Transactions of the Nigerian Association of Mathematical Physics
Volume 18, (January – December, 2022 Issue), **pp81 –86**
© *Trans. of NAMP*

SOME COUPLED FIXED POINT THEOREMS SATISFYING CONTRACTIVE CONDITION OF RATIONAL TYPE VIA MIXED MONOTONE MAPPINGS IN PARTIALLY ORDERED METRIC SPACES

Emmanuella Ehui Aribike[1], Muhammed Raji[2] and Samuel Adamariko Aniki[3]

[1]Department of Mathematical Sciences, Lagos State University of Science and Technology, Ikorodu, Lagos, Nigeria
[2,3]Department of Mathematics and Statistics, Confluence University of Science and Technology, Osara, Kogi, Nigeria

Abstract

In this article, we prove the existence and uniqueness of coupled fixed point for nonlinear contractive mappings of rational type in the context of mixed monotone mappings for metric space, which is endowed with a partial order. The outcome of our findings is an improvement to the available results in the literature.

Keywords: Partially ordered metric space, Coupled fixed point, rational contractions, monotone property.

1. Introduction

Banach's fixed point theorem for contraction mapping is one of the pivotal results of analysis. It is a very popular tool for solving the existence problems in different fields of mathematics. There are a lot of generalizations of the Banach contraction principle in some literature [1 - 12].
Ran and Reurings [11] extended the Banach contraction principle in partially ordered sets with some applications to linear and nonlinear matrix equations, while Nieto and Rodriguez-Lopez [12] extended their result and applied their main theorems to obtain a unique solution for first order ordinary differential equation with periodic boundary conditions. Bhaskar and Lakshmikantham [13] introduced the concept of mixed monotone mappings and obtained some coupled fixed point results. Also, they applied their results to first order differential equations with periodic boundary conditions. Recently, many researchers have obtained fixed point and common fixed point results in metric spaces and partially ordered spaces.
The purpose of this paper is to establish some coupled fixed point results for rational type contraction mappings in metric spaces endowed with a partial order.

2. Preliminary Notes

We start with the following definitions and theorems that motivate our study:
Definition 2.1 [14] The triple $(X, d, \preccurlyeq)$ is called partially ordered metric spaces, if $(X, \preccurlyeq)$ is a partially ordered set and (X, d) is a metric space.
Definition 2.2 [14] If (X, d) is a complete metric space, then the triple $(X, d, \preccurlyeq)$ is called complete partially ordered metric spaces.
Definition 2.3 [15] A partially ordered metric space $(X, d, \preccurlyeq)$ is called ordered complete if for each convergent sequence $\{x_n\}_{n=0}^{\infty} \subset X$, the following condition holds: either

i. if x_n is a non-increasing sequence in X such that $x_n \to x$ implies $x \preccurlyeq x_n$, for all $n \in \mathbb{N}$ that is, $x = \inf\{x_n\}$, or
ii. if x_n is a non-decreasing sequence in X such that $x_n \to x$ implies $x_n \preccurlyeq x$, for all $n \in \mathbb{N}$ that is, $x = \sup\{x_n\}$.

Definition 2.3 [4] Let $(X, \preccurlyeq)$ be a partially ordered set and let $T: X \to X$ be a mapping. Then

i. elements $x, y \in X$ are comparable, if $x \preccurlyeq y$ or $y \preccurlyeq x$ holds;
ii. a non-empty set X is called well ordered set, if every two elements of it are comparable;
iii. T is said to be monotone non-decreasing with respect to $\preccurlyeq$, if for all $x, y \in X$, $x \preccurlyeq y$ implies $Tx \preccurlyeq Ty$.
iv. T is said to be monotone non-increasing with respect to $\preccurlyeq$, if for all $x, y \in X$, $x \preccurlyeq y$ implies $Tx \succcurlyeq Ty$.

Correspondence Author: Emmanuella E. A., Email: aribike.ee@lasutech.edu.ng, Tel: +2348034012186, +2348068525943 (MR)

Theorem 2.4 [16] A mapping $T: X \to X$, defined on a complete metric space (X, d) satisfying the following condition

$$d(Tx, Ty) \leq \alpha \frac{d(x, Tx)[1 + d(y, Ty)]}{1 + d(x, y)} + \beta[d(x, Tx) + d(y, Ty)] + \gamma[d(x, Ty) + d(y, Tx)] + \delta d(x, y) \tag{2.1}$$

for all distinct $x, y \in X$, where $\alpha, \beta, \gamma, \delta$ are non negative real numbers with $\alpha + 2(\beta + \gamma) + \delta < 1$. Then T has a unique fixed point in X.

Theorem 2.5 [4] Let $(X, d, \preccurlyeq)$ be a complete partially ordered metric space. Suppose that $T: X \to X$ be a non-decreasing, continuous self mapping satisfying

$$d \leq \begin{cases} \lambda d(x, y) + \eta[d(x, Ty) + d(y, Tx)] + \mu \dfrac{d(x, Tx)d(x, Ty) + d(y, Tx)d(y, Ty)}{d(y, Tx) + d(x, Ty)}, & if\ A \neq 0 \\ 0 & , if\ A = 0 \end{cases} \tag{2.2}$$

for all distinct $x, y \in X$ with $y \preccurlyeq x$, where $A = d(y, Tx) + d(x, Ty)$ and λ, η, μ are non negative real numbers with $\lambda + 2\eta + \mu < 1$. If there exists $x_0 \in X$ with $x_0 \preccurlyeq Tx_0$, then T has a unique fixed point in X.

Theorem 2.6. [17] Let $(X, d, \preccurlyeq)$ be a complete partially ordered metric space. Suppose that $f: X \to X$ is continuous self-mapping with strict mixed monotone property on X satisfying

$$d\big(f(x, y), f(\mu, v)\big) \leq \alpha \frac{d\big(x, f(x, y)\big)\big[1 + d\big(\mu, f(\mu, v)\big)\big]}{1 + d(x, \mu)} + \beta\big[d\big(x, f(x, y)\big) + d\big(\mu, f(\mu, v)\big)\big] + \gamma d(x, \mu) \tag{2.3}$$

where $\alpha, \beta, \gamma, \delta \in [0,1)$ with $\alpha + 2\beta + \gamma < 1$, and if there exists two points $x_0, y_0 \in X$ with $x_0 < f(x_0, y_0)$ and $y_0 > f(y_0, x_0)$, then f has coupled fixed point $(x, y) \in X^2$.

3. Main Results

Theorem 3.1 Let $(X, d, \leq)$ be a complete partially ordered metric space. Suppose that $f: X^2 \to X$ is continuous self-mapping with strict mixed monotone property on X satisfying

$$d\big(f(x, y), f(\mu, v)\big) \leq \alpha \frac{d\big(x, f(x, y)\big)\big[1 + d\big(\mu, f(\mu, v)\big)\big]}{1 + d(x, \mu)} + \beta \frac{d\big(x, f(x, y)\big)d\big(\mu, f(\mu, v)\big)}{d(x, \mu)} + \gamma\big[d\big(x, f(x, y)\big) + d\big(\mu, f(\mu, v)\big)\big] + \delta d(x, \mu) \tag{3.1}$$

where $\alpha, \beta, \gamma, \delta \in [0,1)$ with $\alpha + \beta + 2\gamma + \delta < 1$, and if there exists two points $x_0, y_0 \in X$ with $x_0 < f(x_0, y_0)$ and $y_0 > f(y_0, x_0)$, then f has coupled fixed point $(x, y) \in X \times X$.

Proof. Suppose f is a continuous map on X. Let $x_0, y_0 \in X$ such that $x_0 < f(x_0, y_0)$ and $y_0 > f(y_0, x_0)$. We construct two sequences $\{x_n\}, \{y_n\}$ in X as follows

$$x_{n+1} = f(x_n, y_n) \text{ and } y_{n+1} = f(y_n, x_n)\ \forall\, n \geq 0. \tag{3.2}$$

Next, we have to show that for all $n \geq 0$,

$$x_n < x_{n+1} \tag{3.3}$$

and

$$y_n > y_{n+1} \tag{3.4}$$

We now use method of mathematical induction. Suppose $n = 0$, since $x_0 < f(x_0, y_0)$ and $y_0 > f(y_0, x_0)$ and (3.2), we have $x_0 < f(x_0, y_0) = x_1$ and $y_0 > f(y_0, x_0) = y_1$ and hence inequalities (3.3) and (3.4) hold for $n = 0$. Suppose that the inequalities (3.3) and (3.4) hold for all $n > 0$ and by using the strict mixed monotone property of f, we get

$$x_{n+1} = f(x_n, y_n) < f(x_{n+1}, y_n) < f(x_{n+1}, y_{n+1}) = x_{n+2} \tag{3.5}$$

and

$$y_{n+1} = f(y_n, x_n) > f(y_{n+1}, x_n) > f(y_{n+1}, x_{n+1}) = y_{n+2}. \tag{3.6}$$

Thus, the inequalities (3.3) and (3.4) hold for all $n \geq 0$ and we obtain that

$$x_0 < x_1 < x_2 < x_3 < \cdots < x_n < x_{n+1} < \cdots \tag{3.7}$$

and

$$y_0 > y_1 > y_2 > y_3 > \cdots > y_n > y_{n+1} > \cdots \tag{3.8}$$

From hypothesis, we have $x_n < x_{n+1},\ y_n > y_{n+1}$ and from (3.2),

$$d(x_{n+1},x_n) = d\big(f(x_n,y_n),f(x_{n-1},y_{n-1})\big)$$
$$\le \alpha \frac{d\big(x_n,f(x_n,y_n)\big)\big[1+d\big(x_{n-1},f(x_{n-1},y_{n-1})\big)\big]}{1+d(x_n,x_{n-1})} + \beta \frac{d\big(x_n,f(x_n,y_n)\big)d\big(x_{n-1},f(x_{n-1},y_{n-1})\big)}{d(x_n,x_{n-1})}$$
$$+ \gamma\big[d\big(x_n,f(x_n,y_n)\big) + d\big(x_{n-1},f(x_{n-1},y_{n-1})\big)\big] + \delta d(x_n,x_{n-1})$$

which implies that

$$d(x_{n+1},x_n) \le \alpha \frac{d(x_n,x_{n+1})[1+d(x_{n-1},x_n)]}{1+d(x_n,x_{n-1})} + \beta \frac{d(x_n,x_{n+1})d(x_{n-1},x_n)}{d(x_n,x_{n-1})} + \gamma[d(x_n,x_{n+1}) + d(x_{n-1},x_n)]$$
$$+ \delta d(x_n,x_{n-1})$$

$$d(x_{n+1},x_n) \le \left(\frac{\gamma+\delta}{1-\alpha-\beta-\gamma}\right) d(x_{n-1},x_n) \tag{3.9}$$

Similarly, we get

$$d(y_{n+1},y_n) \le \left(\frac{\gamma+\delta}{1-\alpha-\beta-\gamma}\right) d(y_{n-1},y_n) \tag{3.10}$$

Adding (3.9) and (3.10), we get

$$d(x_{n+1},x_n) + d(y_{n+1},y_n)$$
$$\le \left(\frac{\gamma+\delta}{1-\alpha-\beta-\gamma}\right)[d(x_{n-1},x_n) + d(y_{n-1},y_n)]$$

Now, we define the sequence $\{T_n\} = \{d(x_{n+1},x_n) + d(y_{n+1},y_n)\}$, by induction, we have

$$0 \le T_n \le kT_{n-1} \le k^2T_{n-2} \le k^3T_{n-3} \le \cdots$$
$$\le k^nT_0$$

where $k = \frac{\gamma+\delta}{1-\alpha-\beta-\gamma} < 1$ and hence, we get

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty}[d(x_{n-1},x_n) + d(y_{n-1},y_n)]$$
$$= 0$$

From this we get

$$\lim_{n\to\infty} d(x_{n-1},x_n)$$
$$= 0$$

and

$$\lim_{n\to\infty} d(y_{n-1},y_n)$$
$$= 0$$

Now, we shall prove that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. For $m \ge n$, by triangular inequality we have

$$d(x_m,x_n) \le d(x_m,x_{m-1}) + d(x_{m-1},x_{m-2}) + \cdots$$
$$+ d(x_{n+1},x_n)$$

and

$$d(y_m,y_n) \le d(y_m,y_{m-1}) + d(y_{m-1},y_{m-2}) + \cdots$$
$$+ d(y_{n+1},y_n)$$

then

$$d(x_m,x_n) + d(y_m,y_n) \le T_{n-1} + T_{n-2} + T_{n-3} + \cdots + T_n \le (k^{m-1} + k^{m-2} + k^{m-3} + \cdots + k^n)T_0$$
$$\le \frac{k^n}{1-k}T_0,$$

as $m,n \to +\infty, d(x_m,x_n) + d(y_m,y_n) \to 0$. Thus, $\{x_n\}$ and $\{x_n\}$ are Cauchy sequences in a complete metric space X. Therefore, there exists $(x,y) \in X \times X$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. By continuity of f, we have

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} f(x_n,y_n) = f\left(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n\right)$$
$$= f(x,y)$$

and

$$y = \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} f(y_n,x_n) = f\left(\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n\right)$$
$$= f(y,x)$$

Hence, $x = f(x,y)$ and $y = f(y,x)$.

Since $\{x_n\}$ is an increasing sequence in X and converges to a point x in X as it is a Cauchy sequence, then $x = sup\{x_n\}$ i.e., $(x_n \leq x, \forall n \in \mathbb{N})$. Therefore, we conclude that $x_n < x$ for all n, otherwise there exists a number $n_0 \in \mathbb{N}$ such that $x_{n_0} = x$, and hence $x < x_{n_0} \leq x_{n_0+1} = x$ which is wrong. Thus, from the strict monotone increasing of f over the first variable, we get

$$f(x_n, y_n) < f(x, y_n). \tag{3.11}$$

Similarly, from above there is a decreasing Cauchy sequence $\{y_n\}$ in X, which converges to a point $y \in X$. Thus, by ordered complete property of X, we have $y = inf\{y_n\}$ i.e., $(y_n \geq y, \forall n \in \mathbb{N})$.

With similar argument above, we have $y_n > y$ for all $n \in \mathbb{N}$. Also, from the strict monotone decreasing of f on the second variable, we get

$$f(x, y_n) < f(x, y). \tag{3.12}$$

Therefore, from equations (3.11) and (3.12), we obtain

$$f(x_n, y_n) < f(x, y) \Longrightarrow x_{n+1} < f(x, y) \text{ for all } n \in \mathbb{N}. \tag{3.13}$$

Since $x_n = x_{n+1} < f(x, y)$ for all $n \in \mathbb{N}$ and $x = sup\{x_n\}$, then we obtain $x \leq f(x, y)$.

Now, let $z_0 = x$ and $z_{n+1} = f(z_n, y_n)$ then, by similar argument above the sequence $\{z_n\}$ is a nondecreasing Cauchy sequence, since $z_0 \leq f(z_0, y_0)$ and converge to a point z in X, implies that $z = sup\{z_n\}$.

Since for all $n \in \mathbb{N}$, $x_n \leq x = z_0 \leq f(z_0, y_0) \leq z_n \leq z$ then from (3.1), we have

$$\begin{aligned} d(x_{n+1}, z_{n+1}) &= d\big(f(x_n, y_n), f(z_n, y_n)\big) \\ &\leq \alpha \frac{d\big(x_n, f(x_n, y_n)\big)\big[1 + d\big(z_n, f(z_n, y_n)\big)\big]}{1 + d(x_n, z_n)} + \beta \frac{d\big(x_n, f(x_n, y_n)\big) d\big(z_n, f(z_n, y_n)\big)}{d(x_n, z_n)} \\ &+ \gamma\big[d\big(x_n, f(x_n, y_n)\big) + d\big(z_n, f(z_n, y_n)\big)\big] \\ &+ \delta d(x_n, z_n). \end{aligned}$$

Taking the limit as $n \to \infty$ in the above inequality, we get

$$d(x, z) \leq (2\gamma + \delta) d(x, z),$$

but $2\gamma + \delta < 1$, then we obtain that $d(x, z) = 0$. Hence $x = z = sup\{x_n\}$, implies that $x \leq f(x, y) \leq x$. Thus, $x = f(x, y)$. Again following similar argument above, we obtain that $y = f(y, x)$. Hence, f has a coupled fixed point in $X \times X$.

To achieve the existence and uniqueness of a coupled fixed point of f over a complete partial ordered metric space X, we have the following partial order relation.

$$(\mu, v) \leq (x, y) \Leftrightarrow x \geq \mu, y \leq v, \text{ for any } (x, y), (\mu, v) \in X \times X.$$

Theorem 3.2 By the hypothesis of Theorem 3.1 and suppose that for every $(x, y), (r, s) \in X \times X$, there exists $(u, v) \in X \times X$ such that $\big(f(u, v), f(v, u)\big)$ is comparable to $\big(f(x, y), f(y, x)\big)$ and $\big(f(r, s), f(s, r)\big)$, then f has a unique coupled fixed point in $X \times X$.

Proof. Following the proof of Theorem 3.1, the set of coupled fixed points of f is non empty. Suppose that (x, y) and (r, s) are two coupled fixed points of the mapping f, then $x = f(x, y), y = f(y, x), r = f(r, s)$ and $s = f(s, r)$. Now, we have to show that $x = r, y = s$ for uniqueness of a coupled fixed point of f.

From hypotheses, there exists $(u, v) \in X \times X$ such that $\big(f(u, v), f(v, u)\big)$ is comparable to $\big(f(x, y), f(y, x)\big)$ and $\big(f(r, s), f(s, r)\big)$. Put $u = u_0, v = v_0$ then choose $u_1, v_1 \in X$ such that $u_1 = f(u_0, v_0)$ and $v_1 = f(v_0, u_0)$. Thus, following the proof of Theorem 3.1, we construct two sequences $\{u_n\}, \{v_n\}$ from $u_{n+1} = f(u_n, v_n)$ and $v_{n+1} = f(v_n, u_n)$ for all $n \in \mathbb{N}$. Similarly by induction from Theorem 3.1, we define the sequences $\{x_n\}, \{y_n\}, \{r_n\}$ and $\{s_n\}$ by setting $x = x_0, y = y_0, r = r_0$ and $s = s_0$. By Theorem 3.1, we have $x_n \to x = f(x, y), y_n \to y = f(y, x), r_n \to r = f(r, s)$ and $s_n \to s = f(s, r)$ for all $n \geq 1$. But $\big(f(x, y), f(y, x)\big) = (x, y)$ and $\big(f(u_0, v_0), f(v_0, u_0)\big) = (u_1, v_1)$ are comparable and then we have $x \geq u_1$ and $y \leq v_1$.

Next we show that (x, y) and (u_n, v_n) are comparable, i.e., to show that $x \geq u_n$ and $y \leq v_n$ for all $n \in \mathbb{N}$. Suppose the inequalities hold for some $n \geq 0$, then from strict mixed monotone property of f, we have $u_{n+1} = f(u_n, v_n) \leq f(x, y) = x$ and $v_{n+1} = f(v_n, u_n) \leq f(y, x) = y$. Therefore, $x \geq u_n$ and $y \leq v_n$ for all $n \in \mathbb{N}$.

Then, from Theorem 1, we have

$$d(x,u_{n+1}) = d\big(f(x,y), f(u_n,v_n)\big) \le \alpha \frac{d\big(x,f(x,y)\big)\big[1+d\big(u_n,f(u_n,v_n)\big)\big]}{1+d(x,u_n)} + \beta \frac{d\big(x,f(x,y)\big)d\big(u_n,f(u_n,v_n)\big)}{d(x_n,u_n)} + \gamma\big[d\big(x,f(x,y)\big)+d\big(u_n,f(u_n,v_n)\big)\big] + \delta d(x,u_n),$$

which implies that

$$d(x,u_{n+1}) \le \left(\frac{\gamma+\delta}{1-\alpha-\beta-\gamma}\right) d(x,u_n).$$

Similarly, we get

$$d(y,v_{n+1}) \le \left(\frac{\gamma+\delta}{1-\alpha-\beta-\gamma}\right) d(y,v_n).$$

Suppose $P = \frac{\gamma+\delta}{1-\alpha-\beta-\gamma} < 1$, then from the above equations, we have

$$d(x,u_{n+1}) + d(y,v_{n+1}) \le P[d(x,u_n)+d(y,v_n)] \le P^2[d(x,u_{n-1})+d(y,v_{n-1})] \le P^3[d(x,u_{n-2})+d(y,v_{n-2})] \ldots \ldots \le P^n[d(x,u_0)+d(y,v_0)].$$

Taking limit as $n \to +\infty$ to the above inequality, we get $\lim_{n\to+\infty} d(x,u_{n+1}) + d(y,v_{n+1}) = 0$.

Consequently, we obtain $\lim_{n\to+\infty} d(x,u_{n+1}) = 0$ and $\lim_{n\to+\infty} d(y,v_{n+1}) = 0$. Similarly, we can prove that $\lim_{n\to+\infty} d(r,u_n) = 0$ and $\lim_{n\to+\infty} d(s,v_n) = 0$.

By triangular inequality, we obtain that

$d(x,r) \le d(x,u_n) + d(u_n,r)$ and $d(y,s) \le d(y,v_n) + d(v_n,s)$.

Take the limit as $n \to \infty$ to the above inequalities, we obtain that $d(x,r) = 0 = d(y,s)$, implies that $x = r$ and $y = s$. Hence, f has a unique coupled fixed point in $X \times X$.

Theorem 3.3 In addition to the hypothesis of Theorem 3.1 (or Theorem 3.2), suppose that x_0, y_0 are comparable then f has a coupled fixed point in $X \times X$.

Proof. Suppose(x,y) is a coupled fixed point of f, then from Theorem 3.1 (or Theorem 3.2), there exists two sequences $\{x_n\}$ and $\{y_n\}$ such that $x_n \to x$ and $y_n \to y$ in X.

Assume that $x_0 \le y_0$, then we show that $x_n \le y_n$, for all $n \ge 0$. Suppose it holds for some $n \ge 0$. By the strict mixed monotone property of f, we get $x_{n+1} = f(x_n,y_n) \le f(y_n,x_n) = y_{n+1}$. Then, from the contraction condition (3.1), we get

$$d(x_{n+1},y_{n+1}) = d\big(f(x_n,y_n), f(y_n,x_n)\big) \le \alpha \frac{d\big(x_n,f(x_n,y_n)\big)\big[1+d\big(y_n,f(y_n,x_n)\big)\big]}{1+d(x_n,y_n)} + \beta \frac{d\big(x_n,f(x_n,y_n)\big)d\big(y_n,f(y_n,x_n)\big)}{d(x_n,y_n)} + \gamma\big[d\big(x_n,f(x_n,y_n)\big)+d\big(y_n,f(y_n,x_n)\big)\big] + \delta d(x_n,z_n).$$

Take the limit as $n \to \infty$ to the above inequalities, we get

$$d(x,y) \le (2\gamma+\delta)d(x,y)$$

Which is a contradiction, since $2\gamma+\delta < 1$. Thus, $d(x,y) = 0$. Therefore, we have $f(x,y) = x = y = f(y,x)$. Similarly, we can also show that $f(x,y) = x = y = f(y,x)$ by considering $y_0 \le x_0$. Hence, (x,y) is a coupled fixed point of f in X^2.

4. Conclusion

In this paper, we prove the existence and uniqueness of some coupled fixed point for nonlinear contractive mappings with rational expressions in the context of metric space endowed with partial order. Hence, further work could be done as an extension of the results herein to tripled fixed point.

Acknowledgment

The authors would like to extend their sincere thanks to all that have contributed in one way or the other for the actualization of the review of this work.

References

[1] R. Kannan; Some results on fixed points, Amer. Math. Soc., 76, (1969), 405-408.
[2] H. Chatterji; On generalization of Banach contraction principle, Indian J. Pure. App. Math., 10, (1979), 400-403.
[3] B. K.Dass and S. Gupta; An extension of Banach contraction principle through rational expression, Indian J. Pure. App. Math., 6, (1975), 1455-1458.
[4] R. N.Seshagiri andK. Kalyani; Fixed point theorems in partially ordered metric spaces with rational expressions, Information Science Letters, 10, (2021), 451-460.
[5] S. Chandok and J.K. Kim;Fixed point theorems in ordered metric spaces for generalized contraction mappings satisfying rational type expressions, Nonlinear Funct. Anal. and Appl., 17, (2012), 301-306.
[6] R. N. Seshagiri, K. Kalyani and M. Belay M; On fixed point theorems of monotone functions in ordered metric spaces, J. of Anal., (2021), 14 pages.
[7] R. N. Seshagiri and K. Kalyani; Coupled fixed point theorems with rational expressions in partially ordered metric spaces, J. of Anal., 28, 4, (2020), 1085-1095.
[8] S. Chandok; Some common fixed point results for rational type contraction mappings in partially ordered metric spaces, Math. Bohem, 138(4), (2013), 407-413.
[9] R. N.Seshagiri, K.Kalyani and K.Khatri; Contractive mappings theorems in partially ordered metric space, Cubo, 22(2), (2020), 203-214.
[10] J. Harjani, B.Lopez and K.Sadarangani; A fixed point theorem for mappings satisfying a contractive condition of rational type on a partially ordered metric space, Abstr. Appl. Anal., Article ID 190701, (2010), 8 pages.
[11] A.C. Ran, M.C.B. Reurings, A fixed point theorem in partially ordered sets and some applications to marix equations, Proceedings of the American Mathematical Society,132, 5, 1435-1443,2004.
[12] J.J. Nieto, R. Rodriguez-Lopez,Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, Oder, 22, 3, 223-239,2005.
[13] T.G. Bhaskar and V.Lakshmikantham; Fixed points theorems in partially ordered metric spaces and applications. Nonlinear Anal., Theory Methods Appl., 65, (2006), 1379-1393.
[14] R. N. Seshagiri and K. Kalyani; Unique fixed point theorems in partially ordered metric spaces, Heliyon, 6, 11, (2020), 7 pages.
[15] M. Arshad, E. Karapinar and J. Ahmad; Some unique fixed point theorems for rational contractions in partially ordered metric spaces, J. of Inequalities and Appl., (2013), 248-255.
[16] K. Kalyani, R. N.Seshagiri and M. Belay On fixed point theorems of monotone functions in ordered metric spaces, J. of Anal., (2021), 12 pages.
[17] R. N. Seshagiri, K. Kalyani and K. Prasad; Coincidence points in ordered metric spaces and its application, Punjab University J. of Math., 54(1), (2022), 33 – 43.