Transactions of the Nigerian Association of Mathematical Physics
***Volume 18, (January – December, 2022 Issue*), pp109 –116**

EFFECTS OF BUOYANCY FORCE ON A HYDRODYNAMIC FLOW OVER A FLAT PLATE IN THE PRESENCE OF CHEMICAL REACTION AND NANOPARTICLES

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Abstract

The free convective heat transfer of an incompressible, viscous, electrically conducting fluid past a vertical impermeable flat plate under containing nanoparticles have been analyzed. Furthermore, using a similarity variable, the governing flow equations are transformed to non-linear coupled differential equations corresponding to a two point boundary value problem, which is solved using symbolic software Mathematica 8.0. A comparison of the solution technique is carried out with previous work and the results are found to be in good agreement. Numerical results for the coefficient of skin friction, local Nusselt number, Sherwood number, as well as the velocity, temperature and nanoparticles concentration profiles are presented for different physical parameters. The analysis of the obtained results show that the field of flow is significantly influenced by these parameters.

Keywords: Mass transfer, heat transfer, nanoparticle, nanofluid, Nusselt number, Sherwood number. impermeable surface, slip flow, free convection, buoyancy, self-similar, similarity solution

1.0 Introduction

The transfer of heat from a warmer body to a colder body with chemical reaction is of most importance in the areas of science and engineering. Application exists in the chemical industry, power and cooling industry for dying, evaporation and energy transfer. The interaction between fluid and body for which the effect of viscosity is significant was presented in [1]. He presented a theoretical result for the boundary layer flow over a flat plate in a uniform stream and on a circular cylinder. Later, numerical and analytical results to the classical Blasius problem studied in [1] were reported. The similarity solution for thermal boundary layer over a flat plate subject to convective surface boundary conditions was studied in [2]. He found out that the similarity solution for a constant convective coefficient of heat transfer does not exist. Instead, the solutions could be used as local similarity solutions. Ishak [3], further expanded [2] with no modification to the flow equations but investigated a system affected by suction and injection. He found that the surface shear stress increased due to suction, which increased the rate of heat transfer at the surface. The effects of chemical reaction on MHD flow with suction and injection was investigated in [4]. The flow equations were solved analytically using perturbation method. The team found that as the Grashof number increased so did velocity distribution. Particularly, temperature increases as the surface convection is increased for a wall that stretches and shrinks [5]. All of the aforementioned research work considered no-slip at the boundary. At the interface between the solid and fluid, there is zero velocity. This is known as the condition of no-slip. However, at very low pressure or for a flow system that is small, slip flow occurs. In micro electro mechanical system (MEMS), the region of imbalance at the interface between the solid and fluid is more accurately described using a slip flow model. There exists an exact solution to a modified micro-channel couette flow [6]. Thermal effects on the boundary layer flow over a flat plate was studied in [7]. The solutions were obtained numerically. The results obtained showed that as the slip increased so did the stream function. In summary for slip flows, as the shear stress at the wall decreases so did the slip velocity, this happens when the slip parameter is increased. In the review thus far, we have considered constant fluid properties. Experimental studies indicate that the thermophysical properties of fluids changes for large temperature variation: while the temperature increases, the transport phenomenon increases. As this happens, the physical properties across the thermal boundary layer is reduced. This consequently affects the heat transfer at the wall. Therefore in order to accurately predict the flow and heat transfer rates, it is imperative that the fluid properties are not taken to be constant all through the flow rather they should be seen as variables. Asogwa[8] studied the numerical solution of hydromagnetic flow

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past an infinite vertical porous plate in presence of constant suction and heat sink. He found out that the suction parameter, heat sink parameter, Hartmann number and permeability parameter significantly affected the flow. Temperature fields are only affected by the reduction in Reynolds number [9]. The effects of viscosity dissipation on a viscoelastic flow over a stretching surface has been investigated in [10]. It was found that the viscosity dissipation is largely dependent on the Brinkmann's constant of the material. In [11], the influence of viscosity dependent on temperature, on a hydrodynamic flow was studied. It was studied over a surface, moving continuously. The results were obtained numerically using Runge-Kutta method to the fourth order. The results obtained revealed that as the variable viscosity parameter decreases the temperature is increased. And the velocity increases as the variable viscosity parameter decreases. In [12], the effects of injection and suction on a boundary layer was considered. The results were obtained numerically using a method similar to the method used in [13]. It was reported that, in the boundary layer the rate at which heat was transferred is greatly govern by the suction and injection parameters. Thermal radiation on a MHD flow over a flat plate of uniform heat source and sink has been investigated in [14]. It was observed that the Nusselt number increases for pertinent flow parameters. In [15], the effects of blowing and suction on a thermal boundary layer was studied. An analytical solution was obtained using perturbation method, after which, graphical representations of pertinent fluid parameters were presented. Heat and mass transfer on a MHD flow over a semi-infinite flat plate in a porous medium was studied in [16]. The results obtained indicated that the concentration of the buoyancy effects was enhanced by the increase in Grashof number. The influence of the heat absorption on the system was that which reduced the temperature of the fluid; this resulted in decrease in the velocity of the fluid. The Prandtl number is directly proportional to the fluid viscosity, it is again inversely proportional to the thermal conductivity of the fluid by definition. With thickness of the boundary layer depending on the Prandtl number, so, as the viscosity and the thermal conductivity of the fluid vary with temperature, in the same way does the Prandtl number. The Prandtl number has to be a variable [17, 18]. In addition, significant errors occur for regimens that use constant Prandtl number and other temperature dependent fluid parameters. In [19, 17], the locally similar equations were solved using a technique put forward in [20]. The results obtained showed that the thermal conductivity parameter increases, as the velocity and temperature increased. The effects of thermal radiation and thermophoresis on a hydromagnetic flow over a flat plate was investigated in [21]. It was reported that increasing Prandtl number decreased the thermal boundary layer. Similarly, the influence of viscous dissipation and internally decaying heat generation on a MHD flow over a flat plate was investigated in [22]. It was found that increasing the Prandtl number decreases both the rate of heat transfer and the temperature at the surface of the plate. Analyzing numerically the chemical reaction on a MHD flow past a flat plate, it was found that the Nusselt number decreased as a result of the increase in slip flow [23]. In [24], the effects of Navier slip together with Newtonian heating on a hydro magnetic flow was considered. The numerical solution of the resulting flow equations was obtained. He found that as the flow became more unsteady and Newtonian heating increased at the boundary layer, the slip parameter significantly affected the skin friction and transfer rate of heat. A nanoparticles fluid suspension is known as nanofluid, obtained by dispersing nanometer sized particles in a conventional base fluid like water, oil, ethylene glycol etc. Nanoparticles are expected to increase the thermal conductivity and consequently the attributes of heat transfer of the base fluid [25]. An extensive analysis into convective transport in nanofluids was presented in [26]. He presented that enhancement in heat transfer was as a decrease in viscosity within the boundary layer necessitated by the introduction of nanoparticles in the fluid. In an international nanofluid property benchmark exercise (INPBE) conducted by 34 organizations participating across the world. A unanimous agreement indicating that thermal conductivity was enhanced consistently across research conducted by participants. The DuFour effects on nanofluids was discussed in [27]. They reported the influence of DuFour effects on Nusselt number for varying fluid parameters. The two dimensional steady flow in the presence of chemical reaction is investigated. The flow equations are solved numerically, having been transformed using a suitable similarity variable. Graphical and tabular representations of important hydrodynamic features of the flow are presented.

2.0 Formulation of the problem

2.1 Analysis of Flow

Consider the boundary layer flow of a steady, two-dimensional, electrically conducting, viscous and incompressible fluid is the presence of nanoparticles that is chemically absorbed. The nanoparticle suspension is assumed to be stable and dilute, i.e the nanoparticle concentration is less than 1%. The fluid of temperature T_∞, nanoparticle concentration C_∞ and uniform velocity U_∞ moves along a vertical impermeable flat plate that is heated.

2. 2 Governing Equations

Take x-axis along the plate to be the direction of flow and y-axis perpendicular to the x-axis. Figure 1 shows the co-ordinate system and the flow configuration.

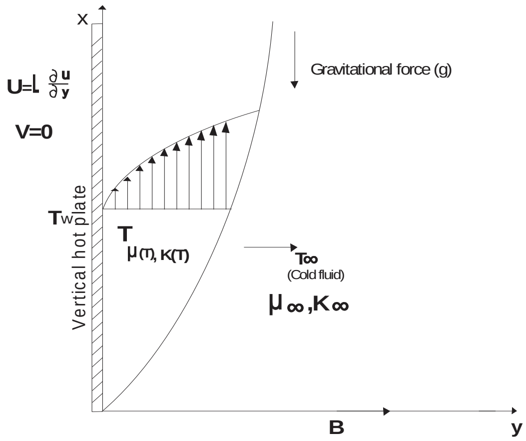

Figure 1 Sketch of flow geometry (*source:*[13]***)***

At the upper surface of the plate, cold fluid flows at temperature T_∞, nanoparticle concentration C_∞ and uniform velocity U_∞, while heat is transferred through convection to the lower surface of the plate causing the temperature to be $T_w(T_w > T_\infty)$. The fluid near the wall transfers heat and mass between the wall and the fluid farther out at coefficients h_f and h_s respectively. The rate of chemical absorption of nanoparticles K_c is taken into account.

The effect of a magnetic field, viscous dissipation, pressure gradient and Joule heating effects in comparison with the effects of heat source/sink are neglected. The flow is due to buoyancy forces necessitated by the density gradients created by temperature differences and nanoparticles distribution in the medium.

As a result of the aforementioned conditions and assumptions, the governing flow equations subject to the Boussineq's approximations are given by:

Equation of continuity :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \qquad (2.1)$$

Equation of motion :

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty) \qquad (2.2)$$

Energy equation :

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (2.3)$$

Nanoparticle concentration equation :

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} - K_c(C-C_\infty) \qquad (2.4)$$

Where u, v are velocity components along x, y-axis respectively, ν =kinematic viscosity, g = acceleration due to gravity, β =coefficient of thermal expansion, $T \,\&\, T_\infty$ = fluid temperature within the boundary and in the free stream respectively, α = thermal diffusivity, $C \,\&\, C_\infty$ = nanoparticle concentration within the boundary and in the free stream respectively, D_B = Brownian diffusion coefficient and K_C = chemical reaction parameter.

2.3 Boundary Conditions

I. on the upper surface of the plate (y=0)

$$u = L\frac{\partial u}{\partial y}, \quad v = 0, \quad k\frac{\partial T}{\partial y} = -h_f(T_w - T), \quad D_B\frac{\partial C}{\partial y} = -h_s(c_w - C) \qquad (2.5)$$

II. matching with the free stream $(y \to \infty)$

$$u(x,\infty)=U_\infty \qquad T(x,\infty)=T_\infty \tag{2.6}$$

where L = slip length and the subscripts w and ∞ are respectively wall and boundary layer edge.

2.4 Introduction to Dimensionless Form

The dimensionless variables for ψ and T with respect to a similarity variable are introduced following [23]as:

$$\psi = \sqrt{\upsilon U_\infty x}\, f(\eta) \tag{2.7}$$

$$T = T_\infty + (T_w - T_\infty)\theta(\eta) \tag{2.8}$$

$$C = C_\infty + (C_w - C_\infty)\phi(\eta) \tag{2.9}$$

$$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}} \tag{2.10}$$

Where,

ψ is the stream function,

η is the similarity variable,

f , θ and N are dimensionless stream function, temperature and nanoparticle concentration respectively.

Using equations (2.7) to (2.10) as necessary, we have

$$u = \frac{\partial \psi}{\partial y} = U_\infty f' \tag{2.11}$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{1}{2}\left(\frac{\upsilon U_\infty}{x}\right)^{\frac{1}{2}}(f - \eta f') \tag{2.12}$$

Here, primes denote differentiation with respect to η. Now for equations (2.11) and (2.12), the continuity equation (2.1) is satisfied automatically. Using equations (2.7) to (2.13) where necessary in equations (2.2) - (2.4), we get

$$f''' + \frac{1}{2} f'' f + \frac{Gr_x}{\mathrm{Re}^2{}_x}\theta = 0 \tag{2.13}$$

$$\theta'' + \frac{1}{2}\Pr f\,\theta' = 0 \tag{2.14}$$

$$\phi'' + \frac{1}{2} Sc\, f\, \phi' - Sc\, R\,\phi = 0 \tag{2.15}$$

where

$Gr_x = \dfrac{g\beta(T_w - T_\infty)x^3}{\upsilon}$ *is the local Grashof number*, $\mathrm{Re}_x = \dfrac{U_\infty x}{\upsilon}$ *is the local Reynolds number*,

$\Pr = \dfrac{\nu}{\alpha}$ *is the Prandtl number*, $Sc = \dfrac{\nu}{D_B}$ *is the Schmidt number*,

$R = \dfrac{x k_c}{U_\infty}$ *is the Chemical reaction parameter*.

The corresponding boundary conditions are:

$$f'(0) = \delta f''(0),\ f(0) = 0,\quad \theta'(0) = -a(1-\theta(0)) \ ,\ \phi(0) = -a_1(1-\phi(0)) \tag{2.16}$$

$$f'(\infty) = 1 \qquad \theta(\infty)=0 \qquad \phi(\infty)=0 \tag{2.17}$$

where

$\delta = L\left(\dfrac{U_\infty}{\upsilon x}\right)^{\frac{1}{2}}$ *is the slip parameter and*

$a = \dfrac{h_f}{k}\left(\dfrac{\upsilon x}{U_\infty}\right)^{\frac{1}{2}}$ *is the surface convection parameter*

$a_1 = \dfrac{h_s}{k}\left(\dfrac{\upsilon x}{U_\infty}\right)^{\frac{1}{2}}$ *is the convective mass transfer parameter*

Notice that Gr_x, δ, a and a_1 are dependent on x. All the aforementioned parameters must be constant if equation (2.13), equation (2.14) and equation (2.15) must have a similarity solution. If we assume that h_f, h_s, β, are proportional to $x^{-\frac{1}{2}}$, then, Gr_x, δ, a and a_1 would not depend on x. Therefore, we assume that $h_f = b x^{-\frac{1}{2}}$, $h_f = c x^{-\frac{1}{2}}$, $\beta = d x^{-\frac{1}{2}}$ where b, c and d are constants.

Observe that δ is a function of x, so for flow with slip over flat plate does not possess self-similar solution. However, the mass and momentum conservation is still preserved using this approach, and as such, it is still valid to study the flow dynamics within the boundary layer. Thus the solution of equation (2.14) subject to equations (2.16) for fixed value of δ would be locally similar. Locally similarity approach implies that the dimensionless quantity δ is determined for any values of x and the upstream history of the flow will be ignored, except as far as it influences the similarity variable.

2.6 Skin Friction Coefficient and Nusselt Number

The parameters of engineering interest for the present problem are the local skin friction, local Nusselt number and the local Sherwood number which indicate physically wall shear stress, rate of heat transfer and rate of mass transfer respectively. In dimensionless forms, these parameters of engineering interest are

Skin-friction coefficient, C_f,

$$\frac{1}{2}\mathrm{Re}_x^{1/2} C_{fx} = f''(0) \qquad (2.18)$$

Nusselt number, Nu,

$$\mathrm{Re}_x^{-1/2} Nu_x = -\theta'(0) \qquad (2.19)$$

Sherwood number, Sh,

$$\mathrm{Re}_x^{-1/2} Sh_x = -\phi'(0) \qquad (2.20)$$

3.0 Numerical Solutions

The governing equations given in equation (2.13) to equation (2.15) subject to the boundary condition given by the equations (2.16) and (2.17) are coupled non-linear equations. The equations cannot be solved analytically thus a numerical solution is to be found. Notice that the set of equations (2.13) -(2.17) form a two-point boundary value problem (BVP). Thus the numerical solutions of the governing equations subject to the boundary conditions equation (2.16) and (2.17) are obtained using symbolic software MATHEMATICA 8.0.

We proceed to confirm the validity of our model. To do this, we set the fluid properties to be constants in order to make them particular forms of the equations known in literature, thereafter a comparison is made. Considering the case of no-slip at the boundary, absence of buoyancy force, magnetic field, nanoparticles and chemical reaction i.e. this present work matches that in [2] and [3]. To actually test our reduced model for accuracy, a comparison of the numerical solutions for various values of the Prandtl number and surface convection parameter $Gr_x = 0$ *and* $\delta = 0$ are calculated for $-\theta'(0)$ and presented on Table 1.

Table 1 Different values of a and Pr when $\theta_r \to \infty$, $\varepsilon = 0$, $Ha = 0$, $Gr_x = 0$ *and* $\delta = 0$ ***for*** $-\theta'(0)$

a	Present results		[2]		[3]	
	Pr =0.1	Pr=0.72	Pr =0.1	Pr=0.72	Pr =0.1	Pr=0.72
20	0.146106	0.291329	0.1461	0.2913	0.139056	0.291329
10	0.145047	0.287146	0.1450	0.2871	0.138096	0.287146
5	0.142973	0.279131	0.1430	0.2791	0.136215	0.279131
1	0.128299	0.228178	0.1283	0.2282	0.122830	0.228178
0.80	0.124311	0.215864	0.1243	0.2159	0.119170	0.215864
0.60	0.118189	0.198051	0.1182	0.1981	0.113533	0.198051
0.40	0.107593	0.169994	0.1076	0.1700	0.103720	0.169994
0.20	0.0847866	0.119295	0.0848	0.1193	0.082363	0.119295
0.10	0.0595439	0.0747242	0.0594	0.0747	0.058338	0.074724
0.05	0.0373213	0.0427669	0.0373	0.0428	0.036844	0.042767

From Table 1 we can deduce that the numerical technique used in this work is justified. This is so because the data given in [2] and [3] and those of the present study show excellent agreement.

3.1 Results and Discussion

Numerical computations have been carried out to analyze the results using the method described in the preceding section for different values of the slip parameter δ, surface convection parameter a, viscosity parameter θ_r, Grashof number Gr_x, thermal conductivity parameter ε, Reynolds number Re_x, Hartmann number Ha and Prandtl number Pr within the boundary layer. Only positive numbers of Grashof number are chosen, since we are considering a cooling problem. Take $R=3,\ Sc=1,\ Gr_x=10,\ \delta=0.5,\ a=0.5,\ a1=0.5, \mathrm{Re}_x=1\ and\ \Pr=1$ to be the default values of the parameters unless otherwise specified.

3.2 Computational Results for Fluid Flow

The non-dimensional velocity, temperature and nanoparticle concentration profiles for increasing Grashof number for both slip flows within the boundary layer are presented in the following figures.

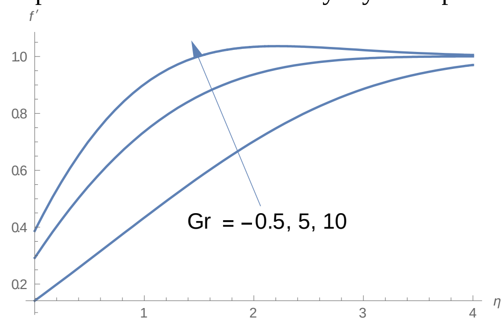

Figure 2 Velocity profiles for different values of Grashof number

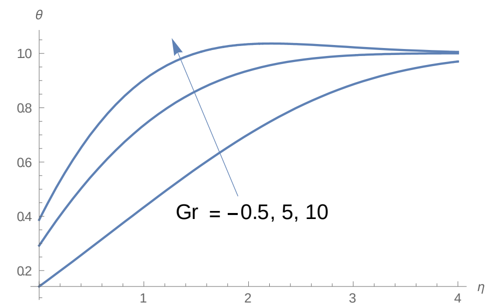

Figure 3 Temperature profiles for different values of Grashof number

Figure 2 and Figure 3 show the effect of Grashof number, Gr on the velocity and temperature profiles respectively. For increasing values of Gr, we see that the velocity increases for slip flows.

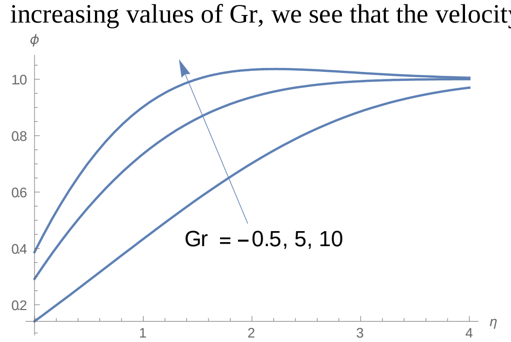

Figure 4 Nanoparticles profiles for different values of Grashof Number

The effect of Grashof number, on the nanoparticles concentration is shown on Figure 4. The nanoparticles concentration increases with increase in Gr asshown on Figure 4 for slip flows.

The parameters of engineering interest are calculated and presented in Table 2 below.

Table 2 Values of f ''(0), $\theta(0)$, $-\theta'(0)$ and $-\phi'(0)$ ***for different values of a, a1,*** δ ***and* Gr *when Sc =1, Re = 1, Pr =1 and R = 1***

Parameters			$\delta=0$			$\delta=0.5$		
Gr	A	a1	f''(0)	$-\theta'(0)$	$-\phi'(0)$	$-\phi'(0)$	$-\theta'(0)$	f''(0)
3	0.2	0.2	0.93182	0.15077	0.15907	0.16072	0.15815	0.6946
3	0.2	0.2	0.93182	0.15077	0.15907	0.16072	0.15815	0.6946
3	0.2	0.2	0.93182	0.15077	0.15907	0.16072	0.15815	0.6946
3	0.2	0.2	0.93182	0.15077	0.15907	0.16072	0.15815	0.6946
3	0.8	0.8	1.47964	0.37146	0.40016	0.41346	0.41796	1.03882
3	0.8	0.8	1.47964	0.37146	0.40016	0.41346	0.41796	1.03882
3	0.8	0.8	1.47964	0.37146	0.40016	0.41346	0.41796	1.03882
3	0.8	0.8	1.47964	0.37146	0.40016	0.41346	0.41796	1.03882
5	0.2	0.2	1.21386	0.15331	0.15959	0.16141	0.160596	0.85781
5	0.2	0.2	1.21386	0.15331	0.15959	0.16141	0.160596	0.85781
5	0.2	0.2	1.21386	0.15331	0.15959	0.16141	0.160596	0.85781
5	0.2	0.2	1.21386	0.15331	0.15959	0.16141	0.160596	0.85781

From Table 2, we observe that the skin friction increased for high values of Gr but decreased during slip flow. However, he the Nusselt number and Sherwood number both increase when there is slip flow as compared to the condition of no-slip. The Nusselt and Sherwood numbers increase when Gr is increased.

4.0 CONCLUSION

Effects of buoyancy forces on a hydrodynamic flow of a viscous incompressible electrically conducting fluid over a vertically flat plate with partial slip at the surface of the boundary in the presence chemical reaction and convective boundary condition is studied. Non-dimensional velocity and temperature profiles within the boundary layer are displayed. Presented in tabular form is the skin friction and the rate of heat and mass transfer from the plate to the fluid for different values of the pertinent parameters governing the flow. The following conclusions can be made from the present study:

1. The velocity, temperature and nanoparticle concentration profiles are increasing for increasing Grashof number.
2. The coefficient of skin friction increases with increase in Grashof number but decrease during slip flow.
3. Nusselt number and Sherwood number both increase when there is slip flow as compared to the condition of no-slip. The Nusselt and Sherwood numbers increase when Grashof number is increased.

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