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INVERSE SUJA DISTRIBUTION AND ITS APPLICATIONS

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Abstract

A new one parameter lifetime distribution named "Inverse Suja distribution" for modeling lifetime data has been introduced. Some important statistical properties of the proposed distribution including its shape characteristics of the density, hazard rate function, survival function, stochastic ordering, entropy measure, stress-strength reliability have been discussed. The maximum likelihood estimation of its parameter has been discussed. Two real data sets were employed in illustrating the usefulness of the new distribution and it was found that the Inverse Suja distribution provides a better fit when compared to the Suja distribution, Rama distribution, Inverse Lindley distribution, and Inverse Akash distribution.

***Keywords*:** Inverse Suja distribution, Stress-strength reliability, Statistical properties, Maximum likelihood estimator and Goodness of fit

1. INTRODUCTION

Statistical distributions are very useful in statistics. For instance, the statistical distribution of a statistic is required for the construction of confidence intervals estimators. Also, the residuals from a linear regression model, analysis of variance model and time series model are often assumed to come from certain statistical distribution. In lifetime analysis, one is required to know the distribution of lifetime data for effective development of lifetime models. Several distributions are used for this purpose like the Exponential distribution, the Gamma distribution, the Weibull distribution, the Lognormal distribution among others.

A new lifetime distribution for modeling lifetime data called Lindley distribution was introduced by [1]. The Lindley and Exponential distributions are popular for modeling lifetime data from biomedical science and engineering. But, [2] have conducted a comparative study on modeling of lifetime data from various fields of knowledge and observed that there are many lifetime data where these two distributions are not suitable due to their shapes, nature of hazard rate functions, and mean residual life, amongst others. In [3,4,5,6,7,8], Shanker introduced some one-parameter lifetime distributions namely; Akash, Shanker, Amarendra, Aradhana, Sujatha, and Devya, and showed that these distributions give better fit than the classical Exponential and Lindley distributions. Each of these lifetime distributions has advantages and disadvantages over one another due to its shape, hazard rate function and mean residual life function. There are many situations where these distributions are not suitable for modeling lifetime data from theoretical or applied point of view. It has been shown that the Suja distribution as a probability model can be a better model than the well-known Akash, Shanker, Amarendra, Aradhana, Sujatha, and Devya distributions in some particular cases [9].

The probability density function (PDF) and the cumulative distribution function (CDF) of Suja distribution are given by;

$$f(x;\theta) = \frac{\theta^5}{\theta^4+24}(1+x^4)e^{-\theta x}, \quad x>0, \theta>0 \qquad (1.1)$$

$$F(x;\theta) = 1-\left[1+\frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x}{\theta^4+24}\right]e^{-\theta x}, \ x>0, \theta>0 \qquad (1.2)$$

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The PDF (1.1) is a two-component mixture of an Exponential distribution having scale parameter (θ) and a Gamma distribution having shape parameter 5 and scale parameter (θ) with their mixing proportions $\frac{\theta^4}{\theta^4+24}$. A detailed study about its various mathematical properties, estimation of parameter and application showing the superiority of Suja distribution over Lindley, Akash, Sujatha, Amarendra, Aradhana, Devya, and Exponential distributions [9].

Since, the Suja distribution is only applicable of modeling to the monotonic increasing hazard rate data, its applicability is restricted to the data that show non-monotone shapes (bathtub and upside-down bathtub) for their hazard rate. Though various article has found in literature that address the analysis of bathtub shape data, limited attention has been paid to the analysis of upside-down bathtub shape data. According to [10], considering the fact that all inverse distribution possesses the upside-down bathtub shape for their hazard rates function, we propose an inverted version of the Suja distribution that can be effectively used to model the upside-down bathtub shape hazard rate data.

If a random variable Y has a Suja distribution SD(θ), then the random variable X = $(^1/_Y)$ is said to be follow the Inverse Suja distribution (ISD) having a scale parameter θ with its probability density function (PDF), is defined by

$$f(x;\theta) = \frac{\theta^5}{\theta^4+24}\left(\frac{1+x^4}{x^6}\right)e^{-\frac{\theta}{x}}, x > 0, \theta > 0 \tag{1.3}$$

It is denoted by ISD(θ). The cumulative distribution function (CDF) of Inverse Akash distribution is given by

$$F(x;\theta) = \left[1 + \frac{\theta^3+4\theta^2x+12\theta x^2+24x^3}{\theta^4+24}\left(\frac{\theta}{x^4}\right)\right]e^{-\frac{\theta}{x}}, x > 0, \theta > 0 \tag{1.4}$$

Since this continuous distribution has the nice closed form expressions for the CDF, hazard function as well as stress-strength reliability, its relevance for survival analysis can never be denied in the literate.

The aim of this paper is in two phases. The first phase is to derive and study the properties of Inverse Suja distribution. The second phase is to compare the Inverse Suja distribution with Suja distribution, Rama distribution, Inverse Lindley distribution, and Inverse Akash distribution.

2. STATISTICAL PROPERTIES OF INVERSE SUJA DISTRIBUTION

2.1 Shape Characteristics of the Density

The first derivation of (1.3) is given by

$$\frac{d}{dx}f(x) = -\left(\frac{\theta^5}{\theta^4+24}\right)\frac{e^{-\frac{\theta}{x}}}{x^8}(2x^5 - (\theta x^3 - 6)x - \theta) \tag{2.1}$$

Thus, the graphs of the PDF and CDF are given in Figure 1 and Figure 2 respectively.

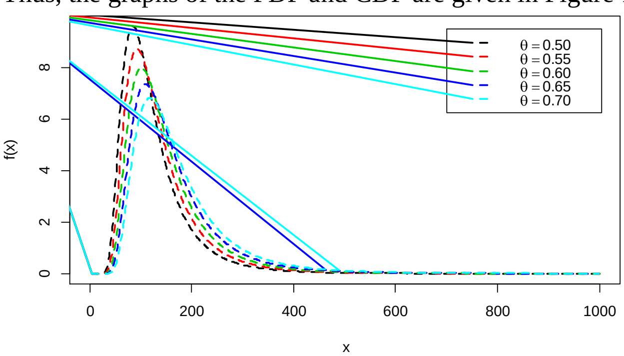

Figure 1. *Plot of the PDF of Inverse Suja distribution for varying values of the parameter θ.*

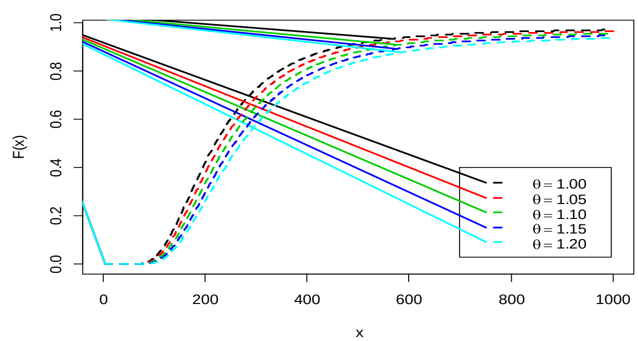

Figure 2. *Plot of the CDF of Inverse Suja distribution for varying values of the parameter θ.*

2.2. Survival Function

Let X be a continuous random variable with CDF $F(x)$. The survival function, $s(x)$, of x is defined as

$$s(x) = 1 - F(x) \tag{2.2}$$

The corresponding survival function $s(x)$, of the Inverse Suja distribution (ISD) is given by

$$s(x) = 1 - \left[1 + \frac{\theta^3+4\theta^2x+12\theta x^2+24x^3}{\theta^4+24}\left(\frac{\theta}{x^4}\right)\right]e^{-\frac{\theta}{x}} \tag{2.3}$$

Thus, the graph of the survival function is showcased in Figure 3.

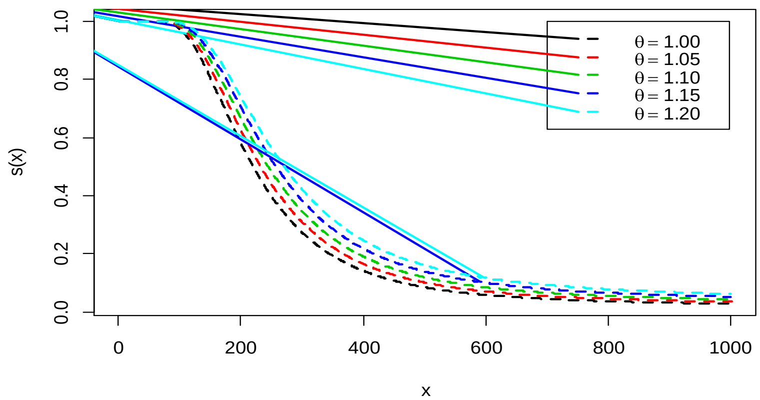

Figure 3. *Plot of the Survival function of Inverse Suja distribution for varying values of the parameter θ.*

2.3. Hazard Rate Function

Let X be a continuous random variable with PDF $f(x)$ and CDF $F(x)$. The hazard rate function, $h(x)$, of x is defined as

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{f(x)}{s(x)} \tag{2.4}$$

The corresponding hazard rate function $h(x)$, of the Inverse Suja distribution (ISD) is given by

$$h(x) = \frac{\theta^5(1+x^4)}{x^2\left[x^4(\theta^4+24)\left(e^{\frac{\theta}{x}}-1\right)-\theta^3(\theta+4x)-12\theta x^2(\theta+2x)\right]} \tag{2.5}$$

Thus, a pictorial representation of the hazard rate function is showcased in Figure 4..

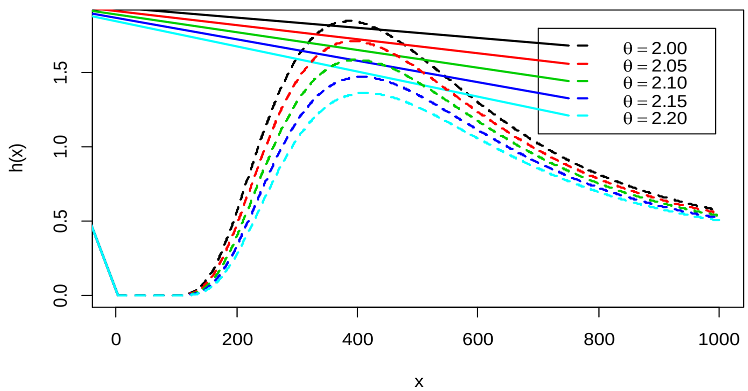

Figure 4. *Plot of the Hazard Rate function of Inverse Suja distribution for varying values of the parameter θ.*

2.4. Stochastic Ordering

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. According to [2], a random variable X is said to be smaller than a random variable Y in the;

i. Stochastic order $(X \leq_{st} Y)$ if $F_X(x) \geq F_Y(x)$ for all x.
ii. Hazard rate order $(X \leq_{hr} Y)$ if $h_X(x) \geq h_Y(x)$ for all x.
iii. Mean residual life order $(X \leq_{mrl} Y)$ if $m_X(x) \leq m_Y(x)$ for all x.
iv. Likelihood ratio order $(X \leq_{lr} Y)$ if $\left(\frac{f_X(x)}{f_Y(x)}\right)$ decreases in x.

The following results due to [11] are well known for establishing stochastic ordering of distributions

$$\begin{array}{ccccc}(X \leq_{lr} Y) & \Rightarrow & (X \leq_{hr} Y) & \Rightarrow & (X \leq_{mrl} Y)\\ & & \Downarrow & & \\ & & (X \leq_{st} Y) & & \end{array}$$

The Inverse Suja distributions are ordered with respect to the strongest likelihood ratio ordering as shown in the following theorem.

Theorem: Let $X \sim ISD(\theta_1)$ and $Y \sim ISD(\theta_2)$.
If $\theta_1 \geq \theta_2$, then $(X \leq_{lr} Y)$ and hence $(X \leq_{hr} Y)$, $(X \leq_{mrl} Y)$ and $(X \leq_{st} Y)$
Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\theta_1^5(\theta_2^4+24)e^{-\frac{\theta_1}{x}}}{\theta_2^5(\theta_1^4+24)e^{-\frac{\theta_2}{x}}}$$

$$= \frac{\theta_1^5(\theta_2^4+24)}{\theta_2^5(\theta_1^4+24)}e^{-\frac{(\theta_1-\theta_2)}{x}}; x > 0 \qquad (2.6)$$

$$\ln\frac{f_X(x)}{f_Y(x)} = \ln\left[\frac{\theta_1^5(\theta_2^4+24)}{\theta_2^5(\theta_1^4+24)}\right] - \left[\frac{(\theta_1-\theta_2)}{x}\right]$$

This gives

$$\frac{d}{dx}\ln\frac{f_X(x)}{f_Y(x)} = \frac{(\theta_1-\theta_2)}{x^2} \qquad (2.7)$$

Thus, for $\theta_1 \geq \theta_2$, $\frac{d}{dx}\ln\frac{f_X(x)}{f_Y(x)} > 0$. This means that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

2.5. Entropy Measure

Entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi entropy [12]. If X is a continuous random variable having probability density function $f(.)$, then Renyi entropy is defined as

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left\{\int f^\gamma(x)\,dx\right\}$$

Where $\gamma > 0$ *and* $\gamma \neq 1$

For the Inverse Suja distribution, the Renyi entropy measure is defined by;

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\int_0^\infty \frac{\theta^{5\gamma}}{(\theta^4+24)^\gamma}\left[\frac{(1+x^4)^\gamma}{x^{6\gamma}}\right]e^{-\frac{\theta\gamma}{x}}dx$$

We know that $(1+z)^j = \sum_{j=0}^{\infty}\binom{\gamma}{j}z^j$ and $\int_0^\infty e^{-\frac{b}{x}}x^{-a-1}dx = \frac{\Gamma(a)}{b^a}$

$$= \frac{1}{1-\gamma}\log\left[\frac{\theta^{5\gamma}}{(\theta^4+24)^\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\int_0^\infty \frac{e^{-\frac{\theta\gamma}{x}}}{x^{6\gamma-4j}}\,dx\right]$$

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left[\frac{\theta^{5\gamma}}{(\theta^4+24)^\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\Gamma(6\gamma-4j-1)}{(\theta\gamma)^{6\gamma-4j-1}}\right] \qquad (2.8)$$

3. STRESS-STRENGTH RELIABILITY AND MAXIMUM LIKELIHOOD ESTIMATION

Let Y and X be independent stress and strength random variables that follow Inverse Akash distribution with parameter θ_1 *and* θ_2 respectively. Then, the stress-strength reliability R is defined as

R = P[Y<X] = $\int_0^\infty$ P[$Y < X \mid X = x$] $f_x(x)$dx =$\int_0^\infty f(x,\theta_1)\ F(x,\theta_2)dx$

$$= \int_0^\infty\left(\left[1 + \frac{\theta_2^3+4\theta_2^2x+12\theta_2x^2+24x^3}{\theta_2^4+24}\left(\frac{\theta_2}{x^4}\right)\right]e^{-\frac{\theta_2}{x}}\right)\frac{\theta_1^5}{\theta_1^4+24}\left(\frac{1+x^4}{x^6}\right)e^{-\frac{\theta_1}{x}}dx$$

$$= \frac{\theta_1^5}{\theta_1^4+24}\int_0^\infty\left(\frac{1+x^4}{x^6}\right)e^{-\frac{(\theta_1+\theta_2)}{x}}dx + \frac{\theta_1^5\theta_2}{(\theta_1^4+24)(\theta_2^4+24)}\int_0^\infty\left(\frac{1+x^4}{x^6}\right)\left(\frac{\theta_2^3+4\theta_2^2x+12\theta_2x^2+24x^3}{x^4}\right)e^{-\frac{(\theta_1+\theta_2)}{x}}dx$$

$$= \frac{\theta_1^5}{\theta_1^4+24}\left[\int_0^\infty x^{-6}e^{-\frac{(\theta_1+\theta_2)}{x}}dx + \int_0^\infty x^{-2}e^{-\frac{(\theta_1+\theta_2)}{x}}dx\right]$$

$$+ \frac{\theta_1^5\theta_2}{(\theta_1^4+24)(\theta_2^4+24)}\int_0^\infty\left(\frac{\theta_2^3+4\theta_2^2x+12\theta_2x^2+24x^3+\theta_2^3x^4+4\theta_2^2x^5+12\theta_2x^6+24x^7}{x^{10}}\right)e^{-\frac{(\theta_1+\theta_2)}{x}}dx$$

Using the definitions of ISD and Inverse Gamma distribution (IGD), we get the expression for the stress-strength reliability as;

$$R = \frac{\theta_1^5}{\theta_1^4+24}\left[\frac{\Gamma(5)}{(\theta_1+\theta_2)^5} + \frac{\Gamma(1)}{(\theta_1+\theta_2)}\right] + \frac{\theta_1^5\theta_2}{(\theta_1^4+24)(\theta_2^4+24)}\left[\theta_2^3\left(\frac{\Gamma(9)}{(\theta_1+\theta_2)^9} + \frac{\Gamma(5)}{(\theta_1+\theta_2)^5}\right) + 4\theta_2^2\left(\frac{\Gamma(8)}{(\theta_1+\theta_2)^8} + \frac{\Gamma(4)}{(\theta_1+\theta_2)^4}\right) + 12\theta_2\left(\frac{\Gamma(7)}{(\theta_1+\theta_2)^7} + \frac{\Gamma(3)}{(\theta_1+\theta_2)^3}\right) + 24\left(\frac{\Gamma(6)}{(\theta_1+\theta_2)^6} + \frac{\Gamma(2)}{(\theta_1+\theta_2)^2}\right)\right]$$

$$R = \theta_1^5 \left[\frac{\begin{array}{c}[\{24+(\theta_1+\theta_2)^4\}\{(\theta_2^4+24)(\theta_1+\theta_2)^4\}]+\theta_2[362880\theta_2^3+24\theta_2^3(\theta_1+\theta_2)^4 \\ +161280\theta_2^2(\theta_1+\theta_2)+24\theta_2^2(\theta_1+\theta_2)^5+60480\theta_2(\theta_1+\theta_2)^2+24\theta_2(\theta_1+\theta_2)^4 \\ +17280(\theta_1+\theta_2)^3+24(\theta_1+\theta_2)^7]\end{array}}{(\theta_1^4+24)(\theta_2^4+24)(\theta_1+\theta_2)^9} \right] \quad (3.1)$$

Since R is the Stress-Strength Reliability function with parameters θ_1 and θ_2, we need to obtain the maximum likelihood estimators (MLEs) of θ_1 and θ_2 to compute the maximum likelihood estimation R under Invariance property of the maximum likelihood estimation.

Suppose $X_1, X_2, \dots, X_n$ is a Strength randon variable sample from Inverse Suja distribution (θ_1) and $Y_1, Y_2, \dots, Y_m$ is a Stress random sample from Inverse Suja distribution (θ_2). Thus, the likelihood function based on the observed sample is given by;

$$L\left(\theta_1, \theta_2 / \underline{x}, \underline{y}\right) = \frac{\theta_1^{5n}\theta_2^{5m}}{(\theta_1^4+24)^n(\theta_2^4+24)^m} \prod_{i=1}^{n}\left(\frac{1+x_i^4}{x_i^6}\right) \prod_{j=1}^{m}\left(\frac{1+y_j^4}{y_j^6}\right) e^{-(\theta_1 S_1+\theta_2 S_2)} \quad (3.2)$$

Where; $S_1 = \sum_{i=1}^{n}\frac{1}{x_i}$, $S_2 = \sum_{j=1}^{m}\frac{1}{y_j}$

The log-Likelihood function is given by;

$$\ln L(\theta_1, \theta_2) = 5n\ln(\theta_1) + 5m\ln(\theta_2) - n\ln(\theta_1^4 + 24) - mln(\theta_2^4 + 24) - \theta_1 S_1 - \theta_2 S_2 + \sum_{i=1}^{n}\ln\left(\frac{1+x_i^4}{x_i^6}\right) + \sum_{j=1}^{m}\ln\left(\frac{1+y_j^4}{y_j^6}\right) \quad (3.3)$$

The Maximum Likelihood Estimators (MLE) of $\theta_1$and θ_2, say $\hat{\theta}_1$ and $\hat{\theta}_2$ respectively can be obtained as the solution of the following eguations;

$$\frac{\partial \log L(\theta_1,\theta_2)}{\partial \theta_1} = \frac{5n}{\theta_1} - \frac{4\theta_1^3 n}{(\theta_1^4+24)} - S_1 \quad (3.4)$$

$$\frac{\partial \log L(\theta_1,\theta_2)}{\partial \theta_2} = \frac{5m}{\theta_2} - \frac{4\theta_2^3 m}{(\theta_2^4+24)} - S_2 \quad (3.5)$$

From (3.4) and (3.5), obtain MLEs as

$$\Rightarrow \theta_1^5 S_1 - \theta_1^4 n + 24\theta_1 S_1 - 120n = 0 \quad (3.6)$$

$$\Rightarrow \theta_2^5 S_2 - \theta_2^4 m + 24\theta_2 S_2 - 120m = 0 \quad (3.7)$$

Hence, using the invariance property of the MLE, the maximum likelihood estimator $\hat{R}_{mle}$ of R can be obtained by substituting $\hat{\theta}_k$ in place of θ_k for k = 1, 2.

$$\boldsymbol{\hat{R}_{mle} = \theta_1^5} \left[\frac{\begin{array}{c}[\{24+(\theta_1+\theta_2)^4\}\{(\theta_2^4+24)(\theta_1+\theta_2)^4\}]+\theta_2[362880\theta_2^3+24\theta_2^3(\theta_1+\theta_2)^4 \\ +161280\theta_2^2(\theta_1+\theta_2)+24\theta_2^2(\theta_1+\theta_2)^5+60480\theta_2(\theta_1+\theta_2)^2+24\theta_2(\theta_1+\theta_2)^4 \\ +17280(\theta_1+\theta_2)^3+24(\theta_1+\theta_2)^7]\end{array}}{(\theta_1^4+24)(\theta_2^4+24)(\theta_1+\theta_2)^9} \right]_{|\theta_k=\theta_k, k=1,2} \quad (3.8)$$

4. PARAMETER ESTIMATION

In this section, the estimations of parameters of ISD using method of maximum likelihood was discussed.

4.1. Method of Maximum Likelihood Estimate

Let $L(x_1, x_2, \dots, x_n;\ \theta)$ be a random sample from the Inverse Suja distribution (1.3). The likelihood function, L of (1.3) is given by

$$L(x_1, x_2, \dots, x_n; \theta) = \prod_{i=1}^{n}[f(x_i; \theta)]$$

$$= \frac{\theta^{5n}}{(\theta^4 + 24)^n}\prod_{i=1}^{n}\left(\frac{1 + x_i^4}{x_i^6}\right)e^{-\theta S}$$

Where; $S = \sum_{i=1}^{n}\frac{1}{x_i}$. The Log-Likelihood is given by;

$$\ln L(\theta) = 5n\ln(\theta) - n\ln(\theta^4 + 24) - \theta S + \sum_{i=1}^{n}\ln\left(\frac{1 + x_i^2}{x_i^4}\right)$$

The MLE of θ, say $\hat{\theta}$ can be obtained as the solution of the equation;

$$\frac{\partial \log L(\theta)}{\partial\theta} = \frac{5n}{\theta} - \frac{4n\theta^3}{(\theta^4+24)} - S \quad (4.1)$$

$$\Rightarrow \theta^5 S - \theta^4 n + 24\theta S - 120n = 0 \quad (4.2)$$

Notably, the analytic solution of (4.2) can be obtained. As a consequence, we can apply a numerical method to solve (4.2). The numerical solution of (4.2) can be found using R software.

5. GOODNESS OF FIT

The Inverse Suja distribution (ISD) was applied to two real life data sets (Data 1 and Data 2) in order to assess its statistical superiority over other models; the Suja distribution, Rama distribution, Inverse Lindley distribution, and Inverse Akash distribution, to demonstrate that the theoretical results in the previous sections can be used in practice. The data sets represent the failure times (in minutes) for a sample of 15 electronic components in an accelerated life test and the lifetimes of 50 devices.

The data sets are as follows;

Data 1:

1.4 5.1 6.3 10.8 12.1 18.5 19.7 22.2 23.0 30.6 37.3 46.3 53.9 59.8 66.2.

Data 2:

0.1 0.2 1 1 1 1 1 2 3 6 7 11 12 18 18 18 18 18 21 32 36 40 45 46 47 50 55
60 63 63 67 67 67 67 72 75 79 82 82 83 84 84 84 85 85 85 85 85 86 86.

First we checked the validity of the Inverse Suja distribution for the given data sets by using Akaike information criterion (AIC), Bayesian information criterion (BIC), Negative Log-Likelihood Function (-L), Consistent Akaike Information Criterion (CAIC), Hannan-Quinn Information Criterion (HQIC), Standard Error Estimate of the Parameter (SE), and The Estimate of the Parameter.

We compared the applicability of Inverse Suja distribution with competing one parameter distributions, Suja distribution (SD), Rama distribution (RD), Inverse Lindley distribution (ILD), and Inverse Akash distribution (IAD), based on real data sets.

For Data 1: The performance of the ISD with respect to the SD, RD, ILD, and IAD using the observations in Data 1 is shown in Table 1.

Table 1: *Performance Ratings of Inverse Suja distribution Using Data 1.*

Model	**MLE**	**Estimates**	**S.E**	**HQIC**	**BIC**	**CAIC**	**AIC**
ISD	68.86353	$\hat{\theta} = 9.66419$	2.429651	139.7195	140.4351	140.0348	139.7271
IAD	68.86451	$\hat{\theta} = 9.94269$	2.428580	139.7215	140.4371	140.0367	139.7290
SD	74.58538	$\hat{\theta} = 0.18150$	0.020956	151.1632	151.8788	151.4784	151.1708
RD	70.45831	$\hat{\theta} = 0.14515$	0.018731	142.9091	143.6247	143.2243	142.9166
ILD	69.13491	$\hat{\theta} = 10.3981$	2.484438	140.2623	140.9779	140.5775	140.2698

From Table 1, the ISD has the lowest -L value of 68.86353, the lowest AIC value of 139.7271, the lowest BIC value of 140.4351, the lowest HQIC value of 139.7195, and the lowest CAIC value of 140.0348 therefore, the ISD provides a better fit than the SD, RD, ILD and IAD.

For Data 2: The performance of the ISD with respect to the SD, RD, ILD, and IAD using the observations in Data 2 is shown in Table 2.

Table 2: *Performance Ratings of Inverse Suja distribution Using Data 2.*

Model	**MLE**	**Estimates**	**S.E**	**HQIC**	**BIC**	**CAIC**	**AIC**
ISD	310.9512	$\hat{\theta} = 3.50234$	0.266722	624.6304	625.8143	623.9856	623.9023
IAD	317.6248	$\hat{\theta} = 3.05607$	0.310793	637.9778	639.1617	637.3330	637.2497
SD	327.0049	$\hat{\theta} = 0.10944$	0.006921	656.7379	657.9219	656.0932	656.0098
RD	299.4706	$\hat{\theta} = 0.08758$	0.006191	601.6693	602.8532	601.0245	600.9412
ILD	324.0412	$\hat{\theta} = 2.84648$	0.334030	650.8105	651.9944	650.1657	650.0824

From Table 2, the ISD has the lowest -L value of 310.9512, the lowest AIC value of 623.9023, the lowest BIC value of 625.8143, the lowest HQIC value of 624.6304, and the lowest CAIC value of 623.9856 therefore, the ISD provides a better fit than the SD, RD, ILD, and IAD.

6. CONCLUSION

A one parameter lifetime distribution named, "Inverse Suja distribution" has been proposed. Its statistical properties including shape characteristics of density, survival function, hazard rate function, stochastic ordering has been discussed. Further, expressions for entropy measure and, Stress-Strength Reliability of the proposed distribution have been derived. The method of maximum likelihood estimation has also been discussed for estimating its parameter. Finally, the goodness of fit test using –L, AIC, BIC, HQIC, and CAIC based on two real lifetime data sets, the applicability and superiority over Suja, Rama, Inverse Lindley, and Inverse Akash distributions while modeling certain lifetime data have been established.

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