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CALCULATION OF FLUID FLOW AND HEAT TRANSFER OVER A NON-ISOTHERMAL HORIZONTAL CIRCULAR CYLINDER IN CROSS FLOW USING THE MODIFIED MERK SERIES OF CHAO AND FAGBENLE

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Abstract

The Merk-Chao-Fagbenle (MCF) method is employed in the calculation of fluid flow and heat transfer over a non-isothermal horizontal circular cylinder in cross flow. The series developed with this method and the results of the wall derivatives of temperature functions was used directly for the heat transfer calculations. At Prandtl number (Pr) = 0.7 and pressure gradient (Λ) =0, the wall derivatives of temperature functions were taken from the table provided. This table, the result derived from the calculations of temperature wall derivatives by the above procedure for the two parameters M-C-F problem, the Prandtl number(Pr) and the pressure gradient($\wedge$) are input into the modified Merk's series of Chao and Fagbenle after the third parameter, the temperature parameter(γ) for the non-isothermal case has been evaluated. The heat transfer calculations were carried out and the results compared with the results of Chao and Fagbenle at a temperature exponent (a) = 0. The results are in good agreement. Other results at various Λ's and a's are presented in tables and graphs. The close agreement of this results with the results of Chao and Fagbenle confirms the correctness of the MCF procedure.

***Keywords*:** Boundary layer, Horizontal Circular cylinder, Merk series, Pressure gradient, Prandtl number.

1. Introduction

The phenomena of flow and heat transfer in the boundary layer over an immersed solid object is encountered in numerous industrial and manufacturing process. Notwithstanding the importance of the detailed kinematics of the flow field, it is readily recognized that reliable information on the rate of momentum and heat transfer between the fluid and the submerged object is frequently needed while performing process engineering and equipment design calculations. This information is conveniently expressed using the relevant dimensionless parameters such as the skin friction coefficient and Nusselt number as functions of the pertinent physical and kinematic variables expressed in dimensionless form as Reynolds and Prandtl numbers and the other system variables. This functional relationship is strongly dependent on the geometry, that is, the shape and orientation of the submerge object. Heat transfer in a Newtonian fluid from external surface of a circular cylinder is the subject of this investigation. Other industrial processes which rely on this thermal boundary layer concept are hot rolling, wire drawing, fiber-glass and paper production, gluing of labels on hot bodies, the design of heat exchangers, etc.

Blasius [1] was the first to apply the series-based expansion methods to the boundary layer fluid flows and the method has continued to be developed rapidly and employed by such researchers as Gortler [2] and Merk [3]. Generally speaking, the Blasius series is quite effective for fluid flows over blunt objects like cylinders. In the case of slender bodies, an excessive number of terms would be required in the polynomial representation and the series surfers from slow convergence. Merk [3] provided a procedure which belongs to the category of 'wedge methods' and which provides a rigorous refinement of the local similarity concept. Merk scheme makes possible rapid calculations of the significant boundary layer quantities (skin friction, heat transfer, mass transfer, etc) with the aid of a limited number of universal functions which can be tabulated once and for all Chao and Fagbenle [4]. An advance in the accuracy of boundary layer series solution was

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therefore made possible by Merk, who refined the 'wedge method' proposed by Meksyn [5] by choosing to treat the wedge parameter (Λ) as an independent variable rather than as a function of the stream wise coordinate, (ξ). An error in the form of the series presented by Merk was found by independent researchers, Chao and Fagbenle [4]. Chao and Fagbenle put forward a corrected form of Merk series and used it to perform a universal, laminar boundary layer analysis for the forced flow of Newtonian fluids over isothermal bodies. Since then, the MCF approach has been used with success for a family of boundary layer solutions

Many researchers have used this method successfully in investigating boundary layer problems. Cameron [6] used the MCF equation to investigate mixed, forced and natural convection from two-dimensional or axisymmetric bodies of arbitrary contour. Meissner [7], used the same method for mixed convection to power-law fluids from two-dimensional or axisymmetric bodies, with huge success. Recently Amoo [8] presented a comparative analysis of numerical methods including the MCF series applied to non-similar boundary layer-derived infinite series equation. By introducing the two-parameter MCF series into the transformed boundary layer equations for non-isothermal surfaces, there resulted a set of ordinary differential equations with three parameters, the pressure gradient (Λ), the Prandtl number (Pr) and the temperature parameter (γ). Therefore, by assigning numerical values to these parameters, this set of equations was solved so that the results for the flow field and the heat transfer was expressed in terms of universal functions, Falana, [9], Falana and Fabgenle, [10]. Solutions were obtained for combinations of the non-isothermal parameters for various combinations of the temperature exponent, the Prandtl number, and the pressure gradient

In this work, the universal wall derivative of temperature functions derived from the MCF series [9] was applied to the calculation of fluid flow and heat transfer over a non-isothermal horizontal circular cylinder in cross flow.

2. Problem Formulation

The Merk-Chao-Fagbenle method is strictly applicable to incompressible, uniform property, laminar boundary layer flows. The MCF equations governing the flow and heat transfer for a non-isothermal surfaces are not re-derived here. They are solved for wall derivatives of universal temperature functions ($\theta'_i,\ i = 0,1,2,3$) using FORTRAN 77 and have been tabulated once and for all [9]. The table for the wall derivative of universal functions have also been presented in [10]. For pressure gradient, (Λ=0), and Prandtl, (Pr=0.7), the table for these values is made available in Table 1.

3. Methodology.

Some amount of theoretical analysis for the two- dimensional boundary layer flow over the front portion of a horizontal circular cylinder can be found in literature. A limited amount of experimental results are also available. Hence the flow configuration provides a good opportunity for ascertaining the limitations of as well as the strength of Merk's method. This case was used by Chao and Fagbenle, [4] in their study of Merk's procedure for constant wall temperature body.

Main Stream Velocity Distribution.

A basic input to the Merk-Chao-Fagbenle's procedure is the information on the mainstream velocity distribution. The velocity distribution for the flow configuration has significant influence on the various computed boundary layer quantities. Therefore, to compute our boundary layer quantities, we first consider the potential velocity distribution [9];

$$\frac{U}{U_\infty} = 2\text{Sin}\left(\frac{2x}{D}\right). \qquad (1)$$

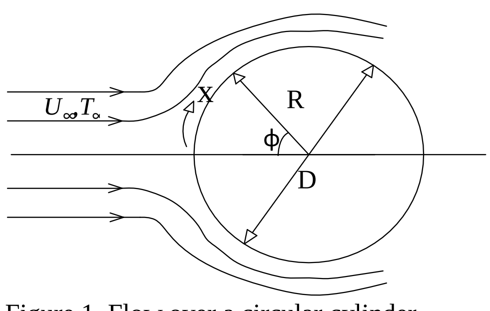


Figure 1. Flow over a circular cylinder

The MCF series employed in this work is stated as follows:

$$\theta(\xi,\eta) = \theta_0((\wedge,\eta) + 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_1(\wedge,\eta) + \gamma\theta_2(\wedge,\eta) + \gamma 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_3(\wedge,\eta) + 4\xi^2\left(\frac{d^2\wedge}{d\xi^2}\right)\theta_1(\wedge,\eta) + \cdots \quad (2)$$

The calculation begins with an evaluation of the dimensionless stream wise coordinate (ξ) and the wedge variable or the pressure gradient ($\wedge$) according to equations (3) and (4).

$$\xi = \int_0^x \frac{U(x)}{U_\infty}\left(\frac{r}{L}\right)^2 \frac{dx}{L} \quad (3)$$

$$\wedge = \frac{2\xi}{U}\frac{du}{d\xi} = 2\left(\frac{L}{r}\frac{U_\infty}{U(x)}\right)^2\left\{\int_0^x\left(\frac{r}{L}\right)^2\frac{U(x)}{U_\infty}\frac{dx}{L}\right\}\frac{d\left(\frac{U}{U_\infty}\right)}{(x/L)} \quad (4)$$

The reference velocity (U_∞) and length (L) may be any convenient quantities appropriate to the problem under consideration; they are constants.

Whenever feasible, it is desirable to express $\frac{U(x)}{U_\infty}$ and $\frac{r}{L}$ as a polynomial of x/L.

Table 1: Wall derivatives of temperature functions for $\Pr = 0.7$

$\wedge$	$\theta_0'(\wedge,0)$	$\theta_1'(\wedge,0)$	$\theta_2'(\wedge,0)$	$\theta_3'(\wedge,0)$
-0.15	0.36437340	-0.73064340E-10	-0.47564870E-05	-0.28152810E-14
-0.10	0.38697190	-0.11286630E-09	-0.70105800E-05	-0.10904380E-13
-0.05	0.40223690	-0.11731800E-09	-0.87854970E-05	-0.13668280E-13
0.00	0.41391300	-0.12072300E-09	-0.10302160E-04	-0.16030270E-13
0.05	0.4234000	-0.12348970E-09	-0.11648920E-04	-0.18127830E-13
0.10	0.43139610	-0.12582120E-09	-0.12872300E-04	-0.20033400E-13
0.20	0.44438450	-0.12960830E-09	-0.15052850E-04	-0.23430270E-13
0.30	0.45469960	-0.13261570E-09	-0.16978330E-04	-0.26430190E-13
0.40	0.46323390	-0.13510350E-09	-0.18719960E-04	-0.29144000E-13
0.50	0.47049320	-0.13721970E-09	-020321000E-04	-0.31639060E-13
0.60	0.47679620	-0.13905670E-09	-0.21810020E-04	-0.33959780E-13
0.70	0.48235520	-0.14067680E-09	-0.23207220E-04	-0.36137600E-13
0.80	0.48731890	-0.14212320E-09	-0.24527370E-04	-0.38195540E-13
0.85	0.48961240	-0.14279140E-09	-0.25162150E-04	-0.39185140E-13
0.90	0.49179500	-0.14342750E-09	-0.25781650E-04	-0.40150950E-13
0.95	0.49387680	-0.14403400E-09	-0.26386930E-04	-0.41094650E-13
1.00	0.49586610	-0.14461340E-09	-0.26978860E-04	-0.42017570E-13

For a two-dimensional boundary layers, one sets $r = L$. Hence, the expression r/L drops out of the computation, as a result of this, a straightforward algebraic equations for ξ and $\wedge$ was obtained and the determination of $2\xi\frac{d\wedge}{d\xi}$ and $4\xi^2\frac{d\wedge^2}{d\xi^2}$ and the expression for the temperature parameter, (γ) in equation (2) were also derived from equations (3) and (4).

At the forward stagnation point, $x = \xi = 0$ and $\wedge = 1$ or $\frac{1}{2}$, corresponding respectively to the two-dimensional or axisymmetrical boundary layers. The local heat transfer coefficient is given by:

$$Nu = \frac{hL}{k} = \frac{q_w L}{k(T_w - T_\infty)} = \frac{-L\frac{dT}{dy} @\, y=0}{T_\infty - T_w} \quad (5)$$

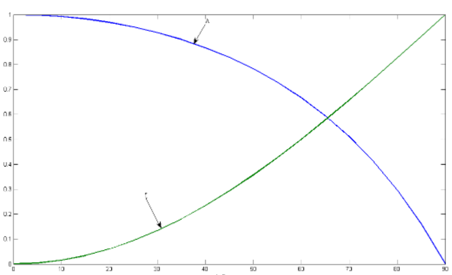
$$Nu/_{Re^{\frac{1}{2}}} = \frac{r}{L}\frac{U(x)}{U_\infty}(2\xi)^{-\frac{1}{2}}[-\theta'(\Lambda,0)] \quad (6)$$

With the aid of Table 1, the local heat transfer coefficient can be obtained by substituting $\eta = 0$ at the wall in equation (2) as follows:

$$NuRe^{-\frac{1}{2}} = \frac{r}{L}\frac{U(x)}{U_\infty}(2\xi)^{-\frac{1}{2}}\Big[\theta_0'(\wedge,0) + 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_0'(\wedge,0) + \gamma\theta_2'(\wedge,0) + +\gamma 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_0'(\wedge,0) + 4\xi^2\left(\frac{d^2\wedge}{d\xi^2}\right)\theta_0'(\wedge,0) + \cdots\Big] \quad (7)$$

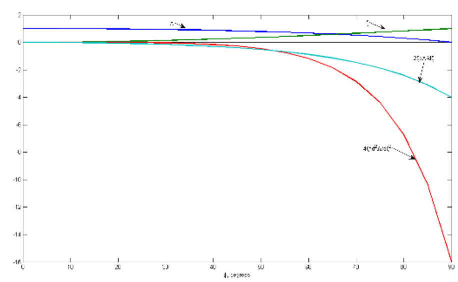
where $Re = U_\infty L/\nu$, Re= the Reynolds number and ν= kinematic viscosity
For a particular Prandtl number, which in this case is Pr = 0.7 (Table 1)
The universal functions ($\theta'_i,\ i = 0,1,2,3$) which are made available in Table1 depend on the Prandtl number as a parameter. Making use of the MCF procedure, the local heat transfer for flow and heat transfer over a horizontal circular cylinder containing three parameters is then calculated.

The local heat transfer coefficient over a surface of non-uniform temperature is given by equation (7). The following expressions $\xi\ 2\xi\frac{d\wedge}{d\xi},\ 4\xi^2\frac{d^2\wedge}{d\xi^2}$, are then computed.
The reason for the consideration of the above equation is the fact that many other numerical works on this flow configuration are based on such distribution. Figure 2 shows the variation of ξ and $\wedge$ *with* ϕ *while Figure* 3 *exhibits the variation of* $2\xi\ \frac{d\wedge}{d\xi}$, *and* $4\xi^2\frac{d^2\wedge}{d\xi^2}$ with ϕ.



Φ, Degree

Figure 2. Variation of ξ and $\wedge$ with ϕ over the front portion of a circular Cylinder in cross flow.



Φ, Degree

Figure 3. Variation of $2\xi\ \frac{d\wedge}{d\xi}$ and $4\xi^2\frac{d^2\wedge}{d\xi^2}$ with ϕ over the front portion of a Circular cylinder in cross flow.

It is worthy of note that the minimum pressure occurs when $\wedge$ becomes zero. A common feature exhibited by the four curves as also noted by Chao and Fagbenle, [4] is that, at small ϕ, they are slowly varying functions of ϕ . As the region of minimum pressure is approached, they become very sensitive to small changes in ϕ . M-C-F procedure has a wider range of applicability when the external flow is described by the potential theory than in the case of measured profiles. This is so because as figure 3 shows, the coefficients $2\xi\ \frac{d\wedge}{d\xi}$ and $4\xi^2\frac{d^2\wedge}{d\xi^2}$ for potential flow velocity attain large values comparatively much later than in the case of measured profiles Chao and Fagbenle, [4].

The temperature parameter for the non-uniform surface for the horizontal circular cylinder in cross flow is defined as follows:

$$\gamma = \frac{2\xi\frac{d}{d\xi}(T_w(x) - T_\infty)}{(T_0 - T_\infty)} \quad (13)$$

Stating $T_w(x) - T_\infty = (T_0 - T_\infty)e^{\frac{ax}{L}}$, the initial temperature variation on the surface of the horizontal circular cylinder in cross flow [9].

4. Results and Discussion

The results were generated from equation (7). The values of the universal wall derivatives of temperature functions (θ'_i, $i = 0,1,2,3$) were taken from Table 1. The equation was programmed using FORTRAN 77, the results were imported into an EXCEL Worksheet for graphing.
The effects of the Prandtl number (Pr) and the temperature exponent**, a,** are shown in figures 4-11 When the temperature exponent, a, was set to zero, that is, a= 0; we recovered the results of Chao and Fagbenle for the isothermal horizontal

cylinder in cross flow, table 2. The heat transfer result of the M-C-F procedure for the potential external velocity distribution for Prandtl of Λ. At low Prandtl numbers, the wall temperature derivatives for i =1, 2, and 3 are relatively small compared to the function $(\theta'_i,\ i = 0)$ and hence reduce the effect of the ssforward stagnation point. This in effect gave rise to low values for the Nusselt numbers for these Prandtl numbers.

In figures 4, 5, 8 and 9 when a, the temperature exponent is positive, heat transfer from the front portion of the horizontal circular cylinder in cross flow decrease slightly and thereafter continue to increase for the particular values of Λ. On the other hand, in figures 6, 7, 10, and 11 when a, the temperature exponent is negative, heat transfer from the same portion of the cylinder increases gradually in the early part of the front portion and decrease rapidly for every value of Λ.

Table 2: Computations showing comparison of the Local Nusselt number, $NuRe^{-1/2}$ for flow over a circular cylinder with Chao and Fagbenle [4] for Pr = 0.7 and temperature exponent, $a = 0$.

x/D	ϕ	Λ	Chao and Fagbenle [1974]	Present Work
0.6205	71.10	-0.15	0.6092	0.5929
0.6136	70.32	-0.10	0.6663	0.6328
0.6064	69.49	-0.05	0.6506	0.6610
0.5908	67.70	0.05	0.6469	0.7033
0.5822	66.72	0.10	0.6512	0.7206
0.5420	62.11	0.30	0.6870	0.7790
0.4876	55.87	0.50	0.7404	0.8314

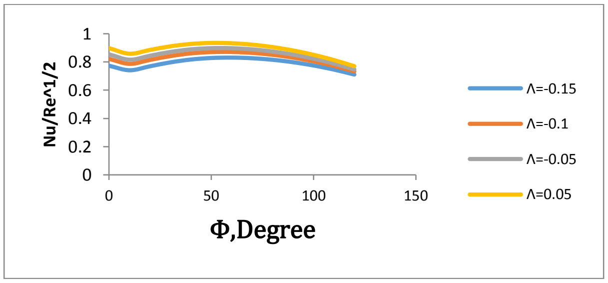


Figure 4. Heat transfer over the front portion of a non-isothermal horizontal circular cylinder in cross flow for Pr = 0.7, a = 0.2

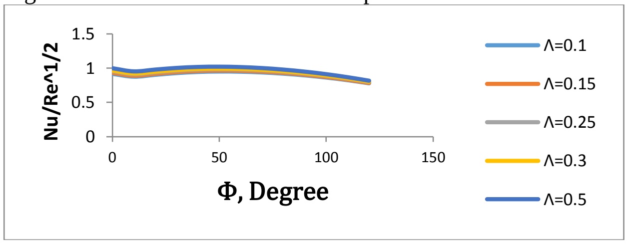


Figure 5. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow for Pr = 0.7, a =0.2

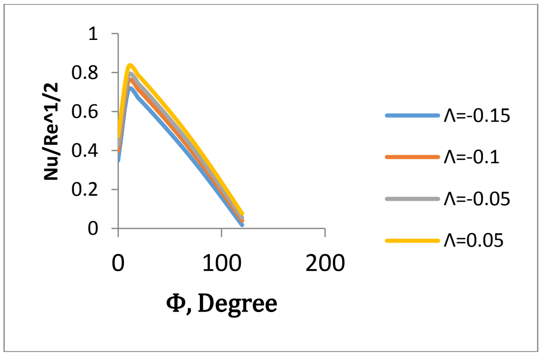


Figure 6. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow for Pr = 0.7, a = -0.2

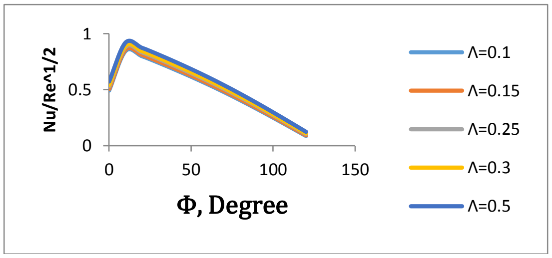


Figure 7. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow, for Pr = 0.7, a = -0.2

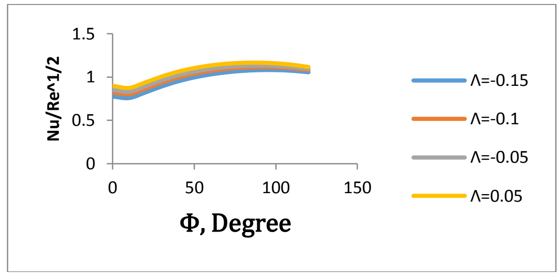


Figure 8. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow, for Pr = 0.7, a = 0.8

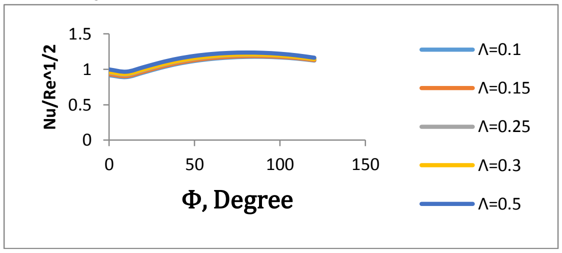


Figure 9. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow, for Pr = 0.7, a = 0.8

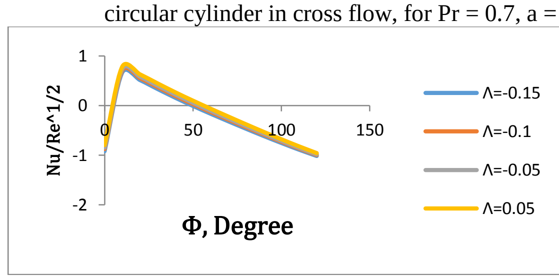


Figure10. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow, for Pr = 0.7, a = - 0.8

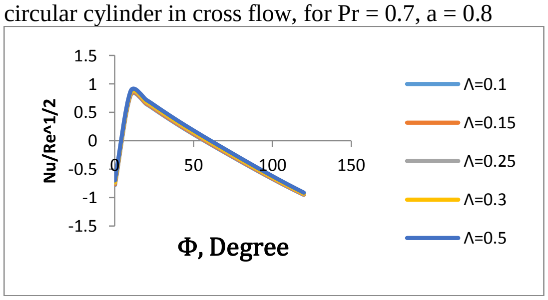


Figure. 11. Heat transfer over the front portion of a non-isothermal circular cylinder in cross flow, for Pr = 0.7, a = -0.8

4. CONCLUSION

The two-parameter Merk's series which was corrected by Chao and Fagbenle and applied to a constant wall temperature or isothermal surface has been applied to a variable or non-isothermal horizontal circular cylinder in cross flow. In addition to the MCF two-parameter problem, a third parameter known as the temperature parameter came up in the non-isothermal case. The temperature parameter was evaluated and incorporated into the MCF series and was used for the calculation of the flow and heat transfer over the non-isothermal horizontal circular cylinder in cross flow. Result were compared with that of Chao and Fagbenle at zero incidence, (Λ=0), and are found to be in good agreement.

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