Transactions of the Nigerian Association of Mathematical Physics
Volume 18, (January – December, 2022 Issue), **pp199 –200**
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THE AFFINITY BETWEEN THE EVENT HORIZON DERIVED FROM HOWUSU METRIC TENSOR AND THAT OF THE SCHWARZSCHILD METRIC TENSOR

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Abstract

A black hole is an object so dense that it sufficiently bends the space-time around it so that nothing can escape. The maximum distance from the center of the black hole for which nothing can escape is called the event horizon. This was first calculated by Schwarzschild metric tensor. In this paper, the Howusu metric tensor for all gravitational fields in nature has been used to calculate the event horizon of a black hole and it is found that $r = R$,which is the same as that of the Schwarzschild metric tensor.

Introduction

Shortly after Einstein introduced his field tensors for general relativity (the geometric theory of gravity) he was surprised when Karl Schwarzschild was able to come up with a solution to these incredibly complex equations in 1916. The Schwarzschild solution, as it became known, described the space-time geometry of an empty region of space. Schwarzschild realized the escape velocity from the surface of an object depends on both its mass and radius [1-2]. Scientists now refer to an object with zero volume but all of its mass as a singularity. Schwarzschild also explained that a singularity was surrounded by a spherical gravitational boundary that forever trapped anything that ventured within. This boundary was called the event horizon. He presented a formula that enabled the size of an event horizon to be calculated which is now known as the Schwarzschild radius. In this paper, a new metric tensor that describes the gravitational field which is regular everywhere continues everywhere including all boundaries, continues normal derivatives everywhere including all boundaries, and its reciprocal decreases at infinite distance from source was introduced in 2012 called the Howusu metric tensor given as

$$\begin{pmatrix} -exp\left(\frac{2GM}{c^2r}\right) & 0 & 0 & 0 \\ 0 & exp\left(-\frac{2GM}{c^2r}\right) & 0 & 0 \\ 0 & 0 & r^2exp\left(-\frac{2GM}{c^2r}\right) & 0 \\ 0 & 0 & 0 & r^2sin^2\theta exp\left(-\frac{2GM}{c^2r}\right) \end{pmatrix} \quad (1)$$

where c is the speed of light; G is the universal constant of gravitation; M is the mass of the object and r is the distance away from the object [3-4] in spherical polar coordinates is used to calculate the event horizon of a black hole.

MATHEMATICAL ANALYSIS

Writing out the line element of the Howusu metric tensor

$$ds^2 = exp\left(\frac{-2GM}{c^2r}\right)c^2dt^2 - exp\left(\frac{2GM}{c^2r}\right)dr^2 -$$
$$r^2exp\left(\frac{2GM}{c^2r}\right)(d\theta^2 + sin^2\theta d\phi^2) \quad (2)$$

As $r \to \frac{2GM}{c^2}$, the coefficient on the radial coordinate tends towards infinity while the coordinate of time tends towards zero. If $r < \frac{2GM}{c^2}$, for a stationary observer held at a fixed position by rockets the metrics become: -

$$ds^2 = c^2d\tau^2 = exp\left(\frac{-2GM}{c^2r}\right)c^2dt^2 - exp\left(\frac{2GM}{c^2r}\right)dr^2 = 0 \quad (3)$$

Let $R = \frac{2GM}{c^2}$

where R is the Schwarzschild radius

$$c^2 = exp\left(\frac{-R}{r}\right)c^2\dot{t}^2 - exp\left(\frac{R}{r}\right)\dot{r}^2 \quad (4)$$

Let $exp\left(\frac{-R}{r}\right)\dot{t} = k$

Multiplying, (4) through by $exp\left(-\frac{R}{r}\right)$, we have

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$c^2 exp\left(-\frac{R}{r}\right) = c^2k^2 - \dot{r}^2$ (5)

$\dot{r}^2 - c^2k^2 + c^2 exp\left(-\frac{R}{r}\right) = 0$ (6)

Note $d\tau = exp\left(-\frac{R}{r}\right)^{\frac{1}{2}} dt$ (7)

Suppose the particle is at rest when $r = r_0 \Rightarrow k^2 = exp\left(-\frac{R}{r_0}\right)$

$\dot{r}^2 - c^2\left[k^2 - exp\left(-\frac{R}{r}\right)\right] = 0$ (8)

$\dot{r}^2 - c^2\left[exp\left(-\frac{R}{r_0}\right) - exp\left(-\frac{R}{r}\right)\right] = 0$ (9)

$\dot{r}^2 = c^2 exp\left(\frac{R}{r} - \frac{R}{r_0}\right)$ (10)

The same form as conservation of energy in Newtonian theory but

1. The dot denotes differentiation with respect to τ and not t
2. r is area (not radial) distance

$\frac{d\tau}{dr} = -c^2\left(exp\left(\frac{R}{r} - \frac{R}{r_0}\right)\right)^{\frac{1}{2}}$ (11)

as the particle falls radially inward.

Integrating the (11),

$\Delta\tau = -c\int_{r_0}^{r}\left(exp\left(\frac{R}{r} - \frac{R}{r_0}\right)\right)^{\frac{1}{2}} dr$ (12)

Clearly, $r = R$ is reached in finite proper time τ.

Now, let's see how much coordinate time elapses during the fall

$\frac{dt}{dr} = \frac{dt}{d\tau}\frac{d\tau}{dr} = kexp\left(\frac{R}{r}\right)\frac{d\tau}{dr}$ (13)

$=c\left[exp\left(\frac{R}{r} - \frac{R}{r_0}\right)\right]^{\frac{1}{2}} exp\left(-\frac{R}{r}\right)\left[exp\left(-\frac{R}{r_0}\right)\right]^{\frac{1}{2}}$ (14)

$=cexp\left(-\frac{R}{r}\right)\left[exp\left(\frac{R}{r} - \frac{R}{r_0} - \frac{R}{r_0}\right)\right]^{\frac{1}{2}}$ (15)

$= cexp\left(-\frac{R}{r}\right)\left[exp\left(\frac{R}{r} - \frac{2R}{r_0}\right)\right]^{\frac{1}{2}}$ (16)

DISCUSSION AND RESULT

For a stationary observer at r $> R$, there is a need to consider the paths of photons leaving the falling particle and arriving at the observer i.e radial null geodesics.

$ds^2 = 0 \Rightarrow exp\left(\frac{-R}{r}\right)c^2t^2 = exp\left(\frac{R}{r}\right)\dot{r}^2$ (17)

$\frac{dr}{dt} = cexp\left(\frac{-R}{r}\right) \to 0 \; as \; r \to R$ (18)

Any photon that the observer sees must have been emitted when the particle was still at $r > R$.outward travelling light just sits still at $r = R$ (not even any angular motion) Also, the frequency observed at $r > R$ is zero.

$ds^2 = exp\left(\frac{-2GM}{c^2r}\right)c^2dt^2 - exp\left(\frac{2GM}{c^2r}\right)dr^2 -$

$r^2 exp\left(\frac{2GM}{c^2r}\right)(d\theta^2 + sin^2\theta d\phi^2)$ (19)

The signs of the dt^2 and dr^2 terms change when r crosses R (r becomes the time coordinates). There is only one direction to time for $r < R$, that's the direction of decreasing r. Any particle at $r < R$ must fall inwards. Some hold for photons (since they must travel forward in time as seen by a particle) nothing can escape from $r < R$. $r = R$ is known as the event horizon.

Conclusion

Interestingly, the Howusu metric tensor gave the exact result that the Schwarzschild metric tensor gave in calculating the black-hole. Now, the size of a black-hole can also be calculated by using the Howusu metric tensor for all gravitational field in nature.

References

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