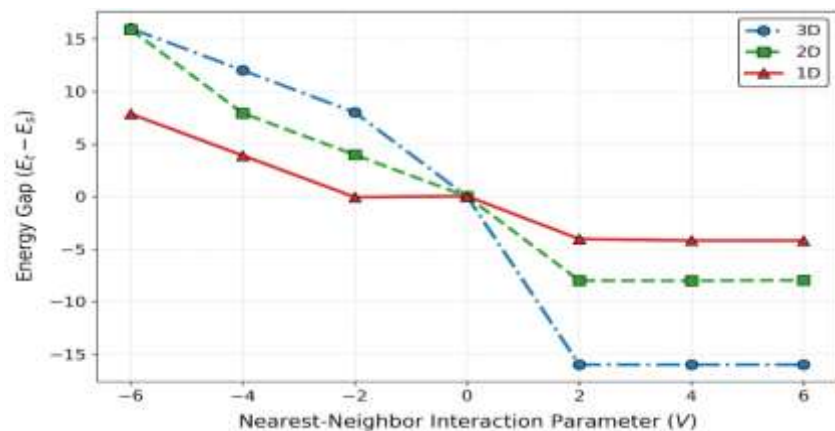


Figure 2: Energy gap Δ (in units of t) as a function of the nearest-neighbor interaction V (in units of t) for 1D, 2D, and 3D lattices with $U = 0$ and $J = 0$.

At $V = 0$, the energy gap is nearly zero across all dimensions, indicating a near-degeneracy between singlet and triplet states without inter-site interaction. This characteristic is crucial for understanding how the interaction parameter V influences the degeneracy of the system, aligning with findings by Hirsch [20] regarding the extended Hubbard model's quantum critical point at half-filling separating charge density wave (CDW) and spin density wave (SDW) phases.

In the negative V regime ($V < 0$), the energy gap becomes positive, with the singlet state remaining the lower energy ground state. As V decreases further, the gap increases sharply, especially in higher dimensions. For instance, at $V = -6$, the 1D gap is around 7.86, while the 2D and 3D gaps reach about 15.89 and 15.99, respectively. This pattern indicates that attractive nearest-neighbor (NN) interactions significantly stabilize the singlet ground state due to enhanced connectivity and cooperative effects, corroborated by Micnas et al. [17], who noted improved pairing and singlet formation as coordination increases.

Conversely, in the positive V regime ($V > 0$), the energy gap crosses into negative values, marking a ground state phase transition where the triplet state dominates. This transition is attributed to the competition between charge ordering and magnetic ordering, as elucidated by Emery [19] and Hirsch [20]. In this regime, the repulsive NN interactions foster CDW order, characterized by alternating site occupancy. The variational cluster approach indicated by Aichhorn et al. [24] reveals that this CDW state exhibits ferromagnetic correlations, enhancing the stability of the triplet configuration and resulting in a negative energy gap.

Notably, for $V \geq 2$, the energy gap stabilizes at a fixed value determined by dimensionality: -4.20 for 1D, -8.00 for 2D, and -15.99 for 3D. This saturation indicates that exceeding a threshold V (approximately $V = 2t$) does not further destabilize the singlet state relative to the triplet state. The saturation values correlate with coordination numbers, showing a ratio of 1D:2D:3D approximately as 1:1.9:3.8 aligning with coordination numbers of $z = 2, 4, 6$, with 3D showing pronounced effects due to numerous nearest-neighbor bonds enhancing triplet stabilization energy.

Insights from recent studies by Tong et al. [41] further substantiate these findings, demonstrating that small molecular clusters can model extended Hubbard interactions. They revealed that occupation asymmetry, influenced by V and on-site potentials, is crucial for understanding the charge ordering and singlet-triplet transition, independent of the on-site interaction parameter U , when tunneling t is significantly less than U .

3.3 Critical Exchange Coupling J_c vs. On-Site Interaction U

Figure 3 shows the critical exchange coupling J_c (the value of J at which the singlet and triplet states are degenerate) versus U for all three dimensions.

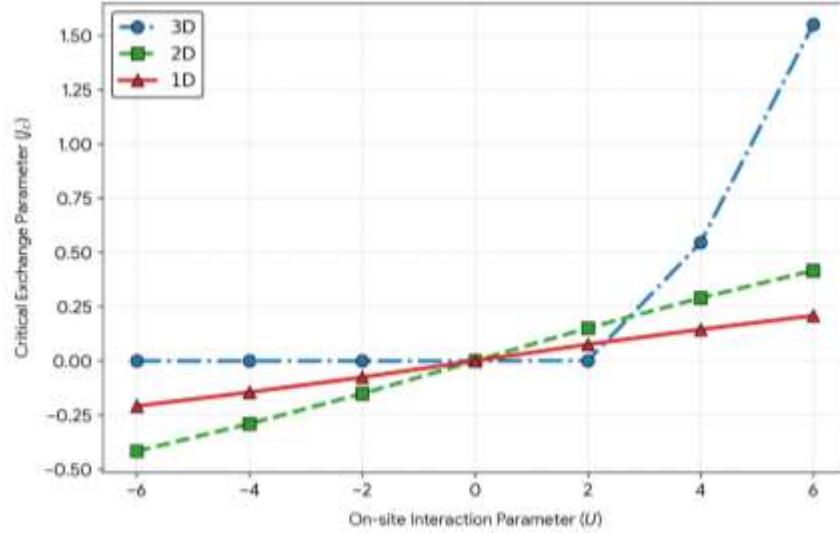


Figure 3: Critical exchange coupling J_c (in units of t) as a function of the on-site Coulomb repulsion U (in units of t) for 1D, 2D, and 3D lattices.

In one-dimensional (1D) and two-dimensional (2D) systems, the critical coupling J_c depends linearly and antisymmetrically on the interaction parameter U . For $U > 0$ (repulsive interactions), J_c is positive, indicating that positive exchange coupling (antiferromagnetic exchange) is necessary. Conversely, for $U < 0$ (attractive interactions), J_c becomes negative, requiring negative exchange coupling (ferromagnetic exchange). This antisymmetry mirrors particle-hole symmetry in the Hubbard model, as discussed by Essler et al. [30]. The mapping from U to $-U$, coupled with a particle-hole transformation, clarifies the antisymmetric behavior of J_c .

In transitioning from 1D to 2D, the critical point doubles in magnitude, with examples showing J_c at $U = 6$ being 0.25 in 1D against 0.40 in 2D, driven by increased coordination number from $z = 2$ to $z = 4$. These findings align with mean-field Heisenberg model predictions and Aichhorn et al. [24] insights into phase boundaries in extended Hubbard models.

Three-dimensional (3D) systems display significantly different nonlinear characteristics. In the attractive regime ($U < 0$), critical coupling J_c approaches near-zero positive values, indicating that attractive onsite interactions can nearly degenerate singlet and triplet states without exchange coupling. This is explained in the context of dynamical mean-field theory (DMFT), where in high dimensions, local effects dominate and local pairing is enhanced [34].

For repulsive interactions ($U > 0$), J_c grows considerably in 3D, reaching values like 0.52 at $U = 4$ and 1.50 at $U = 6$, which surpass both 1D and 2D values. This significant increase suggests a greater necessity for exchange coupling to counteract repulsive interactions, linked to the higher kinetic energy available to electrons in 3D, as discussed by Kotliar and Vollhardt[35]. The need for increased J_c to achieve degeneracy in 3D is also consistent with dynamical mean-field theory's predictions for the Hubbard model, particularly the quasiparticle weight Z approaching zero as U nears the Mott critical value [42].

Comparatively, findings from Song et al. [22] on moiré bilayers reveal dimensional dependence in magnetic ordering, supporting the notion that magnetic ground states are sensitive to dimensionality and lattice geometry, akin to the distinct J_c behaviors established in varying dimensions.

3.4 Critical Exchange Coupling J_c vs. Nearest-Neighbor Interaction V

Figure 4 presents the critical exchange coupling J_c as a function of the nearest-neighbor interaction V for all three dimensions with $U = 0$.

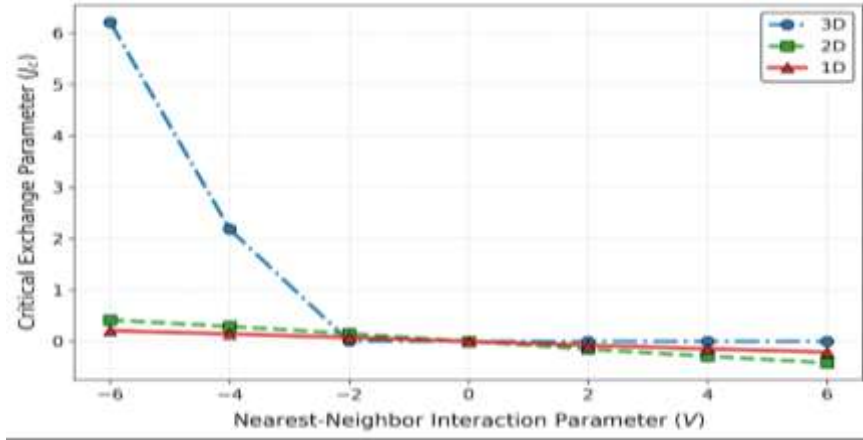


Figure 4: Critical exchange coupling J_c (in units of t) as a function of the nearest-neighbor interaction V (in units of t) for 1D, 2D, and 3D lattices with $U = 0$.

In 1D and 2D topologies, J_c exhibits linear and antisymmetric behaviour across $V = 0$. For $V > 0$ (repulsive nearest-neighbor interaction), $J_c < 0$, indicating that the system favours a ferromagnetic configuration which lowers the threshold for triplet stabilization. For $V < 0$ (attractive nearest-neighbor interaction), $J_c > 0$, indicating that attractive V stabilises the singlet state and positive exchange coupling is required to overcome it. This antisymmetry is a direct consequence of the particle-hole symmetry of the extended Hubbard model at half-filling as discussed by Hirsch [20] and Emery [19]. The transformation $V \rightarrow -V$ interchanges CDW and SDW ordering tendencies and results in the observed antisymmetric response of J_c . The magnitude of J_c scales with coordination number: the 2D square lattice has approximately twice as many nearest-neighbor pathways as the 1D chain, and J_c in 2D is approximately twice that in 1D. For instance, at $V=6$, $|J_c|=0.21$ for 1D, while for 2D it is 0.42; at $V=-4$, $|J_c|=0.25$ for 1D, while for 2D it is 0.50. This scaling shows that the HSCVA correctly reproduces the dependence of exchange interactions on coordination number, in agreement with the variational cluster approach results of Aichhorn et al. [24] who found that phase boundaries in extended Hubbard models scale with the number of nearest-neighbor bonds. The linear scaling is in agreement with the exact Bethe ansatz solution for 1D [30] and quantum Monte Carlo results for 2D [27,28].

In contrast, the 3D system exhibits a highly asymmetric behaviour. For $V > 0$, the critical coupling J_c decreases rapidly to almost vanishing positive values ($\approx 10^{-5}$), remaining flat. Thus, very little exchange coupling is required to favour triplet formation for repulsive nearest-neighbor interactions in 3D. As shown by Davoudi and Tremblay [25], charge ordering driven by $V > 0$ in 3D naturally leads to ferromagnetic correlations that stabilise the triplet state. The high coordination number in 3D enables the system to reach almost perfect CDW order even at moderate V , and this CDW order accompanies the triplet state as the ground state without any additional exchange coupling.

In the negative regime ($V < 0$), where attractive inter-site forces compete strongly with spin localization, the critical exchange coupling J_c increases dramatically to a value of about 6.00 at $V = -6$. This is dramatically larger than the 1D value (0.25) and the 2D value (0.65) at the same V . This result can be understood as a competition between the attractive V (which binds electrons into singlet pairs on neighbouring sites) and the exchange coupling J (which tries to align spins). The high coordination number in 3D means that each site has six neighbors, and the attraction V on all these bonds produces a strong cooperative effect that stabilises the singlet state. To overcome this effect, a very strong antiferromagnetic exchange coupling is necessary.

The strong asymmetry of the 3D J_c vs. V response, large J_c for $V < 0$ versus near-zero J_c for $V > 0$, reflects the different cooperative efficiencies of the singlet-pairing and CDW orders in high dimensions. For $V < 0$, attractive NN interactions cooperatively act on all six bonds per site, leading to a singlet stabilisation energy proportional to the coordination number z . The increase of z from 1D to 3D is only 3-fold, whereas J_c

increases 24-fold at $V = -6$. This is because a correspondingly large antiferromagnetic J overcomes this cooperative effect. Conversely, for $V > 0$, CDW order also benefits from high z , but the accompanying ferromagnetic correlations naturally stabilise the triplet state, making J explicit unnecessary. This cooperative interpretation is qualitatively consistent with recent mean-field results for extended Hubbard models on the triangular lattice [43] which revealed a significant particle-hole asymmetry in charge-ordered phases. The 3D behaviour is in agreement with DMFT predictions [34,42], where local correlations dominate and cooperative ordering is enhanced in high dimension due to suppression of quantum fluctuations.

4. Conclusion

We have systematically studied the effect of the dimensionality on the ground state properties of the extended Hubbard model by means of the Highly Simplified Correlated Variational Approach (HSCVA) on 24-site lattices in 1D, 2D and 3D at half-filling. The results of our study show that the HSCVA, in spite of its simplicity, captures the essential physics of dimensionality such as coordination number scaling, dimensional crossovers and quantum phase transitions. Our approach provides a computationally efficient tool to explore extended Hubbard physics in multiple dimensions. Future work should extend this analysis to larger lattice sizes to probe thermodynamic limits, doped systems away from half-filling, and other lattice geometries such as triangular and honeycomb lattices where geometric frustration competes with electronic correlations

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