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# A COMPARISON OF THE VARIATIONAL ITERATION METHOD AND ADOMIAN DECOMPOSITION METHODS IN SOLVING THE PROBLEM OF SQUEEZING FLOW BETWEEN TWO CIRCULAR DISKS

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***Abstract***

***The aim of this work is to compare two methods adopted in solving nonlinear problems, namely the Variational Iteration Method (VIM) and the Adomian Decomposition Method (ADM). The Navier-Stokes equation for the Unsteady flow between the two circular disks approaching each other symmetrically was solved. The comparison between VIM and ADM is bench-marked against a numerical solution. The results show that the VIM is more reliable and efficient than ADM from a computational viewpoint. The ADM requires slightly more computational effort than the VIM, but the VIM yields more accurate results than the ADM***

## 1. Introduction

In recent years, the Variational Iteration Method (VIM) and the Adomian Decomposition Method (ADM) have been among the source of a lot of research activity. Over the years, different analytical and Numerical schemes have been designed for solving equations owning to the important role played by linear and Non-linear differential equations in modeling real-life problems arising in Physics, Engineering, economics, etc.

Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM) have aided in obtaining approximate solutions to a wide class of such equations [1–4] and [5–10] discussed some of these equations.

However, only a few papers deal with the comparison of these methods [11]. The two methods give rapidly convergent series with specific significant features for each scheme. The main advantage of the two methods is that they can be applied directly to any type of differential and integral equations, homogeneous or inhomogeneous. Another important advantage is that the methods are capable of greatly reducing the size of computational work while still maintaining the high accuracy of the numerical solution. The effectiveness and usefulness of both methods are demonstrated by solving the problem of squeezing flow between two circular disks. Squeezing flows occurs as a result of externally normal stresses or vertical velocities by means of moving boundary. Squeezing flows have many applications in the food industry, especially in chemical engineering. Some practical examples of squeezing flow include polymer processing, compression, and injection molding. In addition, the lubrication system can also be modeled by squeezing flows. The study of squeezing flows has its origins in the 19th century and continues to receive considerable attention due to its practical applications in physical and biophysical areas. Stefan [12] published a classical paper on squeezing flow by using lubrication approximation. Such types of flow exist in lubrication when there is a squeezing flow between two parallel plates. The tackiness of liquid adhesives also reflects squeeze film effects [13] . The squeeze film geometry has been studied extensively since 1947. Other applications in the biomechanics area relate to squeezing flow between parallel plates and the alternation between contraction and expansion of the blood vessels. In addition, polymer extrusion processes are modeled using the squeezing flow of viscous fluids [14]. The theoretical and experimental studies of squeezing flows have been conducted by many researchers [15–25]. In this work, the problem of two circular non-rotating disks that are approaching and receding from each other gives rise to the squeezing flow. We further consider the motion of the plates to be symmetric about the axial line. The fluid flowing between the plates is considered to be a Newtonian incompressible viscous fluid. The systems of partial differential equations are reduced to a fourth-order non-linear differential equation with appropriate boundary conditions. Variational Iteration Method (VIM), Adomian decomposition Method (ADM), and Picard Iterative Method are used to solve the problem. The comparison between the three methods shows that the Variational Iteration Method (VIM) performs better than Adomian decomposition Method (ADM), but the Adomian decomposition method (ADM) maintains its solution for the observed parameter in comparison with the numerical solution.

This paper consists of Section 2, which analyzes the equations as well as the boundary conditions giving the squeezing flow. Sections 3 and 4 apply the Variational Iteration Method (VIM) and Adomian decomposition Method (ADM) to obtain the solution to the problem, respectively. Section 5 is a comparison of the methods and Section 6 is a summary of the results.

## 2. Formulation of the problem

We consider the squeezing flow of an incompressible viscous fluid between two circular disks (see, Fig. 1). The distance between the plates at any time $t$ is $2a(t)$. We select the central axis of the system to be the $r$-axis while the $z$-axis is normal to it. It is assumed that the circular disks are non-rotating and move symmetrically with respect to the central region $z = 0$. The flow is axisymmetric about $r = 0$.

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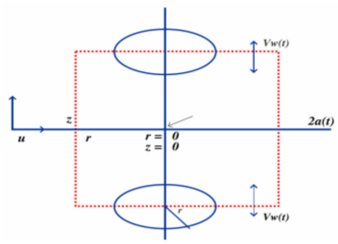


Figure 1: Geometry of the flow.

Now we specify the basic equations for an unsteady axisymmetric flow and assume $v = [u(r,z,t), 0, w(r,z,t)]$, where u and w are the velocity components along the radial and axial directions, respectively. Thus, the unsteady mass and conservation equations become

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(w) = 0 \tag{2.1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \left(\nabla^2 u - \frac{u}{r^2}\right) \tag{2.2}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu(\nabla^2 w) \tag{2.3}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{R}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$. The boundary conditions on $u(r,z,t)$ $w(r,z,t)$ are

$$\left.\begin{array}{lll} \text{at } z = a: & u(r,z,t) = 0 \quad \text{and} & w(r,z,t) = v_w(t) \\ \text{at } z = 0: & \dfrac{\partial u(r,z,t)}{\partial z} = 0 \quad \text{and} & w(r,z,t) = 0, \end{array}\right\} \tag{2.4}$$

where $v_w(t) = \frac{da}{dt}$ denotes the velocity of the circular disks. The conditions (2.4) are due to no-slip conditions at the upper plate $z = a$ and to symmetry at $z = 0$.

If the dimenstionless variable $\eta = \frac{z}{a(t)}$ is introduced, equations (2.1) - (2.3) transform to

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{a}\frac{\partial w}{\partial \eta} = 0 \tag{2.5}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{w}{a}\frac{\partial u}{\partial \eta}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{a^2}\frac{\partial^2 u}{\partial \eta^2 - \frac{u}{r^2}}\right], \tag{2.6}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{w}{a}\frac{\partial w}{\partial \eta}\right) = -\frac{1}{a}\frac{\partial p}{\partial \eta} + \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{a^2}\frac{\partial^2 w}{\partial \eta^2}\right]. \tag{2.7}$$

Introducing the functions $h$ and $\Omega$ respectively as

$$h = \frac{\rho}{2}(u^2 + w^2) + p, \quad \Omega = \frac{\partial w}{\partial r} - \frac{1}{a}\frac{\partial u}{\partial \eta}$$

equations (2.6) - (2.7) can be simplified as

$$\frac{\partial h}{\partial r} + \rho\frac{\partial u}{\partial t} - \rho\Omega w + \frac{\mu}{a}\frac{\partial u}{\partial \eta} = 0$$

$$\frac{1}{a}\frac{\partial h}{\partial \eta} + \rho\frac{\partial w}{\partial t} + \rho\Omega u - \mu\left(\frac{\partial \Omega}{\partial r} + \frac{\Omega}{r}\right) = 0.$$

We can eliminate $h$ from these equations by cross differentiation and obtain the system

$$-\rho\frac{\partial \Omega}{\partial t} - \rho\left[u\frac{\partial \Omega}{\partial r} + \frac{w}{a}\frac{\partial \Omega}{\partial \eta} - \frac{u}{r}\Omega\right] + \mu\left[\nabla^2\Omega - \frac{\partial \Omega}{\partial r^2}\right] = 0 \tag{2.8}$$

along with equation of continuity (2.5). The boundary conditions (2.4) take the form

$$\left.\begin{array}{lll} \text{at } \eta = 1: & u = 0 \quad \text{and} & w = v_w(t) \\ \text{at } \eta = 0: & \dfrac{\partial u}{\partial \eta} = 0 \quad \text{and} & w = 0, \end{array}\right\}$$

Defining velocity components of the form

$$u = -\frac{r}{2a(t)}v_w(t)f'(\eta), \quad w = v_w(t)f(\eta),$$

we find that the equation of continuity (2.5) is identically satisfied and equation (2.8) yields

$$f''''(\eta) + R\big[\big(\eta - f(\eta)\big)f'''(\eta) + 2f''(\eta)\big] = Qf''(\eta) \tag{2.9}$$

where

$$R = \frac{\rho a v_w}{\mu} \quad \text{and} \quad Q = \frac{\rho a^2}{\mu v_w}\frac{dv_w}{dt} \tag{2.10}$$

Both $R$ and $Q$ are functions of $t$ but for the similarity solution $R$ and $Q$ become constants. Integrating the first equation in (2.10) we get

$$a(t) = (2\nu Rt + a_0^2)^{\frac{1}{2}} \tag{2.11}$$

where $\nu = \frac{\mu}{\rho}$ and $2a_0$ is the distance between the two plates at time $t = 0$.When $R > 0$, the plates move apart symmetrically with respect to $\eta = 0$ and when $R < 0$, the plates approach each other and squeezing flow exists with similar velocity profiles as long as $a(t) > 0$.

It follows from equations (2.10) and (2.11) that if $Q = -R$ equation (2.9) is reduced to

$$f''''(\eta) + R\big[\big(\eta - f(\eta)\big)f'''(\eta) + 3f''(\eta)\big] = 0 \tag{2.12}$$

The boundary conditions in terms of $f(\eta)$ can be expressed as

$$\left.\begin{array}{ll} \text{at } \eta = 1: & f'(1) = 0 \quad \text{and} \quad f(1) = 1, \\ \text{at } \eta = 0: & f''(0) = 0 \quad \text{and} \quad f(0) = 0., \end{array}\right\} \tag{2.13}$$

The differential equation (2.12) is nonlinear, we present approximate solutions of this problem using the Variational Iteration Method (VIM), Adomian Decomposition Method (ADM) and numerical method in the succeeding sections. sections.

## 3. Basic Ideas of Variational Iteration Method

To illustrate the basic concept and the idea of the Variational Iteration technique, we consider the following general differential equation.

$$Lf + Nf = g(\eta) \tag{3.1}$$

where $L$ is a linear operator, $N$ a nonlinear operator and $g(\eta)$ is inhomogeneous forcing term. According to the Variational Iteration Method, the construction of correctional function for (3.1) is given as

$$f_{n+1}(\eta) = f_n(\eta) + \int_0^\eta \lambda(s)\{Lf_n(s) + N\tilde{f}_n(s) - g(s)\}ds \tag{3.2}$$

where $\lambda(s)$ is a Lagrange Multiplier which can be express as

$$\lambda(s) = \frac{(-1)^n}{(n-1)!}(s-\eta)^{n-1} \tag{3.3}$$

where (3.3) is known as the approximate Lagrange multiplier and $n$ is the highest derivative. In other for the solution to converge faster to the exact solution, it is required to use the exact Lagrange multiplier, which can be identified optimally via the Variational theory. The subscripts $n$ denoted the $nth$ approximation, $\tilde{f}_n$ is considered as a restricted variation, i.e $\delta\tilde{f}_n = 0$. Consequently, the solution

$$f(\eta) = \lim_{n\to\infty} f_n(\eta)$$

In other words, the correction functional (3.2) will give several approximations, and therefore the exact solution is obtained as the limit of the resulting successive approximations.

Following Variational Iteration Method, the equation

$$f''''(\eta) + R\big[\big(\eta - f(\eta)\big)f'''(\eta) + 3f''(\eta)\big] = 0$$

with the boundary conditions

$$\left.\begin{array}{ll} \text{at } \eta = 1: & f'(1) = 0 \quad \text{and} \quad f(1) = 1, \\ \text{at } \eta = 0: & f''(0) = 0 \quad \text{and} \quad f(0) = 0., \end{array}\right\}$$

can be written as an iteration formular by employing the correctional function in (3.2), which can be given as

$$f_{n+1}(\eta) = f_n(\eta) + \int_0^\eta \lambda(s)[3Rf_n''(s) + R\eta f_n'''(s) - Rf_n(s)f_n'''(s)]ds$$

following the variational theory, $\lambda(s) = \frac{1}{9}\left(\frac{3(s-\eta)}{R} - \frac{\sqrt{3}\sin\left(\sqrt{3}\sqrt{R}(s-\eta)\right)}{R^{3/2}}\right)$, we can choose $f_0(\eta) = \alpha\eta + \frac{\beta}{6}\eta^3$ by using Taylor's expansion and the given boundary conditions, taking into consideration that the value of $\alpha$ and $\beta$ will be determine using the boundary condition at $\eta = 1$. The successive approximation is given as follow.

$$f_0(\eta) = \frac{3\eta}{2} - \frac{\eta^3}{2}$$

$$f_1(\eta) = f_0(\eta) + \int_0^\eta \lambda(s)[3Rf_0''(s) + R\eta f_0'''(s) - Rf_0(s)f_0'''(s)]ds$$

$$= -\frac{\eta^3}{2} + \frac{3\eta}{2} + \frac{(3R-2)\sin(\sqrt{3}\eta\sqrt{R})}{6\sqrt{3}R^{5/2}} + \frac{\eta(3(\eta^2+30)\eta^2R^2 - 20(\eta^2+5)R + 40)}{120R^2} + \frac{\eta\cos(\sqrt{3}\eta\sqrt{R})}{3R}$$

$$f_2(\eta) = f_1(\eta) + \int_0^\eta \lambda(s)[3Rf_1''(s) + R\eta f_1'''(s) - Rf_1(s)f_1'''(s)]ds$$

$$= -\frac{\eta^3}{2} + \frac{3\eta}{2} + \frac{(3R-2)\sin(\sqrt{3}\eta\sqrt{R})}{6\sqrt{3}R^{5/2}} + \frac{\eta(3(\eta^2+30)\eta^2R^2 - 20(\eta^2+5)R + 40)}{120R^2} + \cdots$$

$$\vdots$$

$$f_n(\eta) = f_{n-1}(\eta) + \int_0^\eta \lambda(s)[3Rf_{n-1}''(s) + R\eta f_{n-1}'''(s) - Rf_{n-1}(s)f_n'''(s)]ds$$

Thus, we have the solution of the problem using the Variational Iteration Method (VIM).

## 4. Basic Ideas of Adomian Decomposition Method

Consider the general differential equation

$$Lf + Nf + Rf = g(\eta) \tag{4.1}$$

where $L$ is an operator representing the linear portion of $f$ which is easily invertible, $R$ is a linear operator for the remainder of the linear portion, and $N$ is a nonlinear operator representing the nonlinear term in $f$. Applying the inverse operator $L^{-1}$, the equation (4.1) becomes

$$L^{-1}(Lf) = L^{-1}(g(\eta)) - L^{-1}(Nf) - L^{-1}(Rf) \tag{4.2}$$

since $L$ is linear, $L^{-1}$ would represent integration and with any given initial or boundary conditions, $L^{-1}(Lf)$ will give an equation for $f$ incorporating these conditions, thus giving

$$f(\eta) = h(\eta) - L^{-1}(Nf) - L^{-1}(Rf) \tag{4.3}$$

where $h(\eta)$ represents the function generated by applying the inverse operator on $g(\eta)$ with any given initial or boundary conditions. Then assume that the unknown function can be written as an infinite series.

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta)$$

We set $f_0 = h(\eta)$ and the remaining terms are to be determined by a recursive relationship defined later.

This is found by first decomposing the nonlinear term into a series of Adomian polynomials, $A_n$. The nonlinear term is written as

$$Nf = \sum_{n=0}^{\infty} A_n$$

In order to determine the Adomian polynomials, a grouping parameter, $\lambda$, ia introduced. The series

$$f(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \quad \text{and} \quad Nf(\lambda) = \sum_{n=0}^{\infty} \lambda^n A_n$$

are established. Then $A_n$ can be determined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} Nf(\lambda)\Big|_{\lambda=0}$$

$$\sum_{n=0}^{\infty} f_n(\eta) = h(\eta) - L^{-1}\sum_{n=0}^{\infty} R\,f - L^{-1}\sum_{n=0}^{\infty} A_n$$

The recursive relationship is found to be

$$f_0 = h(\eta)$$

$$f_{n+1} = L^{-1}Rf_n + L^{-1}A_n$$

Following Adomian Decomposition Method, we define the highest order linear operator $L_\eta$ for equation (2.12), where $L_\eta$, $L_\eta^{-1}$ and the nonlinear tern for equation (2.12) is given as

$$L_\eta = \frac{d^4}{d\eta^4}$$

$$L_\eta^{-1} = \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta (\cdot)\, d\eta d\eta d\eta d\eta$$

$$Nf = f(\eta)f'''(\eta)$$

Thus equation (2.12) can be written as

$$L_\eta(f) = -3Rf'' - R\eta f''' + Rff'''$$

Applying the inverse operator to both side gives

$$L_\eta^{-1}(L_\eta f) = -3RL_\eta^{-1}(f'') - RL_\eta^{-1}(\eta f''') + RL_\eta^{-1}(ff''')$$

$$f(\eta) = \frac{3\eta}{2} - \frac{\eta^3}{2} - 3RL_\eta^{-1}(f'') - RL_\eta^{-1}(\eta f''') + RL_\eta^{-1}(ff''') \quad (4.4)$$

Next, we need to determine Adomian polynomials, $A_n$. To find $A_n$, we introduce the scalar $\lambda$

$$f(\lambda) = \sum_{n=0}^{\infty} \lambda^n f_n$$

Such that

$$Nf(\lambda) = \sum_{n=0}^{\infty} \lambda^n f_n \cdot \sum_{n=0}^{\infty} \lambda^n f_n''' = \sum_{n=0}^{\infty} \lambda^n \left( \sum_{j=0}^{\infty} f_j f_{n-j}''' \right)$$

Where

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} Nf(\lambda) \Big|_{\lambda=0}$$

we find the Adomian polynomials

$$\begin{aligned}
A_0 &= f_0 f_0''' \\
A_1 &= f_1 f_0''' + f_0 f_1''' \\
A_2 &= f_2 f_0''' + f_1 f_1''' + f_0 f_2''' \\
A_3 &= f_3 f_0''' + f_2 f_1''' + f_1 f_2''' + f_0 f_3''' \\
A_4 &= f_4 f_0''' + f_3 f_1''' + f_2 f_2''' + f_1 f_3''' + f_0 f_4''' \\
A_5 &= f_5 f_0''' + f_4 f_1''' + f_3 f_2''' + f_2 f_3''' + f_1 f_4''' + f_0 f_5'''
\end{aligned}$$

Returning the Adomian polynomials and the infinite series for the equation (4.4)

$$\sum_{n=0}^{\infty} f(\eta) = \frac{3\eta}{2} - \frac{\eta^3}{2} - 3RL_\eta^{-1}\left(\sum_{n=0}^{\infty} f_n''\right) - RL_\eta^{-1}\left(\eta \sum_{n=0}^{\infty} f_n'''\right) + RL_\eta^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \quad (4.5)$$

We can determine the recursive relationship that will be used to generate the solution.

$$\begin{aligned}
f_0(\eta) &= \frac{3\eta}{2} - \frac{\eta^3}{2} \\
f_{n+1}(\eta) &= -3RL_\eta^{-1}(f_n'') - RL_\eta^{-1}(\eta f_n''') + RL_\eta^{-1}(A_n)
\end{aligned}$$

The first few components of $f_n$ follow immediately upon setting:

$$\begin{aligned}
f_0 &= \frac{3\eta}{2} - \frac{\eta^3}{2} \\
f_1 &= \frac{\eta^5 R}{10} + \frac{1}{560}(\eta^2 - 21)\eta^5 R \\
f_2 &= -\frac{(\eta^2 + 45)\eta^7 R^2}{5040} - \frac{(27\eta^4 + 550\eta^2 - 7425)\eta^7 R^2}{1108800} \\
f_3 &= \frac{(27\eta^4 + 910\eta^2 + 3575)\eta^9 R^3}{14414400} + \frac{(3153\eta^6 + 80465\eta^4 - 50505\eta^2 - 1876875)\eta^9 R^3}{8072064000}
\end{aligned}$$

In view of the above equations, the solution in series form is

$$\begin{aligned}
f(\eta) \approxeq & -\frac{\eta^3}{2} + \frac{3\eta}{2} + \frac{(27\eta^4 + 910\eta^2 + 3575)\eta^9 R^3}{14414400} \\
& + \frac{(3153\eta^6 + 80465\eta^4 - 50505\eta^2 - 1876875)\eta^9 R^3}{8072064000} \\
& - \frac{(\eta^2 + 45)\eta^7 R^2}{5040} - \frac{(27\eta^4 + 550\eta^2 - 7425)\eta^7 R^2}{1108800} + \frac{\eta^5 R}{10} + \frac{1}{560}(\eta^2 - 21)\eta^5 R
\end{aligned} \quad (4.6)$$

## 5. Comparison of the VIM and ADM

To compare the two methods against a numerical solution, in which case we employed the Picard Iteration Method for solving nonlinear problems. In Figures 2-5 we note the comparison between the two methods and the numerical solution of (2.12) along with the boundary conditions (2.13), for the values of parameter $R$. We can observe that The Variational Iteration Method (VIM) is closer to the numerical solution than the Adomian Decomposition Method (ADM). Also, it is worth noting that as the parameter $R$ increases from 0.2 to 0.9 the Adomian Decomposition Method (ADM) solution moves away from the numerical solution whereas the Variational Iteration Method (VIM) maintains its accuracy compared to the numerical solution.

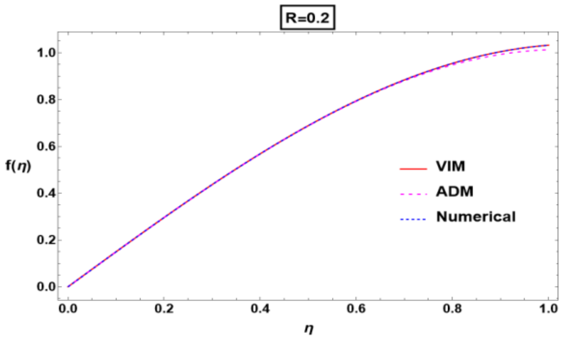


Figure 2: Comparison of solution profiles $f(\eta)$ by VIM and ADM for $R = 0.2$.

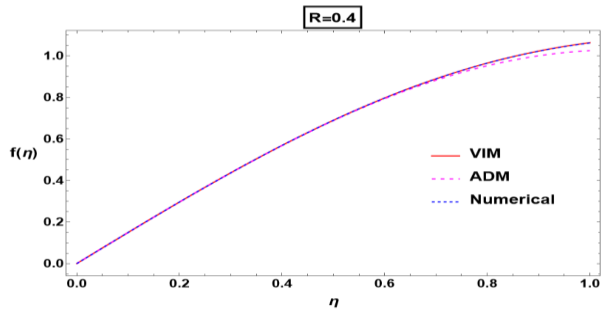


Figure 3: *Comparison of solution profiles* $f(\eta)$ *by VIM and ADM for* $R = 0.4$.

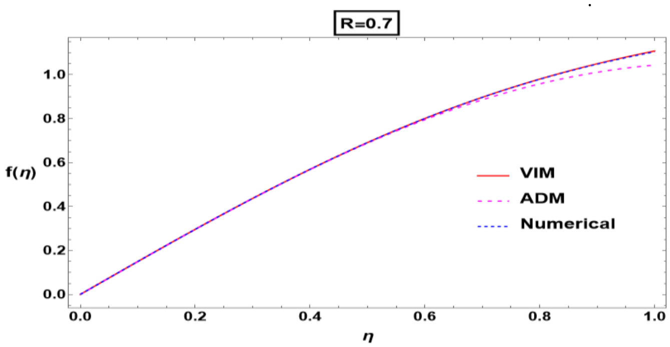


Figure 4: Comparison of solution profiles $f(\eta)$ by VIM and ADM for $R = 0.7$.

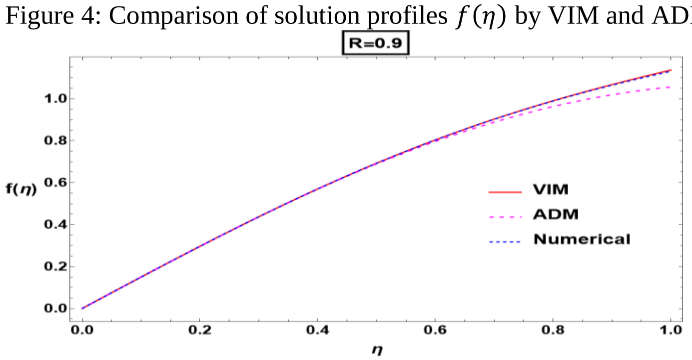


Figure 5: Comparison of solution profiles $f(\eta)$ by VIM and ADM for $R = 0.9$.

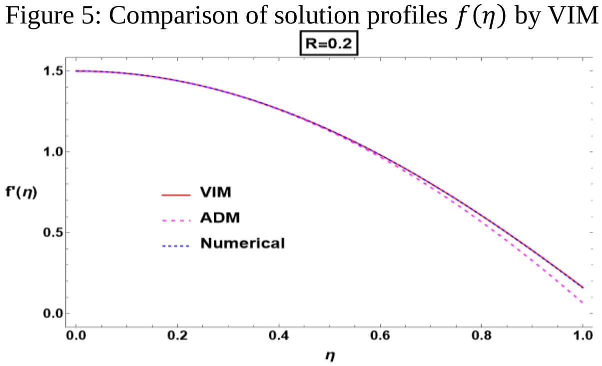


Figure 6: Comparison of velocity profiles $f'(\eta)$ by VIM and ADM for $R = 0.2$.

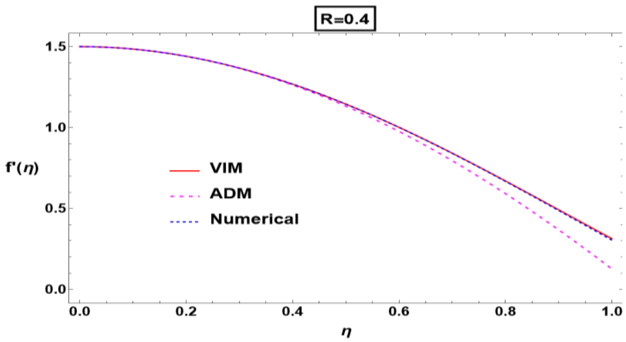


Figure 7: Comparison of velocity profiles $f'(\eta)$ by VIM and ADM for $R = 0.4$.

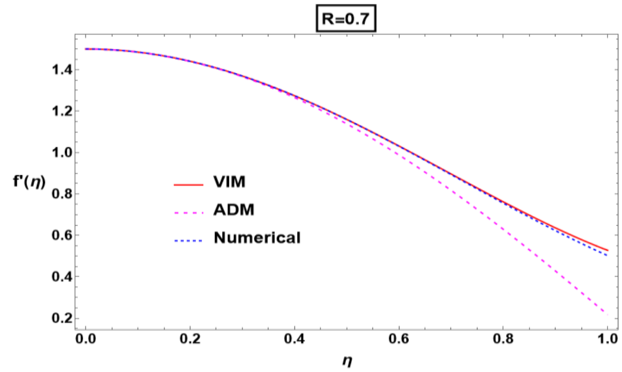


Figure 8: *Comparison of velocity profiles* $f'(\eta)$ *by VIM and ADM for* $R = 0.7$.

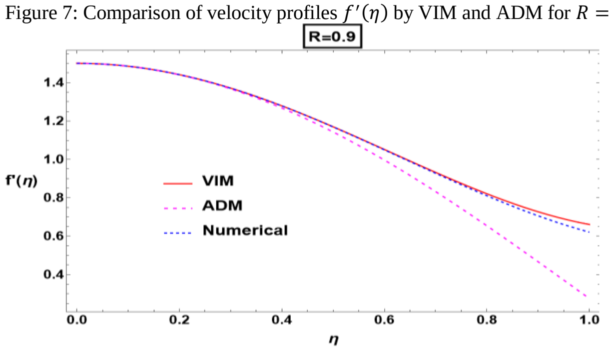


Figure 9: Comparison *of velocity profiles* $f'(\eta)$ *by VIM and ADM for* $R = 0.9$.

*Table 1: Comparison of solution profiles f(η) by VIM and ADM for R = 0.2.*

| | R=0.2 | | |
|---|---|---|---|
| η | VIM | ADM | Numerical |
| 0.0 | 0.000000 | 0.000000 | 0.000000 |
| 0.1 | 0.149500 | *0.149500* | *0.149500* |
| 0.2 | *0.296010* | *0.296004* | *0.296010* |
| 0.3 | *0.436579* | *0.436530* | *0.436579* |
| 0.4 | *0.568332* | *0.568128* | *0.568333* |
| 0.5 | *0.688514* | *0.687893* | *0.688518* |
| 0.6 | *0.794520* | *0.792979* | *0.794537* |
| 0.7 | *0.883942* | *0.880622* | *0.883992* |
| 0.8 | *0.954598* | *0.948148* | *0.954724* |
| 0.9 | *1.004570* | *0.992998* | *1.004860* |
| 1.0 | *1.032250* | *1.012740* | *1.032860* |

*Table2: Comparison of velocity profiles* $f'(\eta)$ *by VIM and ADM for* $R = 0.2$.

| | R=0.2 | | |
|---|---|---|---|
| η | VIM | ADM | Numerical |
| 0.0 | *1.500000* | *1.500000* | *1.500000* |
| 0.1 | *1.485020* | *1.485010* | *1.485020* |
| 0.2 | *1.440260* | *1.440100* | *1.440260* |
| 0.3 | *1.366310* | *1.365510* | *1.366310* |
| 0.4 | *1.264150* | *1.261610* | *1.264150* |
| 0.5 | *1.135130* | *1.128930* | *1.135100* |
| 0.6 | *0.980979* | *0.968183* | *0.980889* |
| 0.7 | *0.803812* | *0.780212* | *0.803577* |
| 0.8 | *0.606106* | *0.566049* | *0.605560* |
| 0.9 | *0.390699* | *0.326894* | *0.389541* |
| 1.0 | *0.160779* | *0.064122* | *0.158499* |

In addition, in Figure 6-9, we also note that the derivative of the solution (velocity profile) shows similar behaviour, once again the Variational Iteration Method (VIM) perform better than the Adomian Decomposition Method (ADM). But nevertheless, both methods provide a solution for the problem considered.


**A Comparison of the…** *Mustapha and Moses* ***J. of NAMP***

## 6. Summary

In this paper, the Variational Iteration Method (VIM) and the Adomian Decomposition Method (ADM) have been successfully applied to solve the non-linear equation (2.12) along with the boundary conditions (2.13) arising in the case of squeezing flow of an incompressible viscous fluid between two circular disks. It was shown that Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM) are efficient in attaining solutions. The comparison between Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM) with the numerical solution when applied to solve the equation (2.12) showed that the Variational Iteration Method (VIM) perform better than Adomian Decomposition Method (ADM) from a computational viewpoint, although both methods provide solutions for the problem.

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