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MATHEMATICAL MODELING OF THE EFFECT OF IVERMECTIN AND CATTLE AVAILABILITY ON MALARIA CONTROL

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Abstract

The fight against malaria is still on with the 2030 elimination goal in view. Several control tools have been in use to achieve the goal and a current attention under research is on the use of some endectocides one of which is known as ivermectin (IVM) drug that serves as mosquitocidal tool. In this study, a mathematical model is formulated for the control of this disease considering IVM with cattle availability. The model consists of ordinary differential equation from which we obtained the basic reproduction number, R_0 ***and then investigated the existence and stability of the disease-free equilibrium (DFE). Analytical estimate based on sensitivity index analysis showed that a 25% reduction in the proportion of vector blood meal on humans with cattle availability corresponds to a 25% reduction in the basic reproduction number. This finding is supported with the result of the numerical simulation. Applying four different combinations of control tools as strategies while varying the degrees of effort, the contribution of cattle availability through the parameter that controls the proportion of vector blood-meal on human is further seen to be positive on malaria control even when no measure of control is applied.***

***Keywords*:** Reproduction Number, Stability, Next Generation Matrix, Disease-free equilibrium points

1. Introduction

Several animals play important roles in the epidemiology of some infectious diseases that affect humans by acting as reservoirs of those disease pathogens. Livestock such as cattle serve as good source of blood-meal to arthropod vectors of human diseases like malaria and in some region some of these malaria vectors referred to as zoophilic vectors prefer taking their blood meals from these livestock than humans [1] while some other species of the vectors feed on both human and livestock in some other regions. Considering such behaviors, an intervention known as zooprophylaxis was proposed in [2,3] and it works in such a way that the attention of the vectors are diverted from humans to livestock for blood-meal given that the malaria parasites that infect humans has no effect on the livestock. An application of this strategy to control malaria was adopted in [1] where the role of livestock was explored. Despite the bulk of research that sought to ascertain the value of this strategy in the eradication or possible elimination of malaria carried out worldwide for over a century yet, no consistency in their findings. Some of these findings include the diversion of the vectors from human host leading to reduction in the vector blood meal on human, another finding has it that mosquito concentration heightened as a result of the closeness of livestock around the human neighbourhood causing the vectors to redirect their bites to the human host instead [1]. In some cases where humans were not available, the availability of cattle served as sustaining host for the vectors [4] which led to the persistence of the vector population. To further clarify the different contributions of livestock on malaria transmission and to also understand the role of zooprophylaxis intervention on controlling malaria, a mathematical model that captured such intervention combining it with data from Pakistan and Ethiopia was formulated in [1]. The model covered the treatment of human and livestock with insecticide and the effects of livestock on malaria transmission was observed to be non-linear. Although the strategy according to the authors is likely to be more beneficial to the people in areas where zoophilic malaria vectors is predominant, applying it under certain conditions has the possibility of bringing down the malaria burden substantially in areas with moderately zoophilic vectors like sub-Saharan Africa. In that study, it was recommended that a community-based trial of insecticide-treated livestock be implemented in Africa region as it is yet to be formally assessed. One gap in malaria control programmes as noted in [5] is the non-inclusion of interventions that considers the interactions between malaria vector biting behaviours and relative availabilities of alternative blood-host species such as livestock. In line with this, combining long-lasting insecticidal nets (LLINs) with the treatment of livestock with insecticidal compounds that is lethal to malaria vectors, a mathematical model was used in [6] to assess the impact of diverse range of host-biting behaviours

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on it. Investigating the efficacy of malaria control programme that involved the use of LLINs as a stand-alone tool and a combination of LLINs and endectocide-treated cattle, it was observed that the period of decay in the efficacy of LLINs over time possibly due to delay in the replacement campaigns which could lead to another rise in the prevalence of malaria is bridged with the use of endectocide. Marked differences across biting ecologies in the efficacy of both control methods was observed from the simulations of the model formulated. Also, considering the effect of relapse of malaria infection in human, a model that explicitly captured a proportion of susceptible human treated with ivermectin drug for the purpose of controlling malaria burden through the reduction of its vector population was formulated in [7]. The result of that study suggests that treatment of infective human as a stand-alone tool in the presence of relapse is not sufficient to bring down malaria burden to a reasonable level. On the other hand, the inclusion of ivermectin usage resulted to a significant positive outcome which suggests that IVM has the potential of bringing down the vector population within a short period of time thereby reducing transmission intensity. In [8] it was estimated that MDA with ivermectin will reduce malaria prevalence and incidence and so highly recommended in areas where the transmission of malaria is highly persistent with existing interventions not sufficient and also in those areas tending towards elimination. Similarly the investigation of our current study is on administering ivermectin to both human and livestock as a strategy that targets zoophilic vectors that escape vector control by long-lasting insecticidal nets (LLINs) and indoor residual spraying (IRS), respectively as a control measure in addition to the use of long-lasting insecticidal nets (LLINs) and the treatment of infective humans. Based on the report from African CDC [9], the drug is in the list of WHO recommended drugs for the treatment of some helminth diseases commonly found in malaria-endemic regions that include scabies, lymphatic filariasis and onchocerciasis (river blindness) [9,10]. In some areas with high-onchocerciasis transmission, the drug is also recommended for pregnant and lactating women [11]. Nevertheless, in some other studies such women are excluded. In addition, with the recent outbreak of covid-19 research finding has it that the drug (IVM) inhibits the replication of SARS-CoV-2 and has the potential for preventing and treating COVID-19 [12]. Nevertheless according to Africa CDC [9] the result of the finding was obtained from laboratory experiment under a higher dose of the drug than that approved for use in human. At present, researches are ongoing with respect to this as WHO recommended that data from well-designed, randomized, controlled clinical trials be used to validate such claims [9]. The method for the investigation of our current study which is aimed at further understanding the effect of treating humans and livestock with ivermectin and availability of alternative biting host (cattle) on malaria control is based a mathematical model that consists of a system of ordinary differential equations. This is developed and analyzed mathematically and numerically.

2. Model formulation

In this section, malaria model consisting of a nonlinear system of ordinary differential equations representing the human and mosquito populations is formulated to study the transmission, spread and control of malaria. The human and mosquito populations are divided into 3 compartments each given as Susceptible (S_h), Exposed/Latent (L_h), Infectious (I_h) compartments for the human population and Susceptible (S_v), Exposed/Latent (L_v), Infectious (I_v) compartments for the mosquito population. It is assumed that the mosquito population consists of only the female Anopheles mosquito and the ages of those susceptible humans that receive the ivermectin drug are 5 years and above. The model parameters and description are represented on Table 1 and the model equations are given as follows:

$$\left.\begin{aligned}
\frac{dS_h}{dt} &= \lambda_h + (\mathrm{v}+\sigma\,\mathrm{u}_2)I_h - \frac{(1-\mathrm{u}_1)abqI_vS_h}{N_h} - \mu_hS_h \\
\frac{dL_h}{dt} &= \frac{(1-\mathrm{u}_1)abqI_vS_h}{N_h} - (\mu_h+\theta)L_h \\
\frac{dI_h}{dt} &= \theta L_h - (\mathrm{v}+\sigma\,\mathrm{u}_2+\delta+\mu_h)I_h \\
\frac{dS_v}{dt} &= \lambda_v - \frac{(1-\mathrm{u}_1)\kappa\,\mathrm{qb}\,I_hS_v}{N_h} - (\tau u_3+\psi)S_v \\
\frac{dL_v}{dt} &= \frac{(1-\mathrm{u}_1)\kappa\,\mathrm{qb}\,I_hS_v}{N_h} - (\tau u_3+\psi+w)L_v \\
\frac{dI_v}{dt} &= wL_v - (\tau u_3+\psi)\,\mathrm{I}_v
\end{aligned}\right\} \qquad (1)$$

Where $\psi = \mu_v + \mu_c,\ \mu_c = bMS_h^{\circ}A_1$

The term "$bMS_h^\circ A_l$" accounts for a reduction in the vector population as a result of bites on *M* portion of the initial susceptible human (S_h°) population and available cattle (A_l) population both having ivermectin in their blood at the level of killing mosquito. This depends on the mosquito biting rate, b. Among other parameters, the force of infection from vectors to humans
and vice versa each depends on the term, *q* which represents the proportion of blood-meal taken on humans by the vectors (human blood index - HBI) [1]. The expression for this term as used in that study, is a function of the livestock and human availability (A_l, A_h respectively), treatment coverage ε, diversion parameter α, and the total human and livestock population, N_h and N_l, respectively. It is assumed here that only susceptible humans partake in the ivermectin treatment as a control strategy. Furthermore, u_1, u_2 and u_3 are the control parameters for the use of treated bed net, treatment of infectious human and the use of ivermectin drug respectively. The total human and mosquito populations are given by $N_h(t) = S_h(t) + L_h(t) + I_h(t)$ and $N_v(t) = S_v(t) + L_v(t) + I_v(t)$ respectively. The initial conditions ($t = 0$) of the system (1) are given as $S_h(0) = S_h^\circ,\ L_h(0) = L_h^\circ,\ I_h(0) = I_h^\circ,\ S_v(0) = S_v^\circ,\ L_v(0) = L_v^\circ,\ I_v(0) = I_v^\circ$ with the initial total human and mosquito populations represented by $N_h(0) = N_h^\circ$ and $N_v(0) = N_v^\circ$ respectively.

3. Analysis of the model

The region for which the malaria model of equation (1) is biologically meaningful and positively invariant is given as

$$\Omega = \begin{Bmatrix} (S_h, L_h, I_h, S_v, L_v, I_v) \in \Re_+^6 : S_h, L_h, I_h, S_v, L_v, I_v \geq 0, \\ S_h + L_h + I_h, \leq \lambda_h/\mu_h, S_v + L_v + I_v \leq \lambda_v/(\tau u_3 + \psi) \end{Bmatrix} \quad (2)$$

It implies that for all time $t > 0$, all feasible solutions of the system (1) remain positive and are attracted in the region Ω Therefore apart from the malaria model being biologically meaningful, it is also mathematically well-posed in that domain.

3.1 Model equilibrium

To obtain the equilibrium points for the malaria model, equate the right hand side of system (1) to zero and solve for the disease free equilibrium (DFE) and endemic equilibrium points as obtained below.

3.2 Existence and stability of disease-free equilibrium (DFE)

Suppose the population is malaria-free $(L_h = I_h = L_v = I_v = 0)$ then we obtain the disease free equilibrium point as

$$E_\circ = \{S_h^\circ,\ L_h^\circ,\ I_h^\circ,\ S_v^\circ,\ L_v^\circ,\ I_v^\circ\} = \{\lambda_h/\mu_h, 0, 0, \lambda_v/\mu_v, 0, 0\}. \quad (3)$$

3.3 The basic reproduction number, (R_0)

Applying the next generation matrix method [13], we derive the expression for the basic reproduction number, R_0 given as

$$R_0 = \sqrt{\frac{(1-u_1)^2 ab^2 kq^2 \omega\theta\lambda_h\lambda_v}{\mu_h N_h^2 (\tau u_3 + \psi)^2 (\mu_h + \theta)(\tau u_3 + \psi + \omega)(\sigma u_2 + \nu + \mu_h + \delta)}} \quad (4)$$

3.4 Existence of endemic equilibrium points

The endemic equilibrium point, $E_1 = \{S_h^*,\ L_h^*,\ I_h^*,\ S_v^*,\ L_v^*,\ I_v^*\}$ is the solution of the model system of equations (1) for the situation where the disease is present in both the human and mosquito population (that is, $L_h, I_h, L_v, I_v \neq 0$). Equating equation (1) to zero and expressing each state vaiables in terms of I_h^*, we obtain the endemic equilibrium as

$$S_v^* = \frac{\lambda_v N_h}{M_1 \kappa qbI_h^* + M_3 N_h} \quad (5)$$

$$L_v^* = \frac{M_1 \kappa qb\lambda_v I_h^*}{(M_3 + \omega)(M_1 \kappa qbI_h^* + M_3 N_h)} \quad (6)$$

$$I_v^* = \frac{M_1\kappa qb\omega\lambda_v I_h^*}{M_3(M_3+\omega)(M_1\kappa qbI_h^* + M_3N_h)} \tag{7}$$

$$L_h^* = \frac{(r+\mu_h+\delta)I_h^*}{\theta} \tag{8}$$

$$S_h^* = \frac{\lambda_h(M_1\kappa qbI_h^* + M_3N_h)}{\mu_h N_h M_3 R_0^2} \tag{9}$$

Where $M_1 = (1-u_1), M_2 = (v+\sigma u_2),$ and $M_3 = (\tau u_3 + \psi)$ and R_0 as given in equation (4).

With further substitutions and simplifications we obtain the expression for endemic equilibrium to be either $I_h^* = 0$ or

$$AI_h^{*2} + BI_h^* + C = 0, \tag{10}$$

$$A = M_1^2M_3N_h\omega\kappa^3b^2q^2\mu_h\lambda_h + M_1^2M_3^2N_h\kappa^3b^2q^2\mu_h\lambda_h + M_1^3\omega ab^3q^3\kappa^2\lambda_v\lambda_h - M_1M_2M_3^2N_h^2\mu_h\kappa qb\omega - M_1M_2N_h^2\mu_h\kappa qbR_0^2 \tag{11}$$

$$B = M_1^2M_3N_hab^2q^2\kappa\omega\lambda_v\lambda_h + 2M_1M_3^3N_h^2\mu_h\lambda_h\kappa qb + 2M_1M_3^2N_h^2\mu_h\omega\lambda_h\kappa qb - M_2M_3^3N_h^3\mu_h\omega R_0^2 - M_3^4M_2N_h^3\mu_hR_0^2 - M_1M_3^3N_h^2\lambda_h\mu_h\kappa qbR_0^2 - M_1M_3^2N_h^2\mu_h\lambda_h\omega\kappa qbR_0^2 \tag{12}$$

$$C = M_3^4N_h^3\mu_h\lambda_h - M_3^4N_h^3\mu_h\lambda_hR_0^2 + M_3^3N_h^3\mu_h\lambda_h\omega - M_3^3N_h^3\mu_h\lambda_h\omega R_0^2 = (c_1+c_2)(1-R_0^2) \tag{13}$$

where $c_1 = M_3^4N_h^3\mu_h\lambda_h$ and $c_2 = M_3^3N_h^3\mu_h\lambda_h\omega$

Equation (10) is an expression that is in a quadratic form and its possible real roots depend on the signs of the terms, *A*, *B* and *C*. Assuming a constant human population, the coefficients *A* and *B* and the constant term, *C* of the quadratic equation (10) are positive whenever $R_0 < 1$. We now consider the possibility of multiple endemic equilibrium for equation (10) base on the theorem below.

Theorem 1:
The malaria model (1) has the following

1. One unique endemic equilibrium if $C < 0$ iff $R_0 > 1$
2. One unique endemic equilibrium if $B < 0$ and $C = 0$ or $B^2 - 4AC = 0$
3. Two endemic equilibrium if $C > 0, B < 0$ and $B^2 - 4AC > 0$
4. Otherwise, no endemic equilibrium.

3.5 Stability of endemic equilibrium

To analyze the stability of the endemic equilibrium for any possible bifurcation we employ the Centre manifold method as described by [14]. To begin, we first transform the system (1) by the following change of variables: $x_1 = S_h,\ x_2 = L_h,\ x_3 = I_h,\ x_4 = S_v,\ x_5 = L_v,\ x_6 = I_v$ so that $N_h = x_1 + x_2 + x_3$ and $N_v = x_4 + x_5 + x_6$ and then denote the right side of the transformed system by $f = (f_1, f_2, f_3, f_4, f_5, f_6)^T$. Choosing '*a*' as the bifurcation parameter with $a = a^*$ and considering the case when $R_0 = 1$, applying these changes to the model equation and linearizing the new system formed at the DFE point, we obtain the Jacobian matrix given as

$$J_{E_{01}} = \begin{pmatrix} -\mu_h & 0 & M_2 & 0 & 0 & -\frac{M_1 a^* bq\lambda_h}{\mu_h N_h} \\ 0 & -(\mu_h+\theta) & 0 & 0 & 0 & \frac{M_1 a^* bq\lambda_h}{\mu_h N_h} \\ 0 & \theta & -(M_2+\delta+\mu_h) & 0 & 0 & 0 \\ 0 & 0 & -\frac{M_1\kappa qb\lambda_v}{\psi N_h} & -M_3 & 0 & 0 \\ 0 & 0 & \frac{M_1\kappa qb\lambda_v}{\psi N_h} & 0 & -(M_3+\omega) & 0 \\ 0 & 0 & 0 & 0 & \omega & -M_3 \end{pmatrix} \quad (14)$$

The eigenvalues of (14) are obtained from the solution of

$$(\lambda+\mu_h)(\lambda+M_3)\left[(\mu_h+\theta+\lambda)(M_1+\delta+\mu_h+\lambda)(M_3+\omega+\lambda)(M_3+\lambda)-\frac{M_1^2 a^* b^2\kappa q^2\omega\lambda_h\lambda_v\theta}{M_3\mu_h N_h^2}\right] \quad (15)$$

Replacing $N_h = \frac{\lambda_h}{\mu_h}$, $a = a^*$ and $R_0 = 1$ in equation (4) and then simplifying yields

$$a^* = \frac{M_3^2\lambda_h(\mu_h+\theta)(M_3+\omega)(M_2+\mu_h+\delta)}{M_1^2 b^2\kappa q^2\omega\theta\mu_h\lambda_v}. \quad (16)$$

Substituting equation (16) into equation (15) and further solving, we obtain one of the eigenvalues obtained as zero (i.e. $\lambda = 0$) and all the others have negative real parts. And so, from the Jacobian matrix (14) for $R_0 = 1$ we obtain the following right eigenvector $w = (w_1, w_2, w_3, w_4, w_5, w_6)^T$ associated with the zero eigenvalue as

$$w = \left(\frac{\mathrm{M2(M3+\omega)}M_3^2 - M_1^2 ab^2\kappa q^2\omega\mu_h}{\mathrm{M1}\omega b\kappa q\mu_h^2}, \frac{M_3^2\lambda_h(\mathrm{M3+\omega})(\mathrm{M2+\delta+\mu h})}{\mathrm{M1}\omega b\kappa q\lambda_v\mu_h\theta}, \frac{\lambda_h(\mathrm{M3+\omega})M_3^2}{\mathrm{M1}\omega b\kappa q\lambda_v\mu_h}, -\frac{(\mathrm{M3+\omega})}{\omega}, \frac{\mathrm{M3}}{\omega}, 1\right).$$

Similarly, the left eigenvector satisfying $v \cdot w = 1$ which corresponds to the zero eigenvalue when $R_0 = 1$ is given as

$$\mathrm{v} = (\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3, \mathrm{v}_4, \mathrm{v}_5, \mathrm{v}_6)^T = \left(0, \frac{M_1\kappa qb\lambda_v\omega\theta\mu_h}{H(\mu_h+\theta)}, \frac{M_1\kappa qb\lambda_v\omega\mu_h}{H}, 0, \frac{\omega}{(M_3+\omega)}, 1\right)^T$$

where $H = M_3\lambda_h(M_3+\omega)(M_2+\mu_h+\delta)$.

Next we compute a_1 and b_1 to examine their signs as required. As a result, we seek the non-zero partial derivatives of the transformed system at DFE given as

$$\frac{\partial^2 f_2}{\partial x_1 \partial x_6} = \frac{\partial^2 f_2}{\partial x_6 \partial x_1} = \frac{M_1 abq}{N_h}, \frac{\partial^2 f_5}{\partial x_3 \partial x_4} = \frac{\partial^2 f_5}{\partial x_4 \partial x_3} = \frac{M_1\kappa bq}{N_h}. \quad (17)$$

For $v_k \neq 0$, applying (17) and the required left and right eigenvectors $v_2, v_5, w_1, w_3, w_4, w_6$ and then simplifying gives the value of a_1 as

$$a_1 = -\left[\frac{\mathrm{M}_1^2\, w\lambda_v\theta\beta_h ab^2q^2k\mu_h + (\mathrm{M}_3+w)\,\mathrm{M}_3^2\,\lambda_h\beta_v - \mathrm{M}_1\mathrm{M}_2(\mathrm{M}_3+w)\,\mathrm{M}_3^2\, w\lambda_v^2 bqk\theta\beta_h}{\mathrm{M}_1\, w\lambda_v bqk\mu_h}\right] \quad (18)$$

where $\beta_h = \frac{\mathrm{M}_1\, abq}{N_h}$, $\beta_v = \frac{\mathrm{M}_1\,\kappa bq}{N_h}$.

From (18) we see that $a_1 < 0$ provided

$$M_1^2 w\lambda_v\theta\beta_h ab^2q^2k\mu_h + (M_3+w)M_3^2\lambda_h\beta_v > M_1M_2(M_3+w)M_3^2 w\lambda_v^2 bqk\theta\beta_h. \quad (19)$$

3.6 Computation of b_1

To compute the value of b_1, we apply similar procedure as above and obtain the required non-zero second partial derivatives at the DFE as

$$\frac{\partial^2 f_2}{\partial x_a \partial x_6} = \frac{\partial^2 f_2}{\partial x_6 \partial x_a} = \frac{\mathrm{M}_1 bqx_1}{N_h} = \mathrm{M}_1 bq. \quad (20)$$

and so,

$$b_1 = v_2 w_6 \frac{\partial^2 f_2}{\partial x_a \partial x_6} + v_6 w_2 \frac{\partial^2 f_2}{\partial x_6 \partial x_a}$$

$$= 2v_2 w_6 \mathrm{M}_1 bq > 0 \text{ since } v_2, w_6 > 0. \quad (21)$$

Provided $a_1 < 0$ and given that $b_1 > 0$ it implies that the malaria model (1) has a unique endemic equilibrium which is locally asymptotically stable when $R_0 < 1$ otherwise, unstable [15].

3.7 Sensitivity analysis

Applying the method of normalized forward sensitivity index [16] on R_0 with respect to each of the parameters reflecting in the expression of equation (4), we obtain the sensitivity indices represented on Table 1. These are indications on how sensitive R_0 is to the relative changes in each of its parameters.

4. Numerical simulations

Figures 1 - 5 are the outcomes of the numerical simulations of our malaria model equations obtained using the baseline parameter values given in Table 1 which also contains the sensitivity indices obtained for each parameters of R_0 using the given baseline parameter values.

Table 1: Parameters Descriptions, Values, References and Sensitivity Indices of the Parameters of R_0

Parameter	Description	Parameter Value/Ref	Sensitivity Index
μ_v	Mosquito natural death	0.0667/[17]	- 1.2728
b	Biting rate	0.2-0.5/[17]	1
q	Proportion of vector blood-meals on humans	0.485/[1]	1
μ_h	Human death rate	0.0004/[18]	- 0.5062
λ_h	Human birth rate	100/[19]	0.5
λ_v	Mosquito birth rate	1000/[19]	0.5
k	Probability of mosquito being infected by an infectious human	0.09/[19]	0.5
a	Probability of human being infected by an infectious mosquito	0.833/[20]	0.5
δ	Disease induce death rate	0.05/[21]	- 0.3551
w	Mosquito Progression rate from Latent to Infectious state	1/18/[19]	0.2728
v	Human Spontaneous Recovery rate	0.02/[20]	- 0.1420
θ	Human Progression rate from Latent to Infectious state	1/17/[19]	0.0034
S_h^0	Initial susceptible human population	7000/(Assumed)	
M	Proportion of Susceptible humans using Ivermectin	0.0001/(Assumed)	
A_l	Proportional availability of livestock to vectors on ivermectin	0.484/[1]	
c	Contact rate	0.6/[22]	
σ	Treatment efficacy	0.01-0.7/[20]	

4.1 Results and discussion

From the sensitivity index on Table 1, the positive index is an indication that an increase in any of those parameter values will result to an increase of the value of R_0 and vice versa. While the negative index is an indication that an increase in the parameter value will result to a decrease in the value of R_0 and vice versa. A verification of this is captured on Table 2 where the first 3 key parameters (q, b and μ_v) that are highly sensitive to the model's basic reproduction number, R_0 with positive (q and b) and negative (μ_v) indices were considered.

Table 2

Corresponding values of R_0 *to the reduction* (*q and b*) *and increment* (μ_v) *in the baseline parameter values for the three key parameters of* R_0 *as reflected in Table 1.*

(%) Reduction/ Increment	Reduction			Increment	
	q	b	R_0	μ_v	R_0
0	0.485	0.2	1.771285891	0.0667	1.771285891
25	0.36375	0.15	1.3284644418	0.083375	1.329272640
50	0.2425	0.1	0.8856429457	0.10005	1.046692607
75	0.12125	0.05	0.4428214726	0.116725	0.8526421237

Since the estimated sensitivity indices for parameters *q* and *b* are both equal to +1, it implies that increasing or decreasing any of these parameter's value by any percentage increases or decreases the value of R_0 by the same percentage respectively. To verify this, from the outcome of Table 2 a 25% reduction in parameters *q* or *b* gives the value of R_0 to be 1.3284644418 which corresponds to a 25% reduction in its initial value obtained using the baseline parameter. The rest of the results followed similar trend for each of the 50% and 75% reduction. For parameter μ_v, 25% increment in its baseline parameter value amounted to a 25% reduction in the initial value of R_0 while a 50% and 75% increment decreased the initial value of R_0 by 41% and 52% respectively. To bring down the value of R_0 below unity, while it requires at least a 50% reduction in the rate at which mosquitoes obtain blood meal from human (involving parameters *q* and *b*), on the other hand a 75% increment in mosquito mortality rate (μ_v) is required to yield similar result. And so we see that decreasing the value of *q* causes a reasonable decrease in the value of R_0.

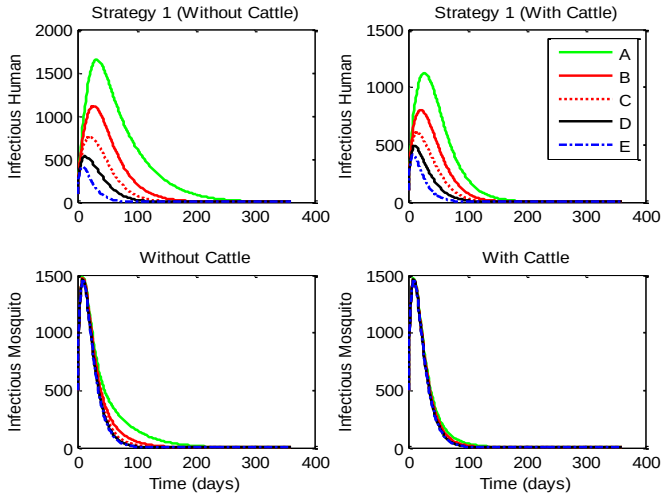

Figure 1: Infectious human (Fig. A & B) and mosquito population (Fig. C & D) with the combination of insecticide treated bed net usage and the treatment of infective human control measures as a strategy (Scenario S1) with (Fig. B & D) and without (Fig. A & C) cattle availability where A = 0%, B = 25%, C = 50%, D = 75% and E = 100% are the different level of control efforts.

It implies that if the proportion of vector blood meals on human is reduced it can result to a reasonable drop in the rate of new infection and this is one key objective in any infectious disease control intervention. As an example, estimate from Table 2 shows that a percent reduction in *q* attracts almost a double reduction in the disease incidence in human (i.e. doubled percent reduction of R_0).

Table 3:
Time (days) taken to reduce the infective mosquito population to less than one (< 1) 'With' and 'Without' cattle availability

Degree of Effort (%)	Control Strategies							
	Strategy 1		**Strategy 2**		**Strategy 3**		**Strategy 4**	
	With	Without	With	Without	With	Without	With	Without
0	215	303	215	303	215	303	215	303
25	172	212	98	115	95	113	87	102
50	131	168	73	85	73	87	63	70
75	131	131	53	58	59	69	50	51
100	131	131	42	42	49	56	42	42

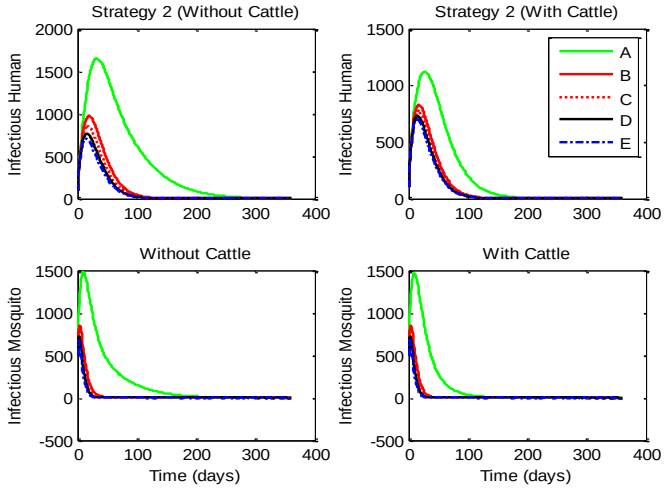

Figure 2: Infectious human (Fig. A & B) and mosquito population (Fig. C & D) with the combination of the treated bed net usage and the use of ivermectin control measures as a strategy (Scenario S2) with (Fig. B & D) and without (Fig. A & C) cattle availability where A = 0%, B = 25%, C = 50%, D = 75% and E = 100% are the different level of control efforts.

To further buttress this findings, the outcome on Figure 5 reveals that for the smallest value of q (q = 0.2) the prevalence of the infection both in the human and mosquito population is at its lowest for all time while for higher values of the parameter the infection prevalence is on the high side through all time and such situation can lead to an outbreak of the disease in the population and so should be avoided. As a result, cattle should be made available in greater proportion for the value of q to be kept low through all time.

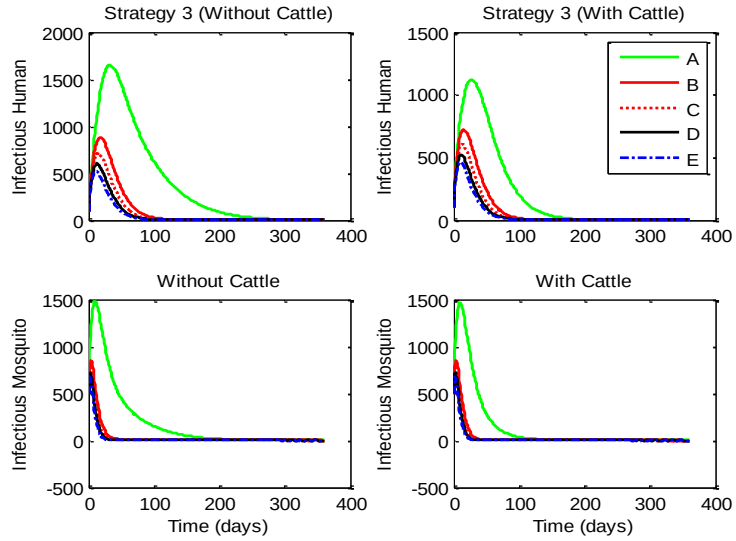

Figure 3: Infectious human (Figs. A & B) and mosquito population (Figs. C & D) with the combination of treatment of infective human and the use of ivermectin control measures as a strategy (Scenario S3) with (Figs. B & D) and without (Figs. A & C) cattle availability where A = 0%, B = 25%, C = 50%, D = 75% and E = 100% are the different level of control efforts.

Considering the outcome on Figures 1-4 and in Tables 3 & 4, the effect of cattle availability for all the strategies is seen to shorten the time taken to bring down the infective population with and without any control applied although at different time rate. On a general note without any control measure applied and no cattle available, from Tables 3 & 4 it shows that while it takes 303 and over 360 days to achieve the set target for the infective mosquito and human population respectively, the presence of cattle fasten the time to 215 days for the infective mosquito and 269 days for the infective human population. Although without any form of control, the presence of cattle act as a control measure of its own but the gap in the time (days) taken to achieve the set target with cattle availability when control is applied at different degrees and when there is no control is huge as captured in Table 5. For just a 25% control effort invested, considering the infective mosquito population we have that apart from strategy 1 which shortened the time taken by 50%, on the average only approximately 0.17% of the time used when there is no control is needed to acquire the desired result for each of the other three strategies as depicted in Table 5. In the case of the infective human population, only 0.44% of the time taken with no control is required for strategy 1 while on the average only approximately 0.14% of the time needed when no control is carried out settles it for each of the remaining strategies. Investing more effort in the control equally reduces the time taken to achieve the set target drastically thereby creating greater gap between when there is control and when there is no control with cattle available.

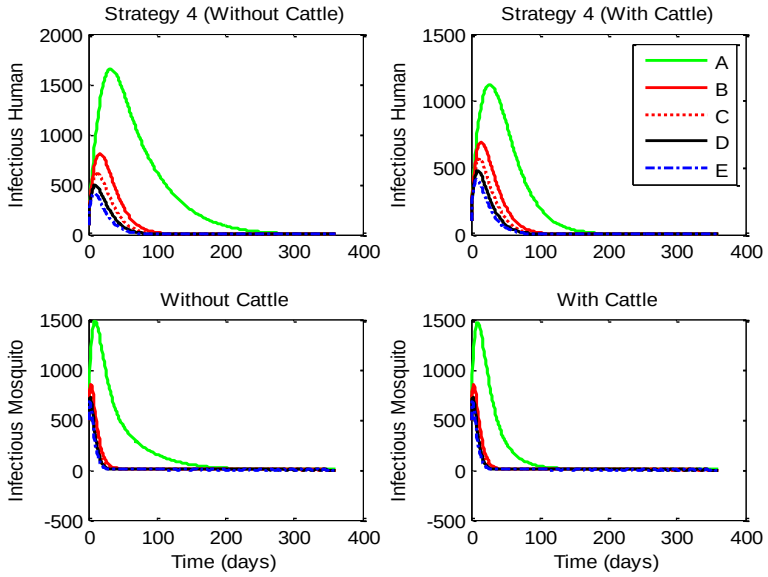

Figure 4: *Infectious human (Figs. A & B) and mosquito population (Figs. C & D) with the combination of all three control measures as a strategy (Scenario S4) with (Figs. B & D) and without (Figs. A & C) cattle availability where A = 0%, B = 25%, C = 50%, D = 75% and E = 100% are the different level of control efforts.*

Table 4:

Time (days) taken to Reduce the infective human population to less than one (< 1) individual 'With' and 'Without' cattle availability.

Degree of Effort (%)	Control Strategies							
	Scenario S1		**Scenario S2**		**Scenario S3**		**Scenario S4**	
	With	Without	With	Without	With	Without	With	Without
0	269	> 360	269	> 360	269	> 360	269	> 360
25	211	258	173	187	159	175	154	165
50	178	201	162	168	143	152	136	141
75	155	166	157	159	133	139	125	127
100	118	118	154	154	126	131	118	118

All of these outcomes are in line with the zooprophylaxis concept whereby the mosquitoes seeking for blood-meal diverts from human to cattle and so bringing down malaria incidence (through R_0) and prevalence of the infection in the population.

Table 5:
The difference in the time (days) taken to reduce the infective human (H) and mosquito (M) population to less than one (< 1) with cattle availability

Degree of Effort (%)	Control Strategies							
	Strategy 1		**Strategy 2**		**Strategy 3**		**Strategy 4**	
	H	M	H	M	H	M	H	M
0	>91	88	>91	88	>91	88	>91	88
25	47	40	14	17	16	18	11	15
50	23	37	6	12	9	14	5	7
75	11	0	2	5	6	10	2	1
100	0	0	0	0	5	7	0	0

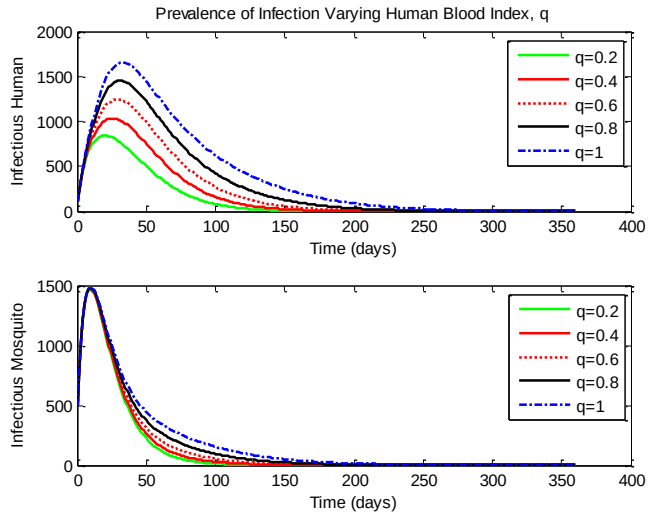

Figure 5: The effect of q (proportion of vector blood meal on human) on the prevalence of the infection in the human and mosquito population.

Our finding on the role of ivermectin in reducing the time taken to bring down the infective populations with the implication of reducing malaria transmission in the population agrees with one of the findings in [10]. In addition, the finding of our study on the role of livestock on malaria transmission also supports an existing finding where animals were used to divert the malaria vector biting from humans [1]. In [6], it was concluded that considering a combination of endectocide (with ivermectin as a member) treated livestock with long lasting insecticide net as a strategy with consistent use of the endectocide can dramatically hasten the goal of attaining local elimination. A similar result is obtained in Table 3 (strategy S2) with such control combination having the next shortest time taken to bring down the infective mosquito population after the strategy that combines all three control measures. The strategy also have such potential among the infective human population too (see Table 4).

5. Conclusion

In this paper, we formulated a malaria model consisting of a system of differential equation and carried out basic analyses that concerns the transmission and control of the disease on it. The model's basic reproduction number was obtained with which it was established that the model is locally asymptotically stable when the number is less than unity. Applying the normalized forward sensitivity index method on the model's basic reproduction number with respect to some parameters of the model it was deduced that the basic reproduction number is highly sensitive to the mosquito biting rate and the proportion of vector blood meal on human with cattle availability. It is such that any percent reduction/increment in any of the two parameters result in an equal percent reduction/increment of the basic reproduction number. The implication therefore is that malaria transmission in the population can be brought down by

reducing the proportion of vector blood meal on human with increase in cattle availability and increase in vector mortality. Applying four different combinations of control tools as strategies at varying degrees of effort, the effect of cattle availability through the parameter that controls the proportion of vector blood-meal on human is further seen to be positive on malaria control even when no measure of control is applied. One of its effect through each of the strategies is that it shortens the time taken to bring down the infective population with and without any control applied although at different time rates. Considering the different control tools applied in this study, significant difference in the outcomes is noticed between the strategies that included the use of ivermectin and those that did not especially for low or average degrees control effort. With the use of ivermectin, the decline of the infective populations is drastic and at a high rate. In summary, firstly the findings of this study suggest that the role of cattle with and without any control intervention is very crucial to the transmission and control of malaria in the population. As a result, cattle should be made more available at all times around the human community in greater proportion so as to provide the vectors with an alternative host for blood meal with the aim of bringing down malaria transmission. Secondly, the use of IVM as a malaria control tool will help to achieve the desired goal within the minimum period of any intervention program. In addition, IVM as earlier mentioned, plays very important role also in fighting some helmiths such as lymphatic filariasis, scabies, onchocersisasis etc. With this added advantage, it is therefore recommended for consideration in intervention programs that is focused on integrated vector management for malaria and even some helmiths control. In conclusion, the presence of livestock (cattle) and the use of ivermectin drug in combination with other control tools have the potential of drastically shortening the time taken to interrupt malaria transmission in the population even with average control effort invested and so poses to be a promising tool for quick elimination of malaria in the population.

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