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# ON INVENTORY MODEL FOR DETERIORATING ITEMS, EXPONENTIAL DECREASING DEMAND AND TIME VARYING HOLDING COST UNDER PARTIAL BACKLOGGING WITH SHORTAGES

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***Abstract***

***In this work, we propose an inventory model for deteriorating items, with exponential decreasing demand and time-varying holding cost. Shortages is allowed and demand partially backlogged. The proposed model is solved analytically. A numerical example is used to illustrate the proposed model.***

***Keywords***: Inventory model, exponential demand, time-varying holding cost, partial backlogging shortages.

## Introduction

Deterioration is one factor to consider when developing inventory models as goods do not always preserve their characteristics while stored for future use. For this reason, researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, blood, medicine, electronic components, fruits and vegetables. Whitin [1] studied deterioration of the storage period especially for fashion goods industry. Dave and Patel [2] were the first to study a deteriorating inventory model with linear increasing demand when shortages are not allowed.

Chang and Dye [3] proposed an inventory model with time-varying demand and partial backlogging. Ouyang and Cheng [4] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Dye *et al*., [5] an inventory model that finds the optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging was proposed. They assumed that the fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases.

Liona [6] gave an economic order quantity (EOQ) model with non-instantaneous receipts and exponential deteriorating item under two levels of credit. Mirzazadeh [7] developed an inventory model for deteriorating items where shortages are partially backlogged under variable inflation and demand with the cost of deterioration considered. Kaur and Sharma [8] studied deteriorating items with price and time dependent demand rate.

In this paper therefore, we extend the work of Mishra *et al*., [9] by considering a time dependent inventory model with exponentially increasing demand and a partially backlogged with shortages. We assumed that the fraction of customers who backlog their orders decreases exponentially as the waiting time for the next replenishment decreases.

## Notations

$h(t)$: Holding cost per unit time

$C_1$ : The unit cost of an item

$C_2$ : Shortage cost per unit time

$S_1$ : The Lost sale cost per unit

$A_1$ :Ordering Cost

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**Assumptions of the Model**

1. Deterioration rate is time proportional $\theta(t) = \theta t$, where $\theta$ is the rate of deterioration, $0 < \theta < 1$
2. The demand rate D at any time is given $D(t) = e^{\alpha t}$; $\alpha > 0$ and is constant
3. The backlogging rate is given by $e^{-\delta t}$; $\delta$ is a constant $0 < \delta \leq 1$
4. Replenishment is Instantaneous, lead time is zero
5. Holding cost $h(t)$ per unit time is time dependent and is assumed $h(t) = h + at$, where $a > 0; h > 0$
6. There is no repair or replenishment of deteriorating item during the period under consideration
7. $I_o$ is the maximum Inventory level at $t = 0$

**Mathematical Analysis and Formulations ot the Model**

The initial inventory level is $I_o$ unit at time $t = 0$; from $t = 0$

$t = t_1$, the Inventory level reduces owing to both demand and deterioration, until it reaches zero level at time $t = t_1$. At this time, Shortage is accumulated which is partially backlogged at the rate $e^{\delta t}$. At the end of the Cycle, the Inventory reaches a maximum shortage level so as to clear the backlogged and again raises the Inventory level to $I_o$ as shown in the graph below

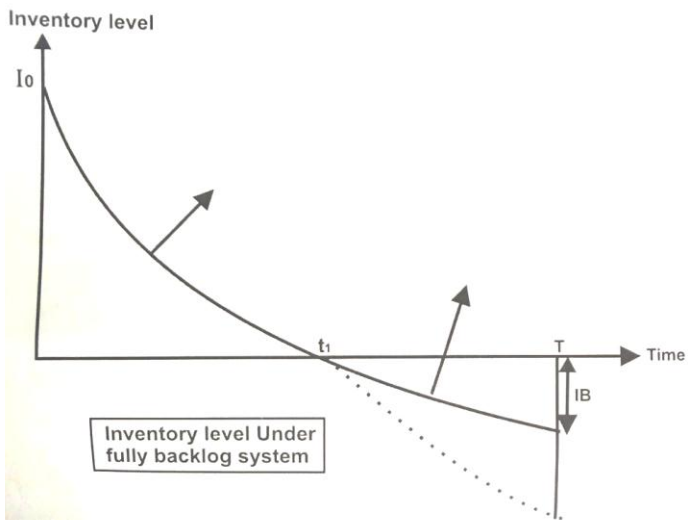


Figure 1: Graphical representation of inventory system

**Mathematical Formulation of the Proposed Model**

The rate of change of the Inventory during the positive Stock period $(0, t_1)$ and shortage period $(t_1, T)$ is governed by the following differential equation

$$\frac{dI_1(t)}{dt} = -D(t) - \theta(t)I_1(t), \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -e^{(\alpha-\delta)t}, \quad t_1 \leq t \leq T \tag{2}$$

Thus the boundary conditions are as follows

$I_1(0) = I_o$

$I_1(t_1) = 0$

$I_2(t_1) = 0$

The solutions of equation (1) and (2) with boundary conditions are as follows

$$I_1(t) = \frac{1}{\alpha + \theta}\left[e^{-\theta t} - e^{(\alpha+\theta)t_1 - \theta t}\right] \tag{3}$$

$$I_2(t) = \frac{1}{\alpha-\delta}\left[e^{(\alpha-\delta)t_1} - e^{(\alpha-\delta)T}\right] \tag{4}$$

From equation (3), we have

$$I_1(t) = \frac{1}{\alpha+\theta}\left[e^{-\theta t} - e^{(\alpha+\theta)t_1-\theta}\right] \tag{5}$$

at $t = 0$ we have in equation (5) as follows

$$I_1(0) = I_o$$

$$I_o = \frac{1}{\alpha+\theta}\left[1 - e^{(\alpha+\theta)t_1}\right] \tag{6}$$

Notice that inventory is available in the system during the time interval $(0, t_1)$. Hence, the cost of holding inventory in stock is computed for time period $(0, t_1)$.

We therefore compute holding cost as follows

$$HC = \int_o^{t_1} h(t) I_1(t) \tag{7}$$

$$HC = \int_o^{t_1} (h + at) I_1(t) \tag{8}$$

$$HC = \int_o^{t_1} (h + at)\left[\frac{1}{\alpha+\theta}\left(e^{-\theta t} - e^{(\alpha+\theta)t_1-\theta t}\right)\right]dt \tag{9}$$

$$HC = \frac{1}{\theta}\left[h - h\,e^{-\theta t_1} - at_1\ e^{-\theta t_1} + h\,e^{\alpha t_1} + at_1\ e^{\alpha t_1} - h\,e^{\alpha t} e^{\theta t_1}\right]$$

$$+\frac{1}{\theta^2}\left[1 - ae^{-\theta t_1} + ae^{\alpha t_1} - ae^{\alpha t_1} e^{-\theta t_1}\right] \tag{10}$$

Shortage cost due to Stock Out is accumulated in the system during the interval $(t_1, T)$.

Therefore the total shortage cost during this time period is given as

$$SC = C_2 \int_{t_1}^{T} -I_2(t)dt \tag{11}$$

$$SC = \frac{C_2}{\alpha-\delta}(T - t_1)\left[e^{(\alpha-\delta)T} - e^{(\alpha-\delta)t_1}\right] \tag{12}$$

Due to stock out during $(t_1, T)$ shortage is accumulated, but not all customers are willing to wait for the next replenishment, hence this result in loss of profit due to lost sales.

Loss sale cost is calculated as follow

$$LSC = S_1 \int_{t_1}^{T} \left(1 - e^{-\delta t}\right)\left(e^{\alpha t}\right)dt \tag{13}$$

$$LSC = \frac{S_1}{\alpha}\left[\frac{1}{\alpha}\left(e^{\alpha T} - e^{\alpha t_1}\right) + \frac{S_1}{\alpha-\delta}\left(e^{(\alpha-\delta)t_1} - e^{(\alpha-\delta)T}\right)\right] \tag{14}$$

Purchase Cost is as follows

$$PC = C_1\left(I_o + \int_{t_1}^{T} e^{\delta t}\ D(t)dt\right) \tag{15}$$

$$PC = C_1 + \frac{C_1}{\alpha-\delta}\left[e^{(\alpha-\delta)T} - e^{(\alpha-\delta)t_1}\right] \tag{16}$$

The total inventory cost for the system is given by

TC = OC + PC + HC + SC + LSC

$$Tc\ = A_1 + C_1 + \frac{C_1 e^{(\alpha-\delta)T}}{\alpha-\delta} - \frac{C_1 e^{(\alpha-\delta)t_1}}{\alpha-\delta} + \frac{1}{\theta}\left[h - he^{-\theta t_1} - at_1 e^{-\theta t_1} - at_1 e^{-\theta t_1} + h\,e^{\alpha t_1} + at_1 e^{\alpha t_1} - he^{\alpha t} e^{\theta t_1}\right] +$$

$$+\frac{1}{\theta^2}\left[1 - ae^{-\theta t_1} + ae^{\theta t_1} - ae^{\alpha t_1} e^{\theta t_1}\right] + \frac{C_2(T - t_1)}{\alpha-\delta}\left[e^{(\alpha-\delta)T} - e^{(\alpha-\delta)t_1}\right]$$

$$+\frac{S_1}{\alpha}\left[e^{\alpha T}-e^{\alpha t_1}\right]+\frac{S_1}{\alpha-\delta}\left[e^{(\alpha-\delta)t_1}-e^{(\alpha-\delta)T}\right] \tag{17}$$

To minimize the total cost of inventory TC per unit time, we obtain the optimal values of $t_1$ and T by resolving the following equations:

$$\frac{\partial TC}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TC}{\partial T}=0 \tag{18}$$

Provided that the following conditions $\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2>0$ and $\left(\frac{\partial^2 TC}{\partial t_1^2}\right)>0$ are satisfied by equations (17)

**Numerical illustration of the model**

The following parameters in proper units are used as inputs for the numerical illustration of our model. They are the same example as used in Mishra *et al* [9]. $A_1=2{,}500, \alpha=0.6, a=20, \delta=0.7, C_2=30, C_1=10, C_2=4, S_1=8$

The output using Mathematical (7.0) for the optimal times when the inventory reaches zero $t_1$, the time when maximum shortage occur T and the optimal value of total cost TC are $t_1=1.027$, $T=1.374$ and TC = 3749.71 respectively.

The inventory total cost here is slightly higher than that of Mishra *et al*., [9]. This is probably due to our assumption of exponentially decreasing partial backlogging; hence fewer customers will wait for the replenishment of inventory in the system.

**Conclusion**

This paper attempt to develop an inventory model for goods that follow constant deterioration with time and time varying holding cost. This is an extended work of Mishra *et al*., [9], but with the assumption that demand and partial backlogging are exponential. Demand is assumed to be exponential, as it is more realistic than other demand patterns especially for new attractive products introduced into the market. In this work, we provided analytic solution to the developed model and illustrated with an example.

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