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A TWO-PARAMETER SUJA DISTRIBUTION AND ITS APPLICATION

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Abstract

This paper proposes a two-parameter Suja distribution (TPSD). This originated from the Lindley and Suja distribution. The mathematical and statistical properties which includes its moments, survival function, hazard rate function, mean residual life function, stochastic ordering, entropy measure, Bonferroni and Lorenz curves were all calculated. The parameters of the distribution were estimated using the method of maximum likelihood. Also, HQIC, BIC, CAIC, AIC, and K-S were used to test for the goodness of fit of the model using a real data set of patients suffering from cancer. The two-parameter Suja distribution was compared with Suja (one parameter Suja) and Rama distributions and the study found out that the two parameter Suja is superior to the other two distributions.

Keywords: Suja distribution, Moments, Hazard rate function, Mean residual life function, Bonferroni and Lorenz Curves, Stress-strength reliability, Estimation of parameter.

1. INTRODUCTION

The one parameter lifetime Lindley distribution proposed by [1] has its probability density function (pdf) and cumulative distribution function (cdf) given below:

$$f(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}, \quad x>0,\theta>0 \tag{1.1}$$

$$F(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x}, \quad x>0,\theta>0 \tag{1.2}$$

The Lindley distribution is a mixture of exponential distribution (θ) and gamma distribution $(2,\theta)$ with their mixing proportions $\frac{\theta}{\theta+1}$. He computed the mathematical and statistical properties of the distribution and the performance was compared with the lindley and exponential distribution, he applied a real-life data set of the waiting (in minutes) before service of 100 bank customers and concluded that the lindley distribution is superior to the exponential distribution.

The Suja distribution for modelling a continuous lifetime data was also proposed by [2], he defined the (pdf) and (cdf) of the distribution as given below:

$$f(x;\theta)=\frac{\theta^5}{\theta^4+24}(1+x^4)e^{-\theta x}, \quad x>0,\theta>0 \tag{1.3}$$

$$F(x;\theta)=1-\left[1+\frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x}{\theta^4+24}\right]e^{-\theta x}, \ x>0,\theta>0 \tag{1.4}$$

The Suja distribution is a mixture of exponential distribution (θ) and gamma distribution $(5,\theta)$ with a mixing proportion of $\frac{\theta^4}{\theta^4+24}$. He discussed the various mathematical properties of the lindley distribution and showed that (1.3) model lifetime data from engineering better than that of Lindley, Akash, Sujatha, Amarendra, Aradhana, Devya, and exponential distributions. The first four moments about the origin of Suja distribution obtained by [2] are given below:

$$\mu_1^\iota=\frac{\theta^4+120}{\theta(\theta^4+24)}, \mu_2^\iota=\frac{2(\theta^4+360)}{\theta^2(\theta^4+24)}, \quad \mu_3^\iota=\frac{6(\theta^4+840)}{\theta^3(\theta^4+24)}, \quad \mu_4^\iota=\frac{24(\theta^4+1680)}{\theta^4(\theta^4+24)}$$

Other important statistical properties of the distribution like, hazard rate function; mean residual life function, stochastic ordering, mean deviations, Bonferroni and Lorenz curves and stress-strength reliability were computed. He has also discussed the maximum likelihood estimation of the parameters and clearly demonstrated the applicability of the Suja distribution.

This study derived a two-parameter Suja distribution (TPSD) using the one-parameter Suja distribution of [2] as a particular

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case. Its moments, hazard rate function, mean residual life function, entropy measures, stochastic ordering, Bonferroni and Lorenz curves and stress-strength reliability of TPSD were derived and discussed. The parameters of the model were estimated using the method of maximum likelihood.
The goodness of fit of the proposed distribution has been discussed and tested with a real-life numerical data set and the fit has been compared with some well-known lifetime distributions.

2. A TWO-PARAMETER SUJA DISTRIBUTION

A two-parameter Rama distribution (TPSD) with parameters θ and α is defined by its (pdf) and (cdf) as follows:

$$f(x;\theta,\alpha)=\frac{\theta^5}{(\alpha\theta^4+24)}(\alpha+x^4)e^{-\theta x}\ ,\ x>0,\theta>0,\alpha>0 \qquad (2.1)$$

where θ is a scale parameter and α is a shape parameter. It reduces to Suja distribution (1.3) for α=1. The pdf (2.1) can be shown as a mixture of exponential (θ) and gamma (5, θ) distributions as shown below:

$$f(x;\theta,\alpha)=gd_1(x)+(1-g)d_2(x) \qquad (2.2)$$

Where $g=\frac{\alpha\theta^4}{\alpha\theta^4+24}$, $\quad d_1=\theta e^{-\theta x}$, and $d_2=\frac{\theta^5x^4e^{-\theta x}}{24}$

The corresponding cumulative distribution function (cdf) of (2.1) can be expressed as:

$$F(x;\theta,\alpha)=1-\left[1+\frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x}{\alpha\theta^4+24}\right]e^{-\theta x}\ ,\ x>0,\theta>0 \qquad (2.3)$$

Thus, the two graphs each representing the pdf and cdf are given below:

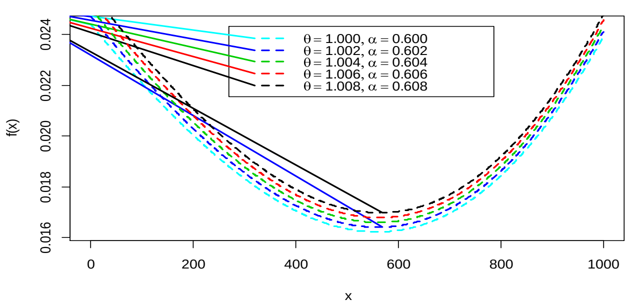

Figure 1. The first plot of the PDF of TPRD for varying values of the parameters θ and α

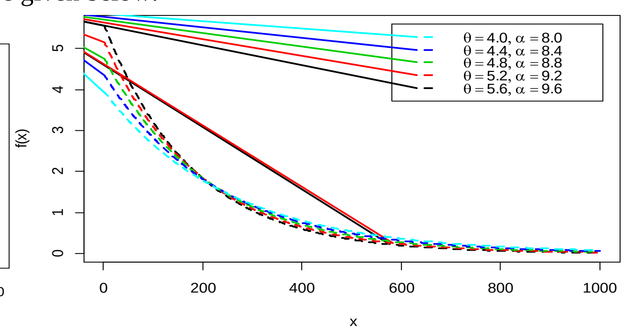

Figure 2. The second plot of the PDF of TPRD for varying values of the parameters θ and α

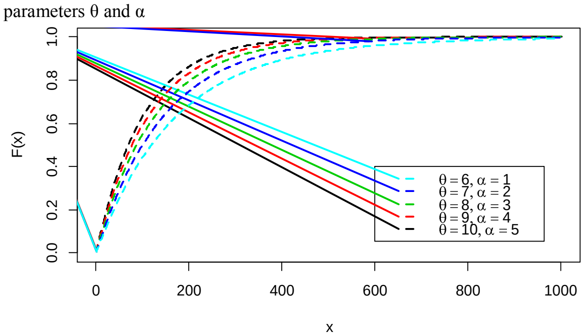

Figure 3. The first plot of the CDF of TPRD for varying values of the parameters θ and α.

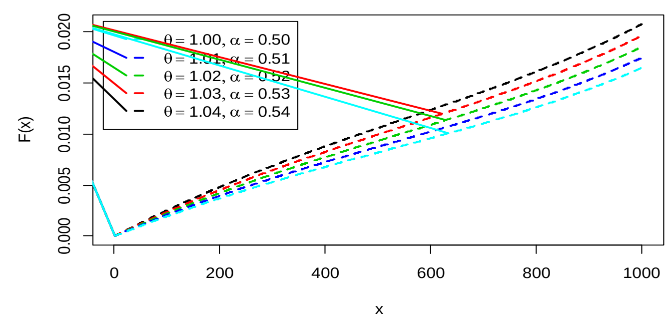

Figure 4. The second Plot of the CDF of TPRD for varying values of the parameters θ and α.

3. MOMENTS

The r^{th} moment about origin of the two-parameter Suja distribution is obtained as

$$E(X^r)=\int_0^\infty x^r f(x)dx$$

$$\mu_r^\prime=\frac{r!\left[\alpha\theta^4+(r+1)(r+2)(r+3)(r+4)\right]}{\theta^r(\alpha\theta^4+24)};\quad r=1,2,3,4, \qquad (3.1)$$

Thus, the first four moments about origin of the two-parameter Suja distribution are given as

$$\mu_1^\prime=\frac{\alpha\theta^4+120}{\theta(\alpha\theta^4+24)},\mu_2^\prime=\frac{2(\alpha\theta^4+360)}{\theta^2(\alpha\theta^4+24)},\quad \mu_3^\prime=\frac{6(\alpha\theta^4+840)}{\theta^3(\alpha\theta^4+24)},\quad \mu_4^\prime=\frac{24(\alpha\theta^4+1680)}{\theta^4(\alpha\theta^4+24)} \qquad (3.2)$$

It can be easily verified that the first four moments of the two-parameter Suja distribution reduces to the corresponding first four moments of the Suja distribution at α=1.

4. SURVIVAL FUNCTION, HAZARD RATE FUNCTION AND MEAN RESIDUAL LIFE FUNCTION

Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. The survival function, $s(x)$, hazard rate function (also known as failure rate function), $h(x)$and the mean residual function, $m(x)$ of x are respectively defined as

$$s(x) = 1 - F(x) \tag{4.1}$$

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{f(x)}{s(x)} \tag{4.2}$$

and

$$m(x) = \frac{1}{1-F(x)}\int_x^\infty [1 - F(t)]dt \tag{4.3}$$

The corresponding survival function $s(x)$,hazard rate function $h(x)$ and the mean residual life function $m(x)$ of the TPSD are thus given as

$$s(x) = 1 - \left[1 - \left[1 + \frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x}{\alpha\theta^4+24}\right]e^{-\theta x}\right] \tag{4.4}$$

$$h(x) = \frac{\theta^5(\alpha+x^4)}{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x+\alpha\theta^4+24} \tag{4.5}$$

$$m(x) = \frac{1}{(\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x+\alpha\theta^4+24)e^{-\theta x}} \times \int_x^\infty (\theta^4t^4+4\theta^3t^3+12\theta^2t^2+24\theta t+\alpha\theta^4+24)e^{-\theta t}dt$$

$$= \frac{\theta^4x^4+8\theta^3x^3+36\theta^2x^2+96\theta x+\alpha\theta^4+120}{\theta(\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x+\alpha\theta^4+24)} \tag{4.6}$$

It can be verified that$h(0) = \frac{\theta^5}{(\alpha\theta^4+24)} = f(0)$and $m(0) = \frac{\alpha\theta^4+120}{\theta(\alpha\theta^4+24)} = \mu_1^{\prime}$.

The expression for $s(x)$, $h(x)$ and $m(x)$ of the two-parameter Suja distribution reduces to the corresponding $s(x)$, $h(x)$ and $m(x)$ of Suja distribution at α =1.

Thus, the graphs of $s(x)$, $h(x)$ and $m(x)$ are given below.

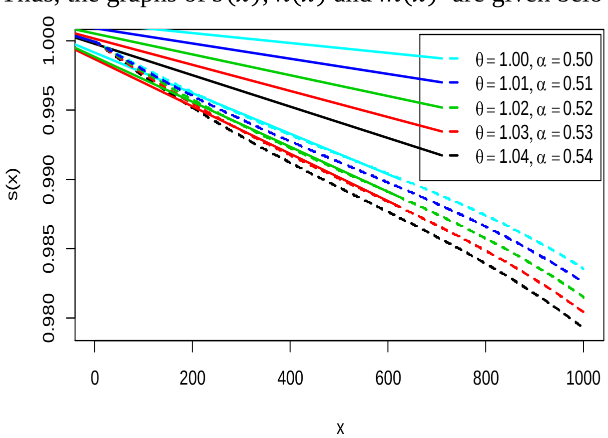

Figure 5. The first plot of the Survival Function of TPRD for varying values of the parameters θ and α.

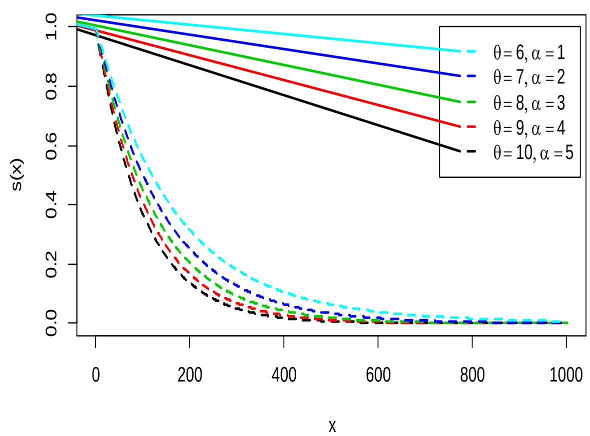

Figure 6. The second plot of the Survival Function of TPRD for varying values of the parameters θ and α.

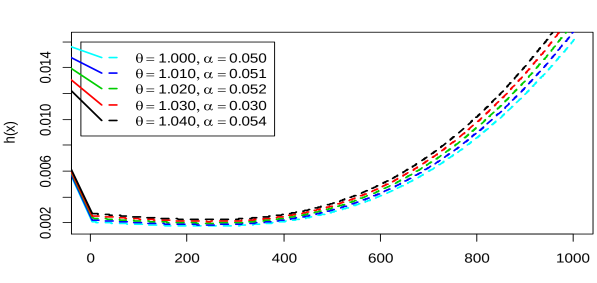

Figure 7. The first plot of the Hazard Rate Function of TPRD for varying values of the parameters θ and α.

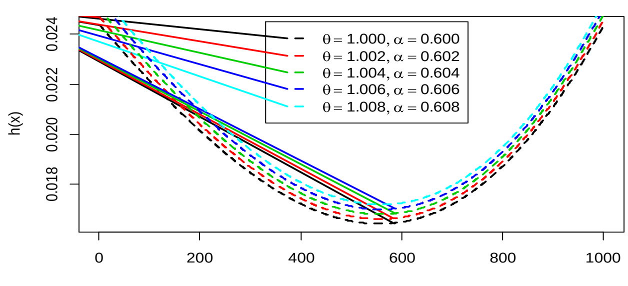

Figure 8. The second plot of the Hazard Rate Function of TPRD for varying values of the parameters θ and α.

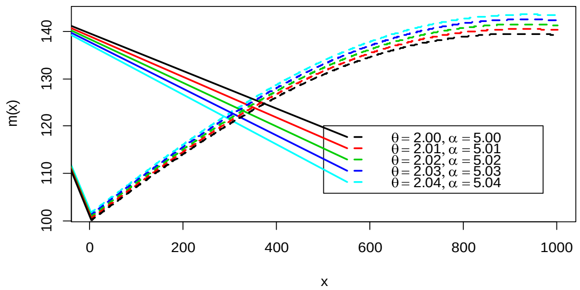

Figure 9. The first plot of Mean Residual Life Function of TPRD for varying values of the parameters θ and α.

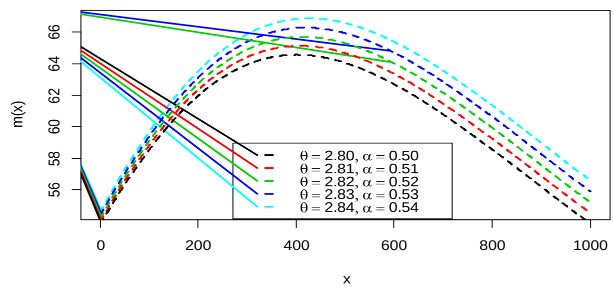

Figure 10. The second plot of Mean Residual Life Function of TPRD for varying values of the parameters θ and α.

5. STOCHASTIC ORDERING

Stochastic ordering of positive continuous random variables is an important tool for judging the comparative behavior. A random variable X is said to be smaller than a random variable Y in the

(i) Stochastic order $X \leq_{st} Y$ if $F_X(x) \geq F_Y(x)$ for all x.
(ii) Hazard rate function$X \leq_{hr} Y$ if $h_X(x) \geq h_Y(x)$ for all x.
(iii) Mean residual life function$X \leq_{mrl} Y$ if $m_X(x) \geq m_Y(x)$ for all x.
(iv) Likelihood ratio order $X \leq_{lr} Y$ if $\frac{f_X(x)}{f_Y(x)}$ decreases in x

The following results due to [3] are well known for establishing stochastic ordering of distributions.

$$(X \leq_{lr} Y) \Rightarrow (X \leq_{hr} Y) \Rightarrow (X \leq_{mrl} Y)$$
$$\Downarrow$$
$$(X \leq_{st} Y)$$

The proposed two-parameter Suja distribution (TPSD) is ordered with respect to the strongest likelihood ratio ordering as shown in the following theorem.

Theorem. Let X be two-parameter Suja distribution(α_1, θ_1) and Y be two-parameter Suja distribution (α_2, θ_2). If $\alpha_1 = \alpha_2$ and $\theta_1 \geq \theta_2$ (or if$\theta_1 = \theta_2$ and $\alpha_1 \geq \alpha_2$), then $X \leq_{lr} Y$and hence $X \leq_{hr} Y, X \leq_{mrl} Y$ and $X \leq_{st} Y$.

Proof. We have

$$\frac{f(x;\theta_1,\alpha_1)}{f(x;\theta_2,\alpha_2)} = \frac{\frac{\theta_1^5}{\alpha_1\theta_1^4+24}(\alpha_1 + x^4)e^{-\theta_1 x}}{\frac{\theta_2^5}{\alpha_2\theta_2^4+24}(\alpha_2 + x^4)e^{-\theta_2 x}}$$

$$\frac{\theta_1^5(\alpha_2\theta_2^4 + 24)}{\theta_2^5(\alpha_1\theta_1^4 + 24)}\frac{(\alpha_1 + x^4)}{(\alpha_2 + x^4)}e^{-(\theta_1-\theta_2)x}; x > 0$$

Now

$$ln\frac{f(x;\theta_1,\alpha_1)}{f(x;\theta_2,\alpha_2)} = ln\left[\frac{\theta_1^5(\alpha_2\theta_2^4 + 24)}{\theta_2^5(\alpha_1\theta_1^4 + 24)}\right] + ln\left[\frac{(\alpha_1 + x^4)}{(\alpha_2 + x^4)}\right] - (\theta_1 - \theta_2)x$$

$$\frac{d}{dx}ln\frac{f(x;\theta_1,\alpha_1)}{f(x;\theta_2,\alpha_2)} = \frac{4x^3(\alpha_2+\alpha_1)}{(\alpha_1+x^4)(\alpha_2+x^4)} - (\theta_1 - \theta_2). \quad (5.1)$$

Thus, for ($\theta_1 > \theta_2$ and $\alpha_1 = \alpha_2$) or ($\alpha_1 > \alpha_2$ and $\theta_1 = \theta_2$)

$$\frac{d}{dx}ln\frac{f(x;\theta_1,\alpha_1)}{f(x;\theta_2,\alpha_2)} < 0$$

This implies that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y, X \leq_{mrl} Y$ and $X \leq_{st} Y$. This shows the flexibility of the two-parameter Suja distribution over the Suja distribution.

6. ENTROPY MEASURE

An entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi entropy by [4]. If X is a continuous random variable having probability density function f(.), then Renyi entropy is defined as:

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\{\int f^{\gamma}(x)\,dx\} \quad (6.1)$$

Where $\gamma > 0$ *and* $\gamma \neq 1$

Thus, the Renyi entropy for the two-parameter Suja distribution (2.1) is obtained as

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left[\int_0^\infty \frac{\theta^{5\gamma}}{(\alpha\theta^4+24)^\gamma}(\alpha+x^4)^\gamma e^{-\theta\gamma x}dx\right]$$

$$= \frac{1}{1-\gamma}log\left[\int_0^\infty \frac{\theta^{5\gamma}\alpha^\gamma}{(\alpha\theta^4+24)^\gamma}\left(1+\frac{x^4}{\alpha}\right)^\gamma e^{-\theta\gamma x}dx\right]$$

$$= \frac{1}{1-\gamma}log\left[\int_0^\infty \frac{\theta^{5\gamma}\alpha^\gamma}{(\alpha\theta^4+24)^\gamma}\sum_{j=0}^\infty \binom{\gamma}{j}\left(\frac{x^4}{\alpha}\right)^j e^{-\theta\gamma x}dx\right]$$

$$= \frac{1}{1-\gamma}log\left[\sum_{j=0}^\infty \binom{\gamma}{j}\frac{\theta^{5\gamma}\alpha^{\gamma-j}}{(\alpha\theta^4+24)^\gamma}\int_0^\infty x^{4j+1-1}e^{-\theta\gamma x}dx\right]$$

$$= \frac{1}{1-\gamma}log\left[\sum_{j=0}^\infty \binom{\gamma}{j}\frac{\theta^{5\gamma}\alpha^{\gamma-j}}{(\alpha\theta^4+24)^\gamma}\frac{\Gamma(4j+1)}{(\theta\gamma)^{4j+1}}\right]$$

$$= \frac{1}{1-\gamma}log\left[\sum_{j=0}^\infty \binom{\gamma}{j}\frac{\theta^{5\gamma-4j-1}\alpha^{\gamma-j}}{(\alpha\theta^4+24)^\gamma}\frac{\Gamma(4j+1)}{(\gamma)^{4j+1}}\right] \quad (6.2)$$

7. BONFERRONI AND LORENZ CURVES

The Bonferroni and Lorenz curves [5] and Bonferroni and Gini indices have applications not only in economics to study income and poverty, but also in other fields like reliability, demography, insurance and medicine. The Bonferroni and Lorenz curves are defined as:

$$B(p) = \frac{1}{p\mu}\int_0^q xf(x)dx = \frac{1}{p\mu}\left[\int_0^\infty xf(x)dx - \int_q^\infty xf(x)dx\right] = \frac{1}{p\mu}\left[\mu - \int_q^\infty xf(x)dx\right] \quad (7.1)$$

And

$$L(p) = \frac{1}{\mu}\int_0^q xf(x)dx = \frac{1}{\mu}\left[\int_0^\infty xf(x)dx - \int_q^\infty xf(x)dx\right] = \frac{1}{\mu}\left[\mu - \int_q^\infty xf(x)dx\right] \quad (7.2)$$

respectively or equivalently

$B(p) = \frac{1}{p\mu}\int_0^p F^{-1}(x)dx$(7.3)

And

$$L(p) = \frac{1}{\mu}\int_0^p F^{-1}(x)dx \quad (7.4)$$

respectively, where $\mu = E(X)$ and$q = F^{-1}(p)$.

The Bonferroni and Gini indices are thus defined as

$B = 1 - \int_0^1 B(p)dp$(7.5)

And

$$G = 1 - 2\int_0^1 L(p)dp \quad (7.6)$$

respectively.

Using the pdf of the two-parameter Suja distribution (2.1), we get

$$\int_q^\infty xf(x)dx) = \frac{\{\theta^5(q^5+\alpha q)+\theta^4(5q^4+\alpha)+20\theta^2q^2(\theta q+3)+120(\theta q+1)\}e^{-\theta q}}{\theta(\alpha\theta^6+24)} \quad (7.7)$$

Now using equation (7.7) in (7.1) and (7.2), we get

$$B(p) = \frac{1}{p}\left[1 - \frac{\{\theta^5(q^5+\alpha q)+\theta^4(5q^4+\alpha)+20\theta^2q^2(\theta q+3)+120(\theta q+1)\}e^{-\theta q}}{(\alpha\theta^6+120)}\right] \quad (7.8)$$

And

$$L(p) = 1 - \frac{\{\theta^5(q^5+\alpha q)+\theta^4(5q^4+\alpha)+20\theta^2q^2(\theta q+3)+120(\theta q+1)\}e^{-\theta q}}{(\alpha\theta^6+120)} \quad (7.9)$$

Now using equations (7.8) and (7.9) in (7.5) and (7.6), the Bonferroni and Gini indices are obtained as

$$B = 1 - \frac{\{\theta^5(q^5+\alpha q)+\theta^4(5q^4+\alpha)+20\theta^2q^2(\theta q+3)+120(\theta q+1)\}e^{-\theta q}}{(\alpha\theta^6+120)} \quad (7.10)$$

$$G = -1 + \frac{\{\theta^5(q^5+\alpha q)+\theta^4(5q^4+\alpha)+20\theta^2 q^2(\theta q+3)+120(\theta q+1)\}e^{-\theta q}}{(\alpha\theta^6+120)} \quad (7.11)$$

8. STRESS-STRENGTH RELIABILITY

The stress-strength reliability describes the life of a component which has random strength X that is subjected to a random stress Y. When the stress applied to it exceeds the strength, the component fails instantly and the component will function satisfactorily till X >Y. Therefore, R= P(Y<X)is a measure of component reliability and in statistical literature it is known as stress-strength parameter. It has wide applications in almost all areas of knowledge especially in engineering such as structures, deterioration of rocket motors, static fatigue of ceramic components, aging of concrete pressure vessels etc.

Let Y and X be independent stress and strength random variables having the two-parameter Suja distribution with parameter (α_1, θ_1) and (α_2, θ_2) respectively. Then, the stress-strength reliability R is defined as

R = P[Y<X] = $\int_0^\infty$ P[Y $< X \mid$ X $= x$] $f_x(x)dx$

$=\int_0^\infty f(x;\theta_1,\alpha_1)\ F(x;\theta_2,\alpha_2)dx$

$$= 1 - \frac{\theta_1^5\begin{bmatrix}\alpha_1\alpha_2\theta_2^4(\theta_1+\theta_2)^8+24\alpha_2\theta_2^4(\theta_1+\theta_2)^4+24\alpha_1\theta_2^4(\theta_1+\theta_2)^4+40320\theta_2^4\\ +24\alpha_1\theta_2^3(\theta_1+\theta_2)^5+20160\theta_2^3(\theta_1+\theta_2)+24\alpha_1\theta_2^2(\theta_1+\theta_2)^6+8640\theta_2^2\\ (\theta_1+\theta_2)^2+24\alpha_1\theta_2(\theta_1+\theta_2)^7+2880\theta_2(\theta_1+\theta_2)^3+24\alpha_1(\theta_1+\theta_2)^8+\\ 576(\theta_1+\theta_2)^4\end{bmatrix}}{(\alpha_1\theta_1^4+24)(\alpha_2\theta_2^4+24)(\theta_1+\theta_2)^9} \quad (8.1)$$

It can be easily verified that at $\alpha_1 = \alpha_2 = 1$, the above expression reduces to the corresponding expression for Suja distribution of [3].

9. PARAMETER ESTIMATION

In this section, the estimations of parameters of TPSD using method of maximum likelihood was discussed.

9.1. Method of Maximum Likelihood Estimates

Let $L(x_1, x_2, \dots, x_n;\ \theta, \alpha)$ be a random sample from the two-parameter Suja distribution in (2.1). The likelihood function, L of (2.1) is given by

$$L(x_1, x_2, \dots, x_n; \theta, \alpha) = \prod_{i=1}^{n}[f(x_i;\theta,\alpha)]$$

$$L = \left(\frac{\theta^5}{\alpha\theta^4+24}\right)^n \prod_{i=1}^{n}(\alpha + x_i^4)e^{-n\theta\bar{x}}$$

Where $\bar{x}$ is the sample mean. The natural log likelihood function is thus obtained as

$$\ln(L) = n\ln\left(\frac{\theta^5}{\alpha\theta^4+24}\right) + \sum_{i=1}^{n}\ln(\alpha + x_i^4) - n\theta\bar{x}$$

The maximum likelihood estimates, $\hat{\theta}$ of θ and $\hat{\alpha}$ of α is the solution of the equation $\frac{d\ln(L)}{d\theta} = 0$ and $\frac{d\ln(L)}{d\alpha} = 0$ and it can be obtained by solving the following non-linear equations

$$\frac{d\ln(L)}{d\theta} = \frac{5n}{\theta} - \frac{4\alpha n\theta^3}{\alpha\theta^4+24} - n\bar{x} = 0 \quad (9.1)$$

Where $\bar{x}$ is the sample mean

$$\frac{d\ln(L)}{d\alpha} = \frac{-n\theta^4}{\alpha\theta^4+24} + \sum_{i=1}^{n}\frac{1}{(\alpha+x_i^4)} = 0 \quad (9.2)$$

These two natural log likelihoods cannot be solved directly because they cannot be expressed in closed forms. The MLE's of θ and α can be computed directly using R-software by solving the natural log likelihood equations using the Newton-Raphson iteration method until sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained.

10. APPLICATION OF THE TWO-PARAMETER SUJA DISTRIBUTION

In this section the study demonstrated the application of TPSD with a real-life data set remission times (in months) of a random sample of 128 bladder cancer patients given in [6] using maximum likelihood estimate. The data set is given as follows:

0.08 2.09 2.73 3.48 4.87 6.94 8.66 13.11 23.63 0.20 2.22 3.52 4.98 6.99 9.02 13.29 0.40 2.26 3.57 5.06 7.09 9.22 13.80 25.74 0.50 2.46 3.64 5.09 7.26 9.47 14.24 25.82 0.51 2.54 3.70 5.17 7.28 9.74 14.76 26.31 0.81 2.62 3.82 5.32 7.32 10.06 14.77 32.15 2.64 3.88 5.32 7.39 10.34 14.83 34.26 0.90 2.69 4.18 5.34 7.59 10.66 15.96 36.66 1.05 2.69 4.23 5.41 7.62 10.75 15.62 43.01 1.19 2.75 4.26 5.41 7.63 17.12 46.12 1.26 2.83 4.33 5.49 7.66 11.25 17.14 79.05 1.35 2.87 5.62 7.87 11.64 17.36 1.40 3.02 4.34 5.71 7.93 11.79 18.10 1.46 4.40 5.85 8.26 11.98 19.13 1.76 3.25 4.50 6.25 8.37 12.02 2.02 3.31 4.51 6.54 8.53 12.03 20.28 2.02 3.36 6.93 8.65 12.63 22.69.

The goodness of fit of TPSD was compared with Suja and Rama distributions. To compare the lifetime distributions, -lnL, AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), CAIC (Consistent Akaike Information Criterion), HQIC (Hannan-Quinn Information Criterion) and K-S Statistics (Kolmogorov-Smirnov Statistics) for the real-life data set remission times (in months) of a random sample of 128 bladder cancer patients was computed. The AIC, BIC, CAIC, HQIC and K-S Statistics were computed using the methods below:

AIC = $-2\log L + 2k$

BIC = $-2\log L + \log(n)k$

$\text{CAIC} = -2\log_e\left[L(\hat{\theta})\right] + k[\log_e(n) + 1]$

$\text{HQIC} = -2L_{\max} + 2k\log(\log(n))$

K-S= $\sup_x |F_n(x) - F_0(x)|$

Where; $\log L$ and $L_{\max}$ are the maximized values of the log-likelihood function of the proposed two parameter Suja distribution, n is the sample size, k is the number of parameters, $F_n(x)$is the empirical distribution function and $\sup_x$is the Supremum of the set of distances. The best model is the distribution with the lower values of -lnL, AIC, BIC, CAIC, HQIC, and K-S Statistics.

The MLE $(\hat{\theta}, \hat{\alpha})$ of θ and α, S.E $(\hat{\theta}, \hat{\alpha})$, standard error of θ and α, -lnL, AIC, BIC, CAIC, HQIC, and K-S Statistics of the proposed distribution are presented in the table below:

Table 1. MLE $(\hat{\theta})$, S.E $(\hat{\theta})$, -lnL, AIC, CAIC, BIC, HQIC and K-S Statistics of the fitted distributions to data set

Model	**-lnL**	**MLE $(\hat{\theta})$**	**S.E**	**HQIC**	**BIC**	**CAIC**	**AIC**	**K.S**
TPSD	428.9638	$\hat{\theta} = 0.3679$ $\hat{\alpha} = 850.04$	0.019946 275.6763	864.2190	867.5682	862.0268	861.9276	0.16058
RD	465.3464	$\hat{\theta} = 0.4255$	0.018931	933.8384	935.5130	932.7255	932.6927	0.25896
SD	505.4894	$\hat{\theta} = 0.5355$	0.021420	1014.125	1015.799	1013.012	1012.979	0.29840

It can be easily deduced from table 1 that the two-parameter Suja distribution (TPSD) out performs and gives a better fit than the one parameter Suja, distribution (SD) and the Rama distribution (RD), hence the two parameter Suja distribution can considered an important lifetime distribution for modelling lifetime data.

11. CONCLUSION

A two-parameter Suja distribution has been proposed. Its statistical properties including moments, survival function, hazard rate function, mean residual life function, stochastic ordering, entropy measure, Bonferroni and Lorenz curves, stress-strength reliability have been discussed. Maximum likelihood estimation has been discussed for estimating its parameters. Finally, the goodness of fit test using -lnL, AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), CAIC (Consistent Akaike Information Criterion), HQIC (Hannan-Quinn Information Criterion) and K-S Statistics (Kolmogorov-Smirnov Statistics) for a real-life data set has been presented and the fit has been compared with other lifetime distributions. The study compared Rama, one parameter Suja and the proposed two parameter Suja distribution and found out that the Two parameter Suja distribution out performs the Rama and the one parameter Suja in modelling life time data.

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