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# COMPARATIVE APPROACH FOR INCREMENTAL OIL DETERMINATION: IMPLEMENTING TRAPEZOIDAL RULE AND CUBIC SPLINE MODELS

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***Abstract***

***Different approach have been applied to estimate the incremental oil recovered in enhanced oil recovery (EOR) flow systems and these models have challenges and limitations in surfactant and polymer flooding methods in EOR incremental oil determination. Two methods of incremental oil determination were applied in this research: cubic spline and trapezoidal rule models. Cubic spline model involves the formulation numerical model for the surfactant and polymer flooding in the determination of incremental oil from EOR flow system application. To achieve this, experimental data for the surfactant and polymer flooding were introduced into the model formulated. Also, trapezoidal rule model was applied to the surfactant and polymer flooding systems to determine incremental oil from EOR flow system. Performance increase in the incremental oil was recorded in the application of cubic spline compared to trapezoidal rule in the surfactant and polymer flooding EOR flow system. The performance differences between cubic spline and trapezoidal rule are due to the errors of approximation associated with the linearity of the function obtained from the trapezoidal rule. The straight line approximation from the trapezoidal rule introduce errors because the actual nature of the line is not a straight line, but a curve and is best represented by a cubic spline, making the cubic spline model to perform better than the trapezoidal rule model in the surfactant and polymer flooding systems. The polymer flooding act as a better sweeping agent than surfactant flooding in EOR system and this determine the incremental oil recovered at the effluents.***

***Keywords*:** Cubic Spline Model, Trapezoidal Rule Model, Surfactant Flooding, Polymer Flooding, Incremental Oil Recovery, Enhanced Oil Recovery.

## 1.0 Introduction

The success of any enhanced oil recovery (EOR) flow system in a porous environment is measured by the volume of the incremental oil recovered [1, 2]. The incremental oil recovered in a typical successful EOR flow system is the shaded region graphically depicted in Figure 1 [3]. Although several techniques have been designed to determine the incremental oil in any successful EOR project [1-5]

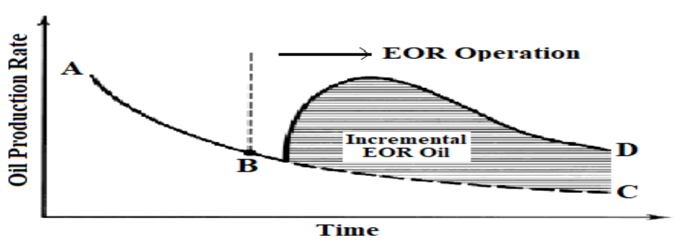


**Figure 1*:*** Incremental Oil Recovery From a typical EOR Response Curve [3]

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The application of enhanced oil recovery (EOR) is considered when primary and secondary recovery methods cannot bring about the expected production performance with the available data indicating oil still in place. The average oil recovery factor is estimated to be 35% which is an indication that more than 60% of the initial oil in place cannot be produced by relying wholly on primary and secondary recovery [6]. Nanoparticles have been used to alter rock wettability and reduce interfacial tension (IFT) so as to increase the oil recovery [7], while the effect of nanofluid and sodium sulphate on surface tension in surfactant solution has also been investigated [8]. Other methods adopted by [9] include chemical flooding like alkaline or polymer flooding, gas injection [10], while cubic spline application in EOR has been investigated and found to give reliable value of the incremental oil [11]. A spline curve adopts a special function of the order of polynomials [12-15]. The polynomial function $f(x)$ involves different intervals representing a specific continuous polynomial having variable x. These polynomials are then joined together at the interval endpoints (knots) in such a way that a certain degree of smoothness of the resulting or composite-function is guaranteed [13]. Cubic spline is a spline constructed of piecewise third-order polynomials which pass through a set of n control data points $(x_1,y_1)$, $(x_2,y_2)$, $(x_n,y_n)$ [11]. As represented in Figure 2, the nodes are the n data points are function of the $x_i$, the nodes ($x_i$ and $y_i$) are the approximations where the contiguous curves meet and it's referred to as the knots of the approximation [14] as graphically depicted in Figure 2 [16] and Figure 3 [17]. The use of cubic spline to this type of composite function to determine the areas of the system involves the use of approximations which is known as the integral function approximations [18].

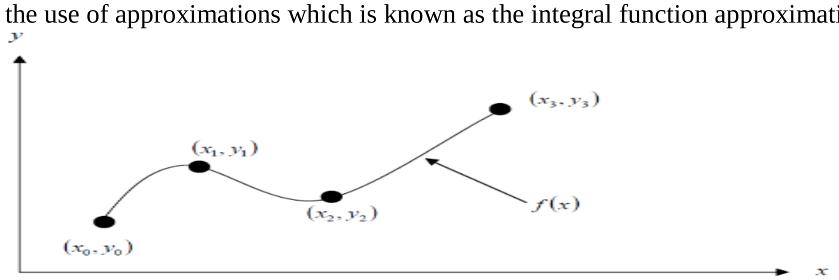


**Figure 2:** Interpolation of Discrete Data [16]

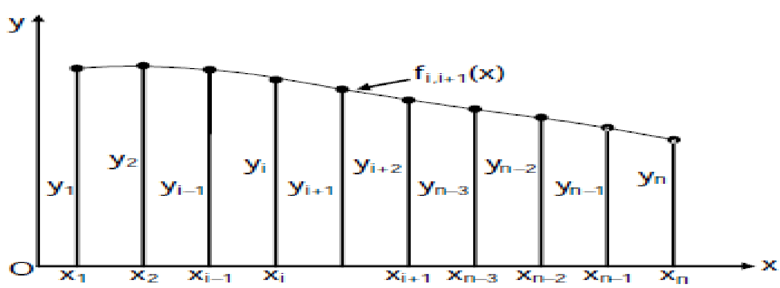


**Figure 3:** Cubic Spline [17]

The trapezoidal rule is a numerical method that approximates the value of a definite integral. We approximate the integral by using n trapezoids formed by using straight line segments between the points $(x_{i-1}, y_{i-1})$ and $(x_i, y_i)$ for $1 \le i \le n$ as shown in the Figure 4 below [19].

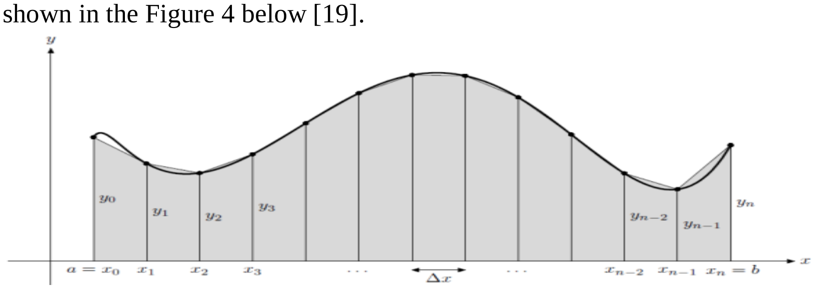


**Figure 4:** Trapezoidal Rule [19]

This study presents the comparative approach to incremental oil determination from polymer and surfactant flooding using cubic spline and trapezoidal models. The main goal of this study is to reveal a means by which better results of the incremental oil recovered in any successful EOR project are obtainable. The incremental oil data obtained from polymer and surfactant flooding were analytically evaluated using trapezoidal and cubic spline models. For the cubic spline model, each curve through the contiguous points is a cubic; the composite curve over the entire interval $x_1$ and $x_n$ must interpolate the data by passing through each knot; the curve itself and the first and second derivatives of the composite curve must be continuous at the nodes $x_i$, conditions must be prescribed at the end points $x_1$ and $x_n$ of the interval, depending on whether the data points indicate that beyond these points the extrapolation curve is required to approach a straight line or a parabola, or exhibit some other behaviors such as periodicity over the interval $x_1 \leq x \leq x_n$ For the trapezoidal rule: each line segment is a trapezium spanning through an entire curve forming a bunch of trapezia. Each of these methods is used to estimate the incremental oil recovery from polymer and surfactant flooding recovery mechanisms.

## 2.0 Model Formulation and Estimation

Trapezoidal rule is an already established numerical integration formula which involves the use of the integral under a straight line to approximate the integral under a curve. While a spline represent a curve having functions in polynomial order. The models formulations for both the cubic spline and the trapezoidal rule are presented below.

### 2.1 The Cubic Spline Model

$$\boldsymbol{f_{i,i+1}(x)} = \frac{ki}{6}\left[\frac{(x-x_{i+1})^3}{xi-x_{i+1}} - (x-x_{i+1})(x_i-x_{i+1})\right] - \frac{ki+1}{6}\left[\frac{(x-x_{i+1})^3}{xi-x_{i+1}} - (x-x_i)(x_i-x_{i+1})\right] + \frac{yi(x+x_{i+1})-y_{i+1}(x-xi)}{x_i-x_{i+1}} \quad (1)$$

$$\boldsymbol{f_{i,+1}(t)} = \frac{ki}{6}\left[\frac{(t-t)^3}{ti-t_{i+1}} - (t-t_{i+1})(t_i-t_{i+1})\right] - \frac{ki}{6}\left[\frac{(t-t)^3}{ti-t_{i+1}} - (t-t_{i+1})(t_i-x_{i+1})\right] + +q_i\frac{(t-t_{i+1})-q_{i+1}(x-ti)}{ti-t_{i+1}} \quad (2)$$

Equations (1) and (2) denotes the cubic polynomial that spans the segment between the knots $i \; and \; (i+1)$.
While the equations for the curvatures are:

$$k_{i-1}(x_{i-1}-x_i) + 2k_i(x_{i-1}-x_{i-1}) + k_{i-1}(x_i-x_{i+1}) = 6\left[\frac{y_{i-1}-y_i}{x_{i-1}-x_i} - \frac{y_i-y_{i+1}}{x_i-x_{i+1}}\right] i = 2,3,\dots\ h - I \quad (3)$$

*"OUR"Algorithm* "OUR" Algorithm is an extremely efficient solution method to tridiagonal system of equations whose formulations are as follows:

$$-A_iK_{i-1} + B_iK_i - C_iK_{i+1} = D_i \quad i = 2,3,,n-1 \quad (4)$$

Equation (4) is the general form of the set of linear simultaneous equations formed by Equation (7), Where $A_i$, $B_i$ and $C_i$are the coefficients of the second derivatives at the knots and $D_i$ is the right hand side of the equation. Setting $\propto(2) = B(2)$ and

$$S(2) = D(2) \quad (5)$$

From $I = 3 \; to \; M$, Where $M$ is the last interior position vector. We can write:

$$\propto(I) = B(I) - \frac{A(I)}{\propto(I-1)} \times C(I-1) \quad (6)$$

And

$$S(I) = D(I) + \frac{A(I)}{\propto(I-1)} \times S(I-1) \quad (7)$$

For the second derivatives $k_i$, at the last interior knot, we have:

$$K(M) = \frac{S(M)}{\propto(M)} \quad (8)$$

Each of the items in the above expression has already been simplified. The incremental oil recovery is determined by applying the following equation:

$$The\ incremental\ oil = N_p(a) - N_p(b) - \int_a^b f(x)dx \quad (9)$$

Where $N_p(a)$ is the cumulative oil produced in cubic meter ($m^3$) at the lower limit, $N_p(b)$ is the cumulative oil produced in ($m^3$) at the upper limit, $a$ is the lower limit in minutes ($mins$), $b$ is the upper limit ($mins$) and $f(x)$ is the equation of the projected decline.

## 2.2 The Trapezoidal Rule

The trapezoidal rule was also applied to formulate model for the surfactant and polymer flooding flow systems in the determination of incremental oil recovery in EOR project as presented in equations (10) and (11).

$$I = \frac{h}{2}[X + 2I] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$I = \frac{h}{2}[y_o + 2(y_1 + y_2 + y_3 \pm - - - - y_n) + y_n] \ldots\ldots\ldots\ldots\ldots \quad (11)$$

Where I = sum of the intermediate points, X = sum of the ends points, $I$ = area under the graph.

## 3.0 Results and Discussion

The results presented in Tables 1 and 2 were obtained from the enhanced oil recovery processes with polymer and surfactant flooding. It is seen in the results that the effluent oil increased as the period of flooding increased, however, starts decreasing when the maximum recoverable volume is obtained.

**Table 1:** Laboratory Data for Polymer Flooding

| Time (min) | Effluent Oil ($m^3$) | Effluent Water/Polymer ($m^3$) | Total Effluent ($m^3$) | Polymer Injected | Cumulative Oil Recovered % OIIP |
|---|---|---|---|---|---|
| 4:00 | 0.00000217 | 0.00000763 | 0.0000098 | 2.572800 | 69.333 |
| 8:00 | 0.00000278 | 0.00000787 | 0.00001065 | 2.798900 | 70.133 |
| 12:00 | 0.00000255 | 0.00000867 | 0.00001122 | 3.025100 | 70.933 |
| 16:00 | 0.00000261 | 0.00000611 | 0.00000872 | 3.246200 | 78.400 |
| 20:00 | 0.00000163 | 0.00000655 | 0.00000818 | 3.454700 | 83.200 |
| 24:00 | 0.00000405 | 0.00000856 | 0.00001261 | 3.680900 | 84.266 |
| 28:00 | 0.00000244 | 0.00000754 | 0.00000998 | 3.896900 | 84.800 |
| 32.00 | 0.00000201 | 0.00000623 | 0.00000824 | 4.113400 | 86.264 |
| **Total** | **0.00002024** | **0.00005916** | **0.0000794** | **26.7889** | **627.329** |

**Table 2:** Laboratory Data for Surfactant Flooding

| Time (min) | Effluent Oil ($m^3$) | Effluent Water/Surfactant ($m^3$) | Total Effluent ($m^3$) | Surfactant Injected | Cumulative Oil Recovered % OIIP |
|---|---|---|---|---|---|
| 4:00 | 0.00000657 | 0.00000785 | 0.00001442 | 1.690955 | 63.3677 |
| 8:00 | 0.00000999 | 0.00000814 | 0.00001813 | 1.919598 | 66.1133 |
| 12:00 | 0.00000568 | 0.00000821 | 0.00001389 | 2.140704 | 67.6733 |
| 16:00 | 0.00000416 | 0.00000816 | 0.00001232 | 2.356784 | 68.7708 |
| 20.00 | 0.00000401 | 0.0000079 | 0.00001191 | 2.43719 | 68.9016 |
| 24.00 | 0.00000398 | 0.00000804 | 0.00001202 | 2.49832 | 69.4172 |
| 28.00 | 0.00000382 | 0.00000798 | 0.0000118 | 2.56710 | 70.8712 |
| 32.00 | 0.00000321 | 0.0000073 | 0.00001051 | 2.70126 | 72.1834 |
| **Total** | **0.00004142** | **0.00006358** | **0.000105** | **18.311911** | **547.2985** |

The results in Figure 5 represent the incremental oil obtained when trapezoidal rule and cubic spline models are applied in polymer and surfactant flooding in enhanced oil recovery process. The difference in the results obtained is due to the graphical assumptions implmented in trapezoidal rule in the formulation of the model in the determination of the incremental, because the trrapezoidal rule model does not captured the apprioprate fitting of the curve obtained. However, the cublic spline model was able to describe the curve as obtained and this resulted in the higher incremental oil shown in Figure 5. This simply indicate that the incremental oil recovered by the trapezoidal rule is always less than the actual volume in cubic meters because errors are introduced in the process of approximating the curve to a straight line segments and consequently does not represent the actual volume of the incremental oil in the EOR system. The use of cubic spline model in the determination of the incremental oil in enhanced oil recovery process is better described and determined when compared to trapezoidal rule model.

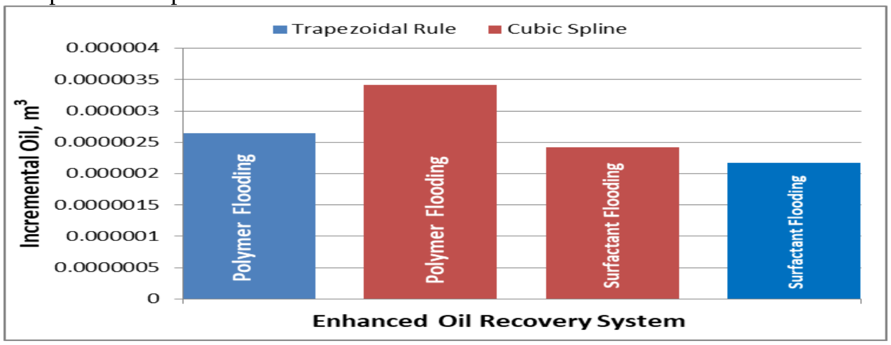


**Figure 5:** Comparison Between Cubic Spline Model and Trapezoidal Rule

## 4.0 Conclusion

The comparism made between the trapezoidal rule and the cubic spline models in the determination of incremental oil at the effluents of an enhanced oil recovery system in the course of this research work gave rise to the following conclusion:

1. The cubic spline model gives a more reliable value of the incremental oil when compareed to trapezoidal rule model in enhanced oil recovery system.
2. The trapezoidal rule model has more accumulated errors in the system than the cubic spline model and this reduces the incremental oil recovered at the effluent for trapezoidal rule.
3. The polymer flooding act as a more efficient sweep material than the surfactant flooding process in both the cubic spline and the trapezoidal rule models.
4. The incremental oil recovered is influenced by the initial oil in place before the commencement of the EOR process and also by the flooding process adopted in the EOR system.

## 5.0 Acknowledgment

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