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STEADY HYDRODYNAMIC VISCOUS FLOW OVER A WEDGE

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Abstract

Analysis is carried out to study the two dimensional steady flow over a wedge. Furthermore, using a similarity variable, the governing flow equations are transformed to a nonlinear coupled differential equation which is solved numerically. A comparison of the solution technique is carried out in previous work and the results are found to be in good agreement. Numerical results for the shear stress and skin friction are presented for different physical parameters. The analysis of the obtained result shows the flow is significantly influenced by these parameters.

Keywords: Hydrodynamics, inviscid, shear stress, slip flow, boundary layer, Blasius, Falkner-Skan, similarity solution

1.0 Introduction

The wide application of fluid dynamics makes it one of the most vital and fundamental of all engineering and applied scientific studies. The flow of fluids in pipes and channels is of importance to civil engineers. The study of fluid machinery such as pumps, compressors, heat exchangers, jet and rocket engines and the like, makes fluid mechanics of importance to mechanical engineers. The flow of air over objects; aerodynamics, is of fundamental interest to aeronautical and space engineers in the design of aircraft, missiles and rockets. In meteorology, hydrology and oceanography, the study of fluids is basic since the atmosphere and the ocean are studied under the field continuum mechanics.

Fluid dynamics is also applicable to the evaluation of mass flux of mineral oils like petroleum through pipelines, prediction of weather patterns, investigation of nebulae in interstellar space, human physiology, food processing, chemical engineering and so on. The importance of fluid flow in the technological enhancement has prompted many researchers to investigate and study the subject. The Pioneer research work in this area can be traced to [1] who presented a theoretical result for the boundary layer flow over a flat plate in a uniform stream and on a circular cylinder. Later, numerical and analytical results to the classical Blasius problem were reported. The similarity solution for thermal boundary layer over a flat plate subject to convective surface boundary conditions was studied in [2]. He found out that the similarity solution for a constant convective coefficient of heat transfer does not exist. Instead, the solutions could be used as local similarity solutions. Ishak [3], expanded [2] with no modification to the flow equations but boundary conditions affected by suction and injection. He found that the surface shear stress increased due to suction, which increased the rate of heat transfer at the surface. The effects of chemical reaction on MHD flow with suction and injection was investigated in [4]. The flow equations were solved analytically using perturbation method. The team found that as the Grashof number increased so did velocity distribution. Particularly, temperature increases as the surface convection is increased for a wall that stretches and shrinks [5]. All of the aforementioned research work considered no-slip at the boundary. At the interface between the solid and fluid, there is zero velocity. This is known as the condition of no-slip. However, at very low pressure or for a flow system that is small, slip flow occurs. In micro electro mechanical system (MEMS), the region of imbalance at the interface between the solid and fluid is more accurately described using a slip flow model. There exists an exact solution to a modified micro-channel couette flow [6]. Thermal effects on the boundary layer flow over a flat plate was studied in [7]. The solutions were obtained numerically. The results obtained showed that as the slip increased so did the stream function. In summary for slip flows, as the shear stress at the wall decreases so did the slip velocity, this happens when the slip parameter is increased. In the review thus far, we have considered constant fluid properties. Experimental studies indicate that the thermophysical properties of fluids changes for large temperature variation: while the temperature increases, the transport phenomenon increases. As this happens, the physical properties across the thermal boundary layer is reduced.

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This consequently affects the heat transfer at the wall. Therefore in order to accurately predict the flow and heat transfer rates, it is imperative that the fluid properties are not taken to be constant all through the flow rather they should be seen as variables. Asogwa[8] studied the numerical solution of hydromagnetic flow past an infinite vertical porous plate in presence of constant suction and heat sink. He found out that the suction parameter, heat sink parameter, Hartmann number and permeability parameter significantly affected the flow. Temperature fields are only affected by the reduction in Reynolds number [9]. The effects of viscosity dissipation on a viscoelastic flow over a stretching surface has been investigated in [10]. It was found that the viscosity dissipation is largely dependent on the Brinkmann's constant of the material. In [11], the influence of viscosity dependent on temperature, on a hydrodynamic flow was studied. It was studied over a surface, moving continuously. The results were obtained numerically using Runge-Kutta method to the fourth order. The results obtained revealed that as the variable viscosity parameter decreases the temperature is increased. And the velocity increases as the variable viscosity parameter decreases. In [12], the effects of injection and suction on a boundary layer was considered. The results were obtained numerically using a method similar to the method used in [13]. It was reported that, in the boundary layer the rate at which heat was transferred is greatly govern by the suction and injection parameters. Thermal radiation on a MHD flow over a flat plate of uniform heat source and sink has been investigated in [14]. It was observed that the Nusselt number increases for pertinent flow parameters. In [15], the effects of blowing and suction on a thermal boundary layer was studied. An analytical solution was obtained using perturbation method, after which, graphical representations of pertinent fluid parameters were presented. Heat and mass transfer on a MHD flow over a semi-infinite flat plate in a porous medium was studied in [16]. The results obtained indicated that the concentration of the buoyancy effects was enhanced by the increase in Grashof number. The influence of the heat absorption on the system was that which reduced the temperature of the fluid; this resulted in decrease in the velocity of the fluid. The Prandtl number is directly proportional to the fluid viscosity, it is again inversely proportional to the thermal conductivity of the fluid by definition. With thickness of the boundary layer depending on the Prandtl number, so, as the viscosity and the thermal conductivity of the fluid vary with temperature, in the same way does the Prandtl number. The Prandtl number has to be a variable [17, 18]. In addition, significant errors occur for regimens that use constant Prandtl number and other temperature dependent fluid parameters. In [19, 17], the locally similar equations were solved using a technique put forward in [20]. The results obtained showed that the thermal conductivity parameter increases, as the velocity and temperature increased. The effects of thermal radiation and thermophoresis on a hydromagnetic flow over a flat plate was investigated in [21]. It was reported that increasing Prandtl number decreased the thermal boundary layer. Similarly, the influence of viscous dissipation and internally decaying heat generation on a MHD flow over a flat plate was investigated in [22]. It was found that increasing the Prandtl number decreases both the rate of heat transfer and the temperature at the surface of the plate. Analyzing numerically the chemical reaction on a MHD flow past a flat plate, it was found that the Nusselt number decreased as a result of the increase in slip flow [23]. In [24], the effects of Navier slip together with Newtonian heating on a hydro magnetic flow was considered. The numerical solution of the resulting flow equations was obtained. He found that as the flow became more unsteady and Newtonian heating increased at the boundary layer, the slip parameter significantly affected the skin friction and transfer rate of heat. The thermophysical properties on a MHD flow was analyzed in [25]. The study was carried out over a flat plate. The equations were solved numerically using symbolic software Mathematica 8.0. They reported that the coefficient of skin friction and local Nusselt number as well as the velocity and temperature profiles were affected by varying values of the fluid flow. In the 1930s a generalization of the Blasius boundary layer flow was developed by V. Falkner and S. Skan named "Falkner - Skan (F-S) equation". It describes the steady two dimensional laminar boundary layer flow on a wedge. Studies of these fluid models have been extended to include the flow of non-Newtonian fluids such as power-law fluids and nanofluids. The two dimensional steady flow over a wedge is investigated. The flow equations are solved numerically, having been transformed using a suitable similarity variable. Graphical and tabular representations of important hydrodynamic features of the flow are presented.

2.0 Formulation of the problem

2.1 Analysis of Flow

The steady two-dimensional laminar boundary layer that forms on a wedge is a generalization of the Blasius boundary layer. It can be modeled as an external flow with pressure gradient given by the inviscid flow solution.

2. 2 Governing Equations

The angle of the wedge is given as $\beta\pi$. Figure 1 shows the co-ordinate system and the flow configuration.

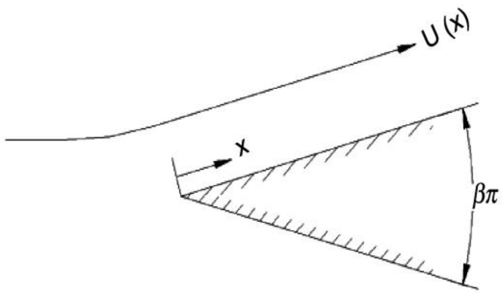

Figure 1 Boundary layer flow over a wedge (*source:* [26]***)***

The external flow velocity and pressure gradients are given by;

$$U(x) = u_0\left(\frac{x}{L}\right)^m \tag{2.1}$$

$$\frac{\partial P}{\partial x} = -\rho U(x)\frac{\partial U}{\partial x} \tag{2.2}$$

Where U is the external velocity, P is the pressure, ρ is the density, x is the position along the wedge, L is the characteristic length. The coefficient u_0 is a function of the flow geometry and the exponent m is a function of the angle β:

$$m = \frac{\beta}{2-\beta} \tag{2.3}$$

The boundary layer equations are:
Equation of Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.4}$$

Equation of Motion:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p(x)}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2.5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{m u_0^2}{x}\left(\frac{x}{L}\right)^{2m} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2.6}$$

Where. y = Cartesian coordinates.

2.3 BOUNDARY CONDITIONS

$$u = v = 0 \qquad at\ \ y = 0 \tag{2.7a}$$

$$u = U(x) \qquad as\ \ y \to \infty \tag{2.7b}$$

2.4 INTRODUCTION TO DIMENSIONLESS FORM

The stream function $\psi(x, y)$ is introduced thus;

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{2.8}$$

$$\Rightarrow \left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial^2 \psi}{\partial x \partial y} \\ \frac{\partial v}{\partial y} &= -\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial^2 \psi}{\partial x \partial y} \\ \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial^2 \psi}{\partial y^2} \\ \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)\right) = \frac{\partial^2 \psi}{\partial x \partial y} \end{aligned}\right\} \tag{2.9}$$

Now for equation (2.8), the continuity equation (2.4) is satisfied automatically. Using equations (2.8) and (2.9), equation (2.6) take the form;

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{m{u_0}^2}{x}\left(\frac{x}{L}\right)^{2m}+\nu\frac{\partial^2\psi}{\partial x\partial y} \qquad (2.10)$$

the velocity components of equation (2.7) equally become:

$$\frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial x}=0 \qquad at\quad y=0 \qquad (2.11)$$

$$\frac{\partial\psi}{\partial y}\to U(x) \qquad at\quad y\to\infty \qquad (2.12)$$

Next, the dimensionless variable for ψ is introduced following [26] as:

$$\psi=\sqrt{\frac{2\nu u_0 L}{m+1}}\left(\frac{x}{L}\right)^{\frac{m+1}{2}} f(\eta) \qquad (2.13)$$

$$\eta=y\sqrt{\frac{(m+1)u_0}{2\nu L}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}} \qquad (2.14)$$

$$\beta=\frac{2m}{m+1} \qquad (2.15)$$

Where η is the similarity variable.

We dimensionalize thus;

$$\frac{\partial\eta}{\partial x}==\frac{(m-1)\eta}{2x}$$

$$\frac{\partial\eta}{\partial y}=\sqrt{\frac{(m+1)u_0}{2\nu L}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}}$$

$$\frac{\partial\psi}{\partial x}==\frac{1}{2x}\sqrt{\frac{2\nu u_0 L}{m+1}}\left(\frac{x}{L}\right)^{\frac{m+1}{2}}\left[(m-1)\eta f'+(m+1)f\right]$$

$$\frac{\partial\psi}{\partial y}=u_0\left(\frac{x}{L}\right)^{m} f'$$

Recall that

$$u=\frac{\partial\psi}{\partial y} \qquad v=-\frac{\partial\psi}{\partial x}$$

$(the\ continuity\ equation\ is\ satisfied)$

$$\Rightarrow \qquad u=u_0\left(\frac{x}{L}\right)^{m} f'$$

$$v=-\frac{1}{2x}\sqrt{\frac{2\nu u_0 L}{m+1}}\left(\frac{x}{L}\right)^{\frac{m+1}{2}}\left[(m-1)\eta f'+(m+1)f\right]$$

$$\frac{\partial u}{\partial x}=\frac{u_0}{x}\left(\frac{x}{L}\right)^{m}\left[m f'+\frac{(m-1)}{2}\eta f''\right]$$

$$\frac{\partial u}{\partial y}=\sqrt{\frac{(m+1)u_0}{2\nu L}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}}u_0\left(\frac{x}{L}\right)^{m} f''$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{(m+1){u_0}^2}{2\nu L}\left(\frac{x}{L}\right)^{2m-1}$$

Substituting in (2.6)

$$f'''+f''f+\beta\left(1-f'^2\right)=0 \qquad (2.16)$$

2.5 DIMENSIONLESS BOUNDARY CONDITIONS

$$u = v = 0 \quad u = u_0\left(\frac{x}{L}\right)^m f' = 0 \Rightarrow f'(0) = 0 \tag{2.17a}$$

$$i.e.\ (m-1)\eta f'(0) + (m+1)f(0) = f(0) = 0 \tag{2.17b}$$

$$u = u_\rho(x)$$

$$\Rightarrow u_0\left(\frac{x}{L}\right)^m f'(\infty) = u_0\left(\frac{x}{L}\right)^m \Rightarrow f'(\infty) = 1 \tag{2.17c}$$

The slip flow boundary conditions are given thus

$$u = L\frac{\partial u}{\partial y}, \qquad v = 0$$

$$u_0\left(\frac{x}{L}\right)^m f'(0) = L\sqrt{\frac{(m+1)u_0}{2\nu L}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}} u_0\left(\frac{x}{L}\right)^m f''(0)$$

$$\Rightarrow f'(0) = N f''(0) \tag{2.18a}$$

$$i.e.\ (m-1)\eta f'(0) + (m+1)f(0) = f(0) = 0 \tag{2.18b}$$

$$u = U(x)$$

$$\Rightarrow u_0\left(\frac{x}{L}\right)^m f'(\infty) = u_0\left(\frac{x}{L}\right)^m \Rightarrow f'(\infty) = 1 \tag{2.18c}$$

$$N = \sqrt{\frac{L(m+1)u_0}{2\nu}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}} \tag{2.19}$$

Where $N = \sqrt{\frac{L(m+1)u_0}{2\nu}}\left(\frac{x}{L}\right)^{\frac{m-1}{2}}$ is the slip parameter.

2.6 SKIN FRICTION COEFFICIENT

The quantity of engineering interest is the coefficient of skin friction which measures the rate of shear stress which we will thus proceed to obtain. The local skin friction coefficient C_f, is given by the formula:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U} \tag{2.20}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \textit{is the local wall shear stress}$$

3.0 Numerical Solutions

Having gotten the dimensionless governing equations; equation (2.16) subject to the boundary conditions contained in section 2.5, analytical solutions are sought. However, since the equation (2.16) is non- linear, the equation cannot be solved analytically. Thus a numerical solution is to be found. Notice that equation (2.16) forms a two-point boundary value problem (BVP) which can be solved using shooting method. But also notice that the boundary condition at the other end (i.e. the final boundary) is at infinity. We need a finite value for $\eta \to \infty$, let that finite value be η_∞. Thus the numerical solution of equation (2.16) subject to the boundary conditions contained in section 2.5 is obtained using symbolic software Mathematica 8.0.

We proceed to confirm the validity of our model. To do this, we set the fluid properties to be constants and compare with known literature, the present model coincides with that of [26].

To actually test our reduced model for accuracy, a comparison of the numerical solutions for various values of the exponent mare calculated for $f''(0)$ and presented on table 1.

Table 1 Values of $f''(0)$ for various values of m

M	Mohammad and Mohammad (2013)	Present results
0	0.4696	0.4696
1/11	0.6549	0.654994
0.2	0.8021	0.802126
1/3	0.9276	0.92768
0.5	1.0389	1.0389
1	1.2325	1.15118

From Table 1 we see that the data produced by the present code and those of [26] show excellent agreement hence justifies the use of the present numerical technique.

3.1 Results and Discussion

Numerical solutions have been carried out to analyze the results the method described in the previous section for various values of m and the slip parameter, N. \

3.2 Computational Results for Velocity Profiles

The non-dimensional velocity profiles within the boundary layer for both slip and no-slip flows corresponding to various values of wedge angle parameter, m are presented in the following figures. Figure 2 displays the effects of wedge angle parameter and slip parameter on the dimensionless velocity. It is found that the dimensionless velocity increases with wedge angle parameter for slip flow. The effect on the dimensionless velocity of increasing the wedge angle parameter is not significant, we can say that the wedge angle parameter needs to be small as possible for there to be a significant effect on the velocity. It is also found that the fluid velocity within the boundary layer increases with the increase in the slip parameter, and as a result, thickness of the momentum boundary decreases.

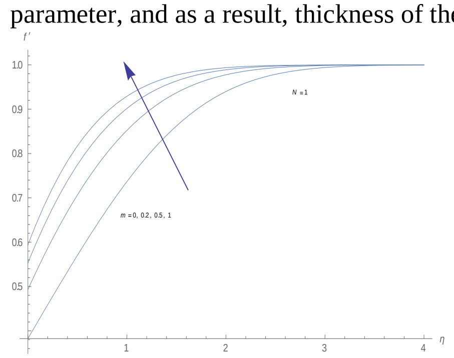

Figure 2 Velocity profiles for various values of wedge angle and fixed slip parameter

Figure 3 expresses the effects of the slip parameter on the dimensionless velocity, for a fixed wedge. The dimensionless velocity increases for higher slip flow conditions, though the effect is not very significant. It is found that the velocity profile increases for both slip and no slip flow.

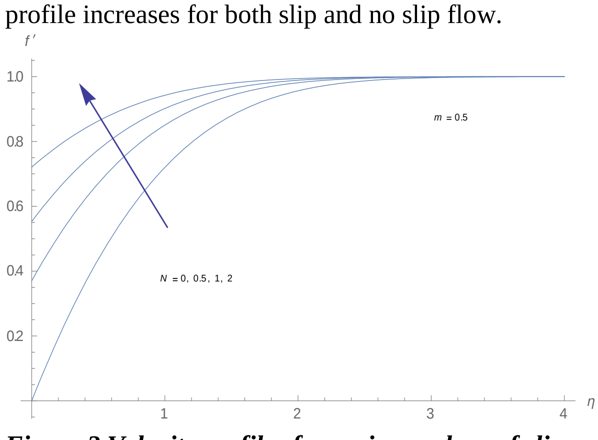

Figure 3 Velocity profiles for various values of slip and fixed wedge angle parameter

The parameter of engineering interest is the skin friction coefficient. It indicates the rate of physical wall shear stress. The value of f''(0) is proportional to skin friction and is presented in table 2.

Table 2 Values of f''(0) and* f'(0) *for different values of m and N

Parameters			
m	N	f'(0)	f''(0)
0.2	0.1	0.0767407	0.767407
	0.4	0.264234	0.660586
	0.7	0.39737	0.567672
	1	0.492995	0.492995
0.5	0.1	0.0972083	0.972083
	0.4	0.31641	0.791024
	0.7	0.457747	0.653924
	1	0.55311	0.55311
0.8	0.1	0.108361	1.08361
	0.4	0.34292	0.857299
	0.7	0.487064	0.695806
	1	0.581473	0.581473
1.5	0.1	0.121876	1.21876
	0.4	0.373338	0.933345
	0.7	0.519601	0.742287
	1	0.612313	0.612313

From table 2, we see that the skin friction decreases with increase in N. But the reverse effect is observed as m increases.

4.0 CONCLUSION

The two dimensional, steady, incompressible flow over a wedge was analyzed incorporating a slip and no-slip boundary condition. A Newtonian fluid was the type of fluid studied in this work. Using a similarity variable, the governing flow equations were transferred to a boundary value problem. The resulting ordinary differential equations were solved using Mathematica software and the results were compared with published papers. These comparisons showed that our results are with good precision. From the present study we can make the following conclusions:

1. The velocity profiles are increasing for increasing values of the wedge angle parameter a, for both slip and no slip flow.
2. The skin friction coefficient decreases with increase of slip parameter but the effect is reversed for wedge angle parameter.

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