

## DERIVATION OF HILLSLOPE LEAKAGE-DEPENDENT EQUATIONS MODELING GROUNDWATER FLOW IN THREE-AQUIFER SYSTEM WITHIN A SEDIMENTARY BASIN

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### Abstract

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*This paper derived hillslope leakage-dependent equations that can be used to study groundwater flow dynamics in leaky sloping three-aquifer system within a sedimentary basin. This was done by the introduction of leakage term(s) and Darcy flux in Boussinesq context into the general groundwater continuity flow equations in each of the aquifers within the three-aquifer system. The derived equations have been shown to be capable of modeling groundwater flow not only in sloping three-aquifer system but also in horizontal three-aquifer system, where the slope angle is zero. These equations, when solved and used for simulations of groundwater flow in three-aquifer system, will be very useful to hydrogeologists in studying the leakage properties of aquifer-aquitard system which is very crucial in estimating long time yields of aquifers within a three-aquifer system. Also, extension of Boussinesq equation into three-aquifer system will help geoscientists to conduct detailed studies and have better understanding of the groundwater flow dynamics in such geological structures.*

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**Keywords:** Hillslope, Leakage Groundwater, three-aquifer, flow, Aquifer, Boussinesq

### 1.0 Introduction

Groundwater has been said to be a vital resource for many sectors and human consumption in the society around the world. Its management, protection and assessment require quantitative knowledge of flow in aquifers [1]. Groundwater hydrology began as a quantitative science when Darcy [2] published a report on the water supply of Dijon, France [3]. Boussinesq derived the general equation to study groundwater flow dynamics within unconfined horizontal aquifers as well as in uniformly sloping (hillslope) aquifers [4, 5]. Since these derivations, different forms of Boussinesq's equations for subsurface flow on a sloping base (or hillslope flow) have been studied extensively. For instance, [6] laid the mathematical foundations for hillslope flow. Applications and analysis of the Boussinesq's equations were done by [7] and [8]. Also, [9, 10, 11, 12] demonstrated the application of fractional partial differential eqs (fPDEs) to groundwater flow problems. Although the results of various applications of Boussinesq's equations have been remarkably accurate yet the assumption of zero flow across the confining layers, while appropriate for many applications, has serious limitations. These limitations are more pronounced at larger scales when the interactions between different components of a system (like flow between a soil mantle and a confined aquifer) play a greater role [13]. But a number of recent field studies, complemented by a handful of modeling studies, have examined the importance of leakage in a hillslope context [7, 13, 14, 15, 16, 17]. [18] and [19] added a leakage term to a version of Boussinesq equation. [19] modified the hillslope storage Boussinesq (hsB) model to include leakage term in a two-aquifer system and used it to explore the sensitivity of the resulted model to a range of constant and variable leakage rate. But the use of horizontal lower confined aquifer in the two-aquifer system limited its

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application. [20] derived groundwater flow equation in layered, anisotropic aquifer but the result is not applicable to aquifer system consisting aquifers separated by aquitards. However, little or nothing is known about the applications of Boussinesq equation (now generally known as hillslope model) to groundwater flow in aquifers within multi-aquifer system. Also, leakages through the aquitards separating aquifers in a three-aquifer system, in Boussinesq context, have not been adequately studied. Since most of the sedimentary basins of the world are multi-aquifer in nature, such geological structures are important long-term storage reservoirs especially in semi-arid and arid countries where it is usually the only perennial water resource [21]. Hence, accurate knowledge of leakages and groundwater flow within multi-aquifer systems are important in determining the water sustainability in such environments. This paper will, therefore, formulate equations that will be leakage-dependent for groundwater flow in aquifers within a three-aquifer as a representative part of the multi-aquifer system. In achieving this, sect 2 of this paper deals with the general processes of deriving the hillslope leakage-dependent equations. Assumptions used in the conceptualization of the equations as well as how leakage terms and Darcy flux in Boussinesq concept are introduced into the general groundwater continuity flow equations for the three-aquifer system is also explained. Sect 3 discusses the benefits and advantages of the derived hillslope leakage-dependent groundwater flow equations over other existing groundwater flow equations. Sect 4 of this paper summarizes the results of the derived equations in sloping three-aquifer system and its equivalent in horizontal three-aquifer system.

**2 Formulation of hillslope leakage-dependent (HL-D) groundwater flow equations**

**2.1 Model conceptualization**

The three-aquifer system, which will be used to represent multi-aquifer system, consists of an upper unconfined aquifer on top of two confined aquifers separated by two aquitards and the base of the lowest aquifer is an aquiclude. As shown in Fig 1, the three aquifers are all within a geological formation and each aquifer and its confining layer (aquitard) are of uniform thickness  $\bar{D}_n$  and  $\bar{d}_n$  respectively. The aquifers are at an angle  $\phi$  to the horizontal and have hillslope length, L.  $L_1$  and  $L_2$  are the leakage terms through the aquitard 1 to/from the middle-confined aquifer to/from the upper unconfined aquifer and leakage through aquitard 2 to/from lower confined aquifer to/from middle confined aquifer respectively. This is in contrast to the existing single hillslope unconfined aquifer resting on an impermeable layer, with no-flow base, at slope angle  $\phi$  to the horizontal as shown in Fig 2.

In formulating the governing equations for groundwater flow in each of the aquifers that made up a leaky three-aquifer system, the following assumptions were made,

1. Each of the aquifers within the three-aquifer system has semi-infinite area extent.
2. The flow through the aquifers is essentially along the hillslope while the flow through the aquitard is mainly in vertical direction.
3. The aquifers as well as the aquitards are saturated.
4. Each of the aquifers within the three-aquifer system has uniform hydraulic conductivity,  $k_n$  along the hillslope.

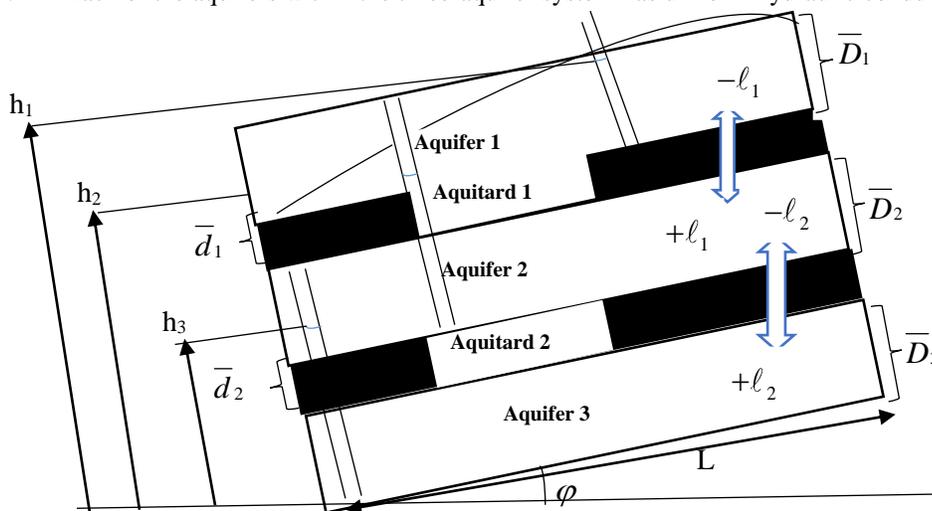


Figure 1: A cross-section of a sloping three-aquifer system of hillslope length, L at an angle  $\phi$  to the horizontal in aquifer 1, aquifer 2 and aquifer 3.

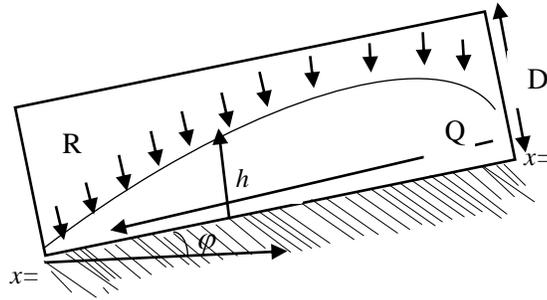


Figure 2: A cross-section of hillslope unconfined aquifer resting on an impermeable layer at an angle  $\phi$  to the horizontal.  $R$  is the vertical recharge of the aquifer from the unsaturated zone above the aquifer.

Assumption (2) above is meant to relax the no-flow boundary condition imposed on the original Boussinesq equation to give way to a more realistic leakage-dependent Hillslope equations. The Darcy groundwater velocity,  $q$  through a single sloping aquifer (hereafter called hillslope aquifer) has been derived by [4] as,

$$q = -k \left( \frac{\partial h}{\partial x} + \sin \phi \right) \tag{1}$$

For more than one aquifer within a sedimentary basin,  $q$  for  $n$ th aquifer can be written as,

$$q_n = -k_n \left( \frac{\partial h_n}{\partial x} + \sin \phi \right) \tag{2}$$

where  $n=1,2,3$  for a three-aquifer system while  $k_n$  and  $h_n$  are the hydraulic conductivity and the hydraulic head in  $n$ th aquifer respectively. The volumetric flux,  $Q$  flowing through a cross-sectional area,  $A_n$  of  $n$ th aquifer can be obtained as,

$$Q_n = q_n A_n \tag{3}$$

The modified form of general mass balance equation in upper unconfined aquifer of width  $w(x)$  and without leakage given by [13] can be written as,

$$fw \frac{\partial h_1}{\partial t} = - \frac{\partial Q_1}{\partial x} + Nw \tag{4}$$

where  $f$  is the drainable porosity of the aquifer (dimensionless),  $w$  is the aquifer's width ( $m$ ),  $Q_1$  is the Darcy flux through the hillslope aquifer 1 ( $m^3/day$ ) and  $N$  is the net vertical flow ( $m/day$ ). Minus sign in eq (4) shows that head decreases in the direction of flow. Its equivalent form in confined aquifer can be written as,

$$S_{t(n)}w \frac{\partial h_n}{\partial t} = - \frac{\partial Q}{\partial x} + Nw \tag{5}$$

where  $S_{t(n)}$  is the storativity in  $n$ th confined aquifer (dimensionless),  $h_n$  is the groundwater head measured perpendicular to the underlying impermeable layer in  $n$ th confined aquifer. For middle aquifer,  $n=2$  while  $n=3$  for lower confined aquifer.

### 2.2 Introduction of leakage term(s)

The leakage-dependent groundwater flow equation in aquifer 1 can be written as,

$$fw \frac{\partial h_{1,x}}{\partial t} = - \frac{\partial Q_{1,x}}{\partial x} + R w(x) - L_1 w \tag{6}$$

where  $h_{1,x}$  is the groundwater head measured perpendicular to the underlying impermeable layer in aquifer 1 at point  $x$  on the hillslope length,  $L_1$  represents all possible forms of leakages (both upwards and downwards) through aquitard 1,  $R$  is the vertical recharge into aquifer 1 from the unsaturated zone above aquifer 1. Modified form of Darcy-type leakage through aquitard 1,  $L_1$  was given by [19] as,

$$L_1 = -k_{v1} \frac{h_2 - h_1}{d_1} \tag{7}$$

where  $h_1$  and  $h_2$  are the groundwater heads measured perpendicular to the underlying impermeable layer in aquifer 1 and 2 respectively,  $k_{v1}$  is the vertical hydraulic conductivity of aquitard 1,  $d_1$  is the thickness of aquitard 1. The general groundwater flow equation for aquifer 2 with leakage terms can then be written as,

$$S_{t,2}w \frac{\partial h_{2,x}}{\partial t} = - \frac{\partial Q_{2,x}}{\partial x} + L_1 w - L_2 w \tag{8}$$

where  $S_{t,2}$  is storativity of aquifer 2,  $Q_{2,x}$  is the Darcy flux through the hillslope aquifer 2 ( $m^3/day$ ) at point  $x$  on the horizontal axis,  $L_2$  represents all possible forms of leakages (upwards and/or downwards) through aquitard 2.

$$L_2 = -k_{v2} \left( \frac{h_3 - h_2}{d_2} \right) \quad (9)$$

where  $d_2$  is the thickness of aquitard 2,  $h_3$  is the groundwater heads measured perpendicular to the underlying impermeable layer in aquifer 3. Also, the leakage-dependent groundwater flow equation in aquifer 3 can be written as,

$$S_{t,3} w \frac{\partial h_{3,x}}{\partial t} = -\frac{\partial Q_{3,x}}{\partial x} + L_2 w(x) \quad (10)$$

where  $S_{t,3}$  is storativity of aquifer 3,  $Q_{3,x}$  is the Darcy flux through the hillslope aquifer 3 ( $m^3/day$ ) at point  $x$  on the hillslope.

Leakage term  $L_1$  is subtracted from the horizontal flow term in eq (6) because this ensured that a downwards leakage (leakage away from the aquifer), which is positive, was deducted from the total water flowing through aquifer 1 giving the net water flowing through the aquifer 1. But if there is an upwards leakage (leakage into aquifer 1 from the middle confined aquifer) the negative in front of  $L_1$  will turn the leakage term (which is originally negative due to upwards leakage) into positive, thereby, adding the leaking water from the middle confined aquifer to the water flowing in aquifer 1 to give the water balance flowing in aquifer 1. Therefore, eq (6) is the correct expression for groundwater flowing through aquifer 1 with due consideration of both downwards and upwards leakages. Similarly, since no-flow boundary condition has been imposed on the base of aquifer 3, the only means of leakage is through aquitard 2 above it. Addition of leakage term,  $L_2$  to the groundwater flowing through aquifer 3 in eq (10) ensured that downwards leakages, which are positive, would be added to the volume of water flowing horizontally through aquifer 3 to give the net water flowing through aquifer 3. But if it is upwards leakage, the negative leakages that result automatically deduct the quantity of water leaking from the water flowing through aquifer 3 to give the net water flowing through aquifer 3 after leakage. Therefore, eq (10) is the correct expression for groundwater balance flowing through aquifer 3 after due consideration of both downwards and upwards leakages along the hillslope. Also, due consideration has been given to the fact that the direction of leakages may vary from point to point along the hillslope. The distance along the hillslope, which was at angle  $\phi$  to the horizontal, was taking as the  $x$ -direction. Cross-sectional area which the groundwater flow through in aquifer 1 is,

$$A_1 = h_1 w \quad (11)$$

Substituting eqs (2) and (11) into eq (3), it becomes:

$$Q_1 = -k_1 h_1 w \left( \cos \phi \frac{\partial h_1}{\partial x} \sin \phi \right) \quad (12)$$

Substituting eqs (7) and (12) into eq (6) and dividing through by  $fw$ , it becomes,

$$\begin{aligned} fw \frac{\partial h_1}{\partial t} &= -\frac{\partial}{\partial x} \left[ -k_1 h_1 w (\cos \phi) \frac{\partial h_1}{\partial x} + \sin \phi \right] + R w + k_{v1} \left( \frac{h_2 - h_1}{d_1} \right) w \\ \frac{\partial h_1}{\partial t} &= \frac{k_1 \cos \phi}{f} \frac{\partial}{\partial x} \left( h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_1 \sin \phi}{f} \frac{\partial h_1}{\partial x} + \frac{R}{f} + k_{v1} \left( \frac{h_2 - h_1}{fd_1} \right) \end{aligned} \quad (13)$$

For a perfectly horizontal aquifer,  $\phi = 0$ ,  $\cos 0 = 1$  and  $\sin 0 = 0$ . Then eq (13) will reduce to,

$$\frac{\partial h_1}{\partial t} = \frac{k_1}{f} \frac{\partial}{\partial x} \left( h_1 \frac{\partial h_1}{\partial x} \right) + \frac{R}{f} + \frac{k_{v1} (h_2 - h_1)}{fd_1} \quad (14)$$

In aquifer 2,  $n=2$  and the cross-sectional area through which groundwater flow in aquifer 2 is,

$$A_2 = D_2 w \quad (15)$$

eq (2) becomes,

$$q_2 = -k_2 \left( \cos \phi \frac{\partial h_2}{\partial x} + \sin \phi \right) \quad (16)$$

Substituting eqs (15) and (16) into eq (3), it becomes,

$$Q_2 = -k_2 D_2 w \left( \cos \phi \frac{\partial h_2}{\partial x} + \sin \phi \right) \quad (17)$$

Also, substituting eqs (7), (9) and (17) into eq (8) and dividing through by  $S_{t2}w$ , we will obtain,

$$\frac{\partial h_2}{\partial t} = \frac{T_2 \cos \phi}{S_{t2}} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v1}}{S_{t2}} \left( \frac{h_2 - h_1}{d_1} \right) + \frac{k_{v2}}{S_{t2}} \left( \frac{h_3 - h_2}{d_2} \right) \quad (18)$$

Also, for a perfectly horizontal aquifer 2 with leakages,  $\phi = 0$  and  $\cos 0 = 1$ . Then eq (18) reduces to,

$$\frac{\partial h_2}{\partial t} = \frac{T_2}{S_{t2}} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v1}}{S_{t2}} \left( \frac{h_2 - h_1}{d_1} \right) + \frac{k_{v2}}{S_{t2}} \left( \frac{h_3 - h_2}{d_2} \right) \quad (19)$$

where  $T = k_2 \bar{D}_2$  is the transmissivity in aquifer 2 and  $\bar{D}_2$  is the average thickness of aquifer 2. Also, since  $\sin \phi$  is a not functions of time and  $\bar{D}_2$  is a constant, then their derivatives with respect to  $x$  is zero.

Following the same procedures as above, leakage-dependent hillslope groundwater flow equation in aquifer 3 can be obtained as,

$$\frac{\partial h_3}{\partial t} = \frac{T_3 \cos \phi}{S_{t_3}} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v2}}{S_{t_3}} \left( \frac{h_3 - h_2}{d_2} \right) \tag{20}$$

where  $T_3 = k_3 \bar{D}_3$  is the transmissivity in aquifer 3.

Also, for a perfectly horizontal aquifer 3 with leakages,  $\phi = 0$  and  $\cos 0 = 1$ . Then eq (20) reduces to,

$$\frac{\partial h_3}{\partial t} = \frac{T_3}{S_{t_3}} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v2}}{S_{t_3}} \left( \frac{h_3 - h_2}{d_2} \right) \tag{21}$$

**3. Discussion**

Equation (13), (18) and (20) are the newly derived hillslope leakage-dependent (HL-D) groundwater flow equations in a uniform width homogeneous and isotropic sloping upper unconfined aquifer (aquifer 1), middle semi-confined aquifer (aquifer 2) and lower semi-confined aquifer (aquifer 3) respectively found within the same sedimentary basin. Each of them will be needed for the computation of hydraulic/piezometric heads at different points along the simple hillslope for their respective isotropic and homogeneous aquifer within the three-aquifer. Also, eqs (14), (19) and (21) are the leakage-dependent groundwater flow equations in a homogeneous and isotropic horizontal upper unconfined aquifer (aquifer 1), middle semi-confined aquifer (aquifer 2) and lower semi-confined aquifer (aquifer 3) respectively found within the same sedimentary basin. This shows that the derived hillslope leakage-dependent (HL-D) groundwater flow equations can also handle groundwater flow in horizontal three-aquifer system, where  $\phi = 0$ . These equations, when applied to sloping or perfectly horizontal three-aquifer system and the resulted equations are solved either analytically or numerically, will be very useful to hydrogeologists in studying the leakage properties of aquifer-aquitard system. A good knowledge of leakage properties of aquifer-aquitard system, which can be obtained from the derived HL-D equations, is very crucial in estimating long time yields of aquifers within any multi-aquifer system. Also, the derived HL-D equations will prove to be very useful tools for geoscientists in conducting detailed studies of the groundwater flow dynamics in sedimentary basins containing three-aquifer system.

**4. Conclusion**

New hillslope leakage-dependent, HL-D equations that model groundwater flow in sloping three-aquifer system have been derived. These equations have been shown within the paper and in table 1 not only to be capable of modeling groundwater flow in sloping three-aquifer system but also groundwater flow in perfectly horizontal three-aquifer system, where the slope angle is zero, within a sedimentary basin. These equations, when solved and used for simulations of groundwater flow in three-aquifer system, will be very useful to hydrogeologists in studying the leakage properties of aquifer-aquitard system which is very crucial in estimating long time yields of aquifers within a three-aquifer system. Also, extension of Boussinesq equation into three-aquifer system will help geoscientists to study in detail and have a better understanding of the groundwater flow dynamics in such geological structures.

**Table 1: Summary of Newly Derived HL-D Groundwater Equations for sloping Three-aquifer System and the Resultant L-D Groundwater flow Equations when  $\phi = 0$  (Perfectly horizontal Three-aquifer System)**

Aquifer Types	Groundwater Equations	Remark
Sloping Three-aquifer System	$\frac{\partial h_1}{\partial t} = \frac{k_1 \cos \phi}{f} \frac{\partial}{\partial x} \left( h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_1 \sin \phi}{f} \frac{\partial h_1}{\partial x} + \frac{R}{f} + k_{v1} \left( \frac{h_2 - h_1}{fd_1} \right)$	Newly Derived HL-D Groundwater Flow Equations
	$\frac{\partial h_2}{\partial t} = \frac{T_2 \cos \phi}{S_{t_2}} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v1}}{S_{t_2}} \left( \frac{h_2 - h_1}{d_1} \right) + \frac{k_{v2}}{S_{t_2}} \left( \frac{h_3 - h_2}{d_2} \right)$	
	$\frac{\partial h_3}{\partial t} = \frac{T_3 \cos \phi}{S_{t_3}} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v2}}{S_{t_3}} \left( \frac{h_3 - h_2}{d_2} \right)$	
Perfectly horizontal Three-aquifer System	$\frac{\partial h_1}{\partial t} = \frac{k_1}{f} \frac{\partial}{\partial x} \left( h_1 \frac{\partial h_1}{\partial x} \right) + \frac{R}{f} + \frac{k_{v1}}{fd_1} (h_2 - h_1)$	L-D(leakage-dependent) Groundwater Flow Equations
	$\frac{\partial h_2}{\partial t} = \frac{T_2}{S_{t_2}} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v1}}{S_{t_2}} \left( \frac{h_2 - h_1}{d_1} \right) + \frac{k_{v2}}{S_{t_2}} \left( \frac{h_3 - h_2}{d_2} \right)$	
	$\frac{\partial h_3}{\partial t} = \frac{T_3}{S_{t_3}} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v2}}{S_{t_3}} \left( \frac{h_3 - h_2}{d_2} \right)$	

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