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AN EOQ MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH TWO PHASE DEMAND RATES AND TWO-LEVEL PRICING STRATEGIES UNDER TRADE CREDIT POLICY.

B. Babangida[1*] and Y. M. Baraya[2]

[1]Department of Mathematics and Statistics, Umaru Musa Yar'adua University, P.M.B. 2218, Katsina, Nigeria.
[2]Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria.

Abstract

In some classical inventory models for non-instantaneous deteriorating items, it is tacitly assumed that the selling price before and after deterioration sets in is the same. However, in real practice, when deterioration sets in, the retailer may decide to reduce the selling price in order to encourage more sales, reduces the cost of holding stock, attracts new customers and reduces lost due to deterioration. In this paper, an EOQ model for non-instantaneous deteriorating items with two phase demand rates and two-level pricing strategies under trade credit policy is considered. It is assumed that the unit selling price before deterioration sets in is greater than that after deterioration sets in. Also, the demand rate before deterioration sets in is assumed to be continuous time-dependent quadratic and that after deterioration sets in is considered as constant and shortages are not allowed. The main purpose of this research work is to determine the optimal cycle length and corresponding economic order quantity such that the total profit of the inventory system is optimise. The necessary and sufficient conditions for the existence and uniqueness of the optimal solutions have been established. Some numerical examples have been given to illustrate the theoretical result of the model. Sensitivity analysis of some model parameters on the decision variables has been carried out and suggestions toward maximising the total profit were also given.

***Keywords*:** economic order quantity; non-instantaneous deteriorating items; two phase demand rate rates; two-level pricing strategies; trade credit policy.

1 Introduction

Most inventory models for non-instantaneous deteriorating items assumed that the unit selling price before and after deterioration sets in is the same. However, in real practice, the unit selling price before and after deterioration sets in differs and this assumption need to be considered in developing inventory policies for non-instantaneous deteriorating items, where the objective function is to maximise the total profit of the inventory system. Tsao and Sheen [1] presented dynamic pricing, promotion and replenishment policies for deteriorating items when a delay in payments is permissible. Lee and Hsu [2] developed a two warehouse production model for instantaneous deteriorating inventory items with time-dependent demands rates, a finite replenishment rate within a finite planning horizon. Tsao [3] developed two-phase pricing and inventory decisions for deteriorating and fashion goods under permissible delay in payments. The demand rate vary with price or time, shortages are allowed and partially backlogged, and the objective function is to maximise profit. Chen and Kang [4] developed integrated inventory models considering the two-level trade credit policy and a price negotiation scheme, in which customers' demand is sensitive to the buyers' price. Wang *et al.* [5] proposed a dynamic pricing inventory model for non-instantaneous deteriorating items, where the objective function is to maximise the total

Corresponding Author**:** Babangida B., Email: bature.babangida@umyu.edu.ng, Tel: +2347067704150

profit per unit time, and both uniform pricing and two-stage pricing models are developed and a comparative study between dynamic and uniform pricing shows the advantage of dynamic pricing over uniform pricing. Tsao *et al.* [6] developed two-tiered pricing and ordering policies for non-instantaneous deteriorating items with price-sensitive demand rates under permissible delay in payments. Moreover, some related studies on inventory models with two-phase or two-period pricing strategies can be found in Dye [7], Dye and Hsieh [8], Sainathan [9], Herbon [10], Tayal *et al.* [11] and so on.

In this paper model, an EOQ model for non-instantaneous deteriorating items with two phase demand rates and two-level pricing strategies under trade credit policy is considered. The demand rate before deterioration sets in is assumed to be time-dependent quadratic and that is considered as constant after deterioration sets in. It is also assumed that the unit selling price before deterioration sets in is greater than that after deterioration sets in. Shortages are not allowed. To the best of authors' knowledge, this type of EOQ model, with the above assumptions, has not yet been discussed in inventory literature. The main aim of this research is to develop an EOQ model that will determine the optimal cycle length and lot size so as to maximise the total profit per unit. The necessary and sufficient conditions for the existence and uniqueness of the optimal solutions will be discussed. Some numerical examples have been given to illustrate the theoretical result of the model. Sensitivity analyses of some model parameters on optimal solutions will also be carry out and suggestions toward maximising the total profit per unit time will also be given.

2. Model Descriptions and Formulation

This section describes model notation, assumptions and formulation.

2.1 Notation

The units and parameters used in this model are described in Table 1 below:

Table 1 Notation used in the proposed model

Parameters	Description	Units
A	Cost of placing an order	\$/order
C	Unit purchasing cost	\$/unit/ unit time
S_1	Unit selling price during the interval $[0,\ t_d]$	\$/unit/ unit time
S_2	Unit selling price during the interval $[t_d, T]$, where $S_1 > S_2 > C$	\$/unit/ unit time
h	Inventory holding cost (excluding interest charges)	\$/unit/ unit time
I_c	Interest charged by the supplier	%/\$/ unit time
I_e	Interest earned by the retailer, where $I_c \geq I_e$	%/\$/ unit time
M	Trade credit period offered by the supplier/manufacturer to the retailer for settling accounts	unit time
θ	Deterioration rate, where $0 < \theta < 1$.	%
t_d	Length of time in which the product exhibits no deterioration	unit time
T	Length of the replenishment cycle	unit time
$TP(T)$	Total profit	\$/unit time
Q	Quantity of items received at the begging of the inventory system	Units
T^*	Optimal length of the replenishment cycle (decision variable)	unit time
$TP(T^*)$	Optimal total profit (decision variable)	\$/unit time
EOQ^*	Optimal order quantity (decision variable)	Units
$I_1(t)$	Inventory level before deterioration sets in	Units
$I_2\ \ (t)$	Inventory level after deterioration sets in	Units

Assumptions

This model is developed under the following assumptions:

1. The replenishment rate is infinite, i.e., the replenishment rate is instantaneous and the lead time is zero.
2. The planning horizon is infinite and only one single type of non-instantaneous deteriorating item is modelled.
3. During the fixed period, t_d, there is no deterioration and at the end of this period, the inventory item deteriorates at the rate θ.
4. There is no replacement or repair for deteriorated items.
5. Demand rate before deterioration begins is assumed to be continuous time-dependent quadratic and is given by $\alpha + \beta t + \gamma t^2$ where $\alpha \geq 0, \beta \neq 0, \gamma \neq 0$.
   Here α is the initial demand rate, β is the rate at which the demand rate changes and γ is the rate of change at which the demand rate changes itself.
6. Demand rate after deterioration sets in is assumed to be constant and is given by $\lambda, \lambda > 0$.

7. During the trade credit period M $(0 < M < 1)$, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock.
8. No interest earn after the trade credit periods.
9. The unit selling price is not the same as unit purchasing cost. It is assumed that the unit selling price before deterioration sets in is greater than that after deterioration sets in $(S_1 > S_2 > C)$.
10. Shortages are not allowed to occur.

2.2 Formulation of the model

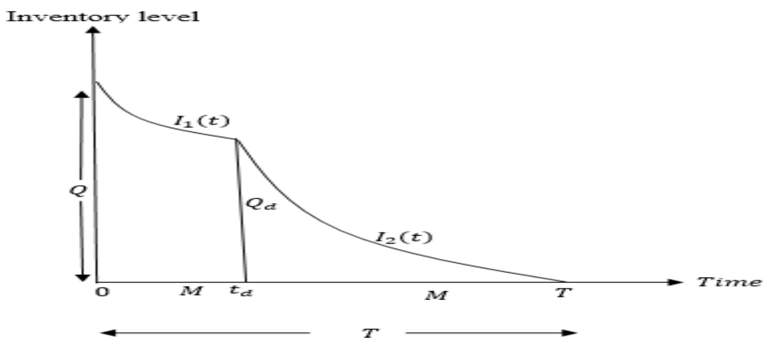

Figure 1: Graphical representation of the Inventory system

The behaviours of the inventory system are described in figure 1. Q units of a single product from the manufacturer/supplier are received at the beginning of each cycle (i.e., at time $t = 0$). During the time interval $[0, t_d]$, the inventory level $I_1(t)$ is depleting gradually due to market demand only and is assumed to be continuous quadratic function of time. At time interval $[t_d, T]$, the inventory level $I_2(t)$ is depleting due to combined effects of demand from the customers and deterioration and the demand rate at time t_d is reduced to λ. At time $t = T$, the inventory level depletes to zero. The differential equations which describe the change of the inventory level during $[0, T]$ are given below:

$$\frac{dI_1(t)}{dt} = -(\alpha + \beta t + \gamma t^2), \qquad 0 \le t \le t_d \qquad (1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -\lambda, \qquad t_d \le t \le T \qquad (2)$$

with boundary conditions $I_1(0) = Q$, $I_1(t_d) = Q_d$, $I_2(T) = 0$ and $I_2(t_d) = Q_d$.
The solution of equations (1) and (2) are using the above conditions as follows:

$$I_1(t) = \frac{\lambda}{\theta}\left(e^{\theta(T-t_d)} - 1\right) + \alpha(t_d - t) + \frac{\beta}{2}(t_d^2 - t^2) + \frac{\gamma}{3}(t_d^3 - t^3), \qquad 0 \le t \le t_d \qquad (3)$$

and

$$I_2(t) = \frac{\lambda}{\theta}\left(e^{\theta(T-t)} - 1\right), \qquad t_d \le t \le T \qquad (4)$$

From Figure 1, using the condition $I_1(0) = Q$ at $t = 0$ in equation (3), the initial stock level in any cycle, Q, is obtained as follows:

$$Q = \frac{\lambda}{\theta}\left(e^{\theta(T-t_d)} - 1\right) + \left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) \qquad (5)$$

Moreover, from Figure 1, the stock level after deterioration sets in, Q_d, could be derived at $t = t_d$ from equation (4) as follows:

$$Q_d = \frac{\lambda}{\theta}\left(e^{\theta(T-t_d)} - 1\right) \qquad (6)$$

a The ordering cost per order is given by A

b The inventory holding cost during the period $[0, T]$ is given by

$$C_H = h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right] \tag{7}$$

Substituting equations (3) and (4) into equation (7) yields

$$C_H = h\left[\frac{\lambda t_d}{\theta} e^{\theta(T-t_d)} + \frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 + \frac{\lambda}{\theta^2}\left(e^{\theta(T-t_d)} - 1 - \theta T\right)\right] \tag{8}$$

c Purchasing cost *(PC)*

$$PC = CQ$$
$$= \frac{\lambda}{\theta}\left(e^{\theta(T-t_d)} - 1\right) + \left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) \tag{9a}$$

d Sale revenue *(SR)*

$$SR = S_1\left[\int_0^{t_d}(\alpha + \beta t + \gamma t^2)dt\right] + S_2\left[\int_{t_d}^{T}\lambda dt\right]$$

$$= S_1\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + S_2\lambda(T - t_d) \tag{9b}$$

e The total profit per unit time for a replenishment cycle *(*denoted by $TP(T\ )$ is given by

$$TP(T\ ) = \begin{cases} TP_1(T\ ) & 0 < M \le t_d \\ TP_2(t_{1,}T\ ) & t_d < M \le T \end{cases} \tag{10}$$

where $TP_1(T\ )$ and $TP_2(T\ )$ are discussed as follows:

Case 1 $(0 < M \le t_d)$

The Interest Payable

This is the period before deterioration sets in, and payment for goods is settled with the capital opportunity cost rate I_c for the items in stock. Thus, the interest payable is computed as follows:

$$I_{P1} = cI_c\left[\int_M^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right]$$

$$= cI_c\left[\frac{\lambda(t_d - M)}{\theta}\left(e^{\theta(T-t_d)} - 1\right) + \frac{\alpha}{2}(t_d - M)^2 + \frac{\beta}{6}(2t_d + M)(t_d - M)^2 + \frac{\gamma}{12}(3t_d^2 + 2t_dM + M^2)(t_d - M)^2 + \frac{\lambda}{\theta^2}\left(e^{\theta(T-t_d)} - 1 - \theta(T - t_d)\right)\right] \tag{11}$$

The interest Earn

In this case, the retailer can earn interest on revenue generated from the sales up to the trade credit period M. Although, the retailer has to settle the accounts at period M, for that money has to arrange at some specified rate of interest in order to get his remaining stocks financed for the period M to t_d. The interest earn is computed as follows:

$$I_{E1} = S_1 I_e\left[\int_0^{M}(\alpha + \beta t + \gamma t^2)tdt\right]$$
$$= S_1 I_e\left(\alpha\frac{M^2}{2} + \beta\frac{M^3}{3} + \gamma\frac{M^4}{4}\right) \tag{12}$$

The total profit per unit time for case 1 ($0 < M \leq t_d$) is given by

$TP_1(T) = \frac{1}{T}$\{ Sale revenue-ordering cost - inventory holding cost - purchasing cost - interest payable during the permissible delay period + interest earned during the cycle\}

$$= \frac{1}{T}\left\{(S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + S_2\lambda(T - t_d) - A - h\left[\frac{\lambda t_{\mathrm{d}}}{\theta}e^{\theta(T-t_{\mathrm{d}})} + \frac{\alpha}{2}t_{\mathrm{d}}^2 + \frac{\beta}{3}t_{\mathrm{d}}^3 + \frac{\gamma}{4}t_{\mathrm{d}}^4 + \frac{\lambda}{\theta^2}\left(e^{\theta(T-t_{\mathrm{d}})} - 1 - \theta T\right)\right]\right.$$
$$- C\left[\frac{\lambda}{\theta}\left(e^{\theta(T-t_d)} - 1\right)\right]$$
$$- cI_c\left[\frac{\lambda(t_{\mathrm{d}} - M)}{\theta}\left(e^{\theta(T-t_{\mathrm{d}})} - 1\right) + \frac{\alpha}{2}(t_{\mathrm{d}} - M)^2 + \frac{\beta}{6}(2t_{\mathrm{d}} + M)(t_{\mathrm{d}} - M)^2\right.$$
$$\left.+ \frac{\gamma}{12}(3t_{\mathrm{d}}^2 + 2t_{\mathrm{d}}M + M^2)(t_{\mathrm{d}} - M)^2 + \frac{\lambda}{\theta^2}\left(e^{\theta(T-t_{\mathrm{d}})} - 1 - \theta(T - t_{\mathrm{d}})\right)\right]$$
$$\left.+ S_1I_e\left(\alpha\frac{M^2}{2} + \beta\frac{M^3}{3} + \gamma\frac{M^4}{4}\right)\right\} \quad (13)$$

Case 2 $(t_d < M \leq T)$

The interest payable

This is when the end point of credit period is greater than the period with no deterioration but shorter than or equal to the length of period with positive inventory stock of the items. The interest payable is computed as follows:

$$I_{P2} = cI_c\left[\int_M^T I_2(t)dt\right]$$

$$= cI_c\left[\frac{\lambda}{\theta^2}\left(e^{\theta(T-M)} - 1 - \theta(T - M)\right)\right] \quad (14)$$

The interest earned

In this case, the retailer can earn interest on revenue generated from the sales up to the trade credit period M. Although, the retailer has to settle the accounts at period M, for that money has to arrange at some specified rate of interest in order to get his remaining stocks financed for the period M to T. The interest earn is computed as follows:

$$I_{E2} = S_1I_e\left[\int_0^{t_d}(\alpha + \beta t + \gamma t^2)tdt\right] + S_2I_e\left[\int_{t_d}^M \lambda tdt\right]$$

$$= S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) + S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right) \quad (15)$$

The total profit per unit time for case 2 ($t_d < M \leq T$) is given by

$TP_2(T) = \frac{1}{T}$\{ Sale revenue-Ordering cost - inventory holding cost - purchasing cost - interest payable during the permissible delay period + interest earned during the cycle\}

$$= \frac{1}{T}\left\{(S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + S_2\lambda(T - t_d) - A - h\left[\frac{\lambda t_{\mathrm{d}}}{\theta}e^{\theta(T-t_{\mathrm{d}})} + \frac{\alpha}{2}t_{\mathrm{d}}^2 + \frac{\beta}{3}t_{\mathrm{d}}^3 + \frac{\gamma}{4}t_{\mathrm{d}}^4 + \frac{\lambda}{\theta^2}\left(e^{\theta(T-t_{\mathrm{d}})} - 1 - \theta T\right)\right]\right.$$
$$- C\left[\frac{\lambda}{\theta}\left(e^{\theta(T-t_d)} - 1\right)\right] - cI_c\left[\frac{\lambda}{\theta^2}\left(e^{\theta(T-M)} - 1 - \theta(T - M)\right)\right] + S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right)$$
$$\left.+ S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right)\right\} (16)$$

Since $0 < \theta < 1$, by utilizing a quadratic approximation for the exponential terms in equations (13) and (16) yields

$$TP_1(T) = \frac{1}{T}\Bigg\{(S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda T - (S_2 - C)\lambda t_d - A$$
$$- h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2} + \frac{\lambda t_d\theta T^2}{2} + \frac{\lambda T^2}{2} - \lambda t_d^2\theta T\right) - \frac{C\lambda\theta}{2}(t_d^2 + T^2 - 2t_d T)$$
$$- cI_c\Bigg[\frac{\alpha}{2}(t_d - M)^2 + \frac{\beta}{6}(2t_d + M)(t_d - M)^2 + \frac{\gamma}{12}(3t_d^2 + 2t_d M + M^2)(t_d - M)^2 - \frac{\lambda t_d^2}{2} + M\lambda t_d$$
$$+ \frac{\lambda\theta}{2}(t_d - M)t_d^2 + \frac{\lambda T^2}{2} + \frac{\lambda\theta}{2}(t_d - M)T^2 - MT\lambda - \lambda\theta(t_d - M)t_d T\Bigg]$$
$$+ S_1 I_e\left(\alpha\frac{M^2}{2} + \beta\frac{M^3}{3} + \gamma\frac{M^4}{4}\right)\Bigg\} \quad (17)$$

and

$$TP_2(T) = \frac{1}{T}\Bigg\{(S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda T - (S_2 - C)\lambda t_d - A$$
$$- h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2} + \frac{\lambda t_d\theta T^2}{2} + \frac{\lambda T^2}{2} - \lambda t_d^2\theta T\right) - \frac{C\lambda\theta}{2}(t_d^2 + T^2 - 2t_d T)$$
$$- cI_c\frac{\lambda}{2}[M^2 + T^2 - 2MT] + S_1 I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) + S_2 I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right)\Bigg\} \quad (18)$$

3 Optimal Decision

This section determines the optimal ordering policy that maximises the total profit per unit time. The necessary and sufficient conditions for the existence and uniqueness of solutions are established. The necessary condition for the total profit per unit time $TP_i(T)$ to be maximum is $\frac{TP_i(T)}{dT} = 0$ for $= 1, 2$. The value of T obtained from $\frac{TP_i(T)}{dT} = 0$ and for which the sufficient condition $\frac{d^2TP_i(T)}{dT^2} < 0$ is satisfied gives maximum total profit per unit time $TP_i(T)$.

For $(0 < M \leq t_d)$

The necessary and sufficient conditions for the total profit $TP_1(T)$ to be maximum are $\frac{dTP_1(T)}{dT} = 0$ and $\frac{d^2TP_1(T)}{dT^2} < 0$ respectively, which give

$$\frac{dTP_1(T)}{dT} = \frac{1}{T^2}\Bigg\{-\frac{T^2}{2}\lambda[h(t_d\theta + 1) + C\theta + cI_c(\theta(t_d - M) + 1)]$$
$$+ \Bigg[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2}\right) + \frac{C\lambda\theta}{2}t_d^2$$
$$+ cI_c\left(\frac{\alpha}{2}(t_d - M)^2 + \frac{\beta}{6}(2t_d + M)(t_d - M)^2 + \frac{\gamma}{12}(3t_d^2 + 2t_d M + M^2)(t_d - M)^2 - \frac{\lambda t_d^2}{2} + M\lambda t_d\right.$$
$$\left.+ \frac{\lambda\theta}{2}(t_d - M)t_d^2\right) - S_1 I_e\left(\alpha\frac{M^2}{2} + \beta\frac{M^3}{3} + \gamma\frac{M^4}{4}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d\Bigg]\Bigg\}$$
$$= 0 \quad (19)$$

And this implies that

$$\frac{1}{T^2}\left\{-\frac{T^2}{2}\lambda[h(t_d\theta+1)+C\theta+cI_c(\theta(t_d-M)+1)]\right.$$
$$+\left[A+h\left(\frac{\alpha}{2}t_d^2+\frac{\beta}{3}t_d^3+\frac{\gamma}{4}t_d^4-\frac{\lambda t_d^2}{2}+\frac{\lambda t_d^3\theta}{2}\right)+\frac{C\lambda\theta}{2}t_d^2\right.$$
$$+cI_c\left(\frac{\alpha}{2}(t_d-M)^2+\frac{\beta}{6}(2t_d+M)(t_d-M)^2+\frac{\gamma}{12}(3t_d^2+2t_dM+M^2)(t_d-M)^2-\frac{\lambda t_d^2}{2}+M\lambda t_d\right.$$
$$\left.\left.\left.+\frac{\lambda\theta}{2}(t_d-M)t_d^2\right)-S_1I_e\left(\alpha\frac{M^2}{2}+\beta\frac{M^3}{3}+\gamma\frac{M^4}{4}\right)-(S_1-C)\left(\alpha t_d+\beta\frac{t_d^2}{2}+\gamma\frac{t_d^3}{3}\right)+(S_2-C)\lambda t_d\right]\right\}$$
$$=0 \qquad (20)$$

From equation (20), for notational convenience, let

$X_1=\lambda[h(t_d\theta+1)+C\theta+cI_c(\theta(t_d-M)+1)]$ and

$$X_2=\left[A+h\left(\frac{\alpha}{2}t_d^2+\frac{\beta}{3}t_d^3+\frac{\gamma}{4}t_d^4-\frac{\lambda t_d^2}{2}+\frac{\lambda t_d^3\theta}{2}\right)+\frac{C\lambda\theta}{2}t_d^2\right.$$
$$+cI_c\left(\frac{\alpha}{2}(t_d-M)^2+\frac{\beta}{6}(2t_d+M)(t_d-M)^2+\frac{\gamma}{12}(3t_d^2+2t_dM+M^2)(t_d-M)^2-\frac{\lambda t_d^2}{2}+M\lambda t_d\right.$$
$$\left.\left.+\frac{\lambda\theta}{2}(t_d-M)t_d^2\right)-S_1I_e\left(\alpha\frac{M^2}{2}+\beta\frac{M^3}{3}+\gamma\frac{M^4}{4}\right)-(S_1-C)\left(\alpha t_d+\beta\frac{t_d^2}{2}+\gamma\frac{t_d^3}{3}\right)+(S_2-C)\lambda t_d\right].$$

Since $M\le t_d$, it should be noted that $X_1=\lambda[h(t_d\theta+1)+C\theta+cI_c(\theta(t_d-M)+1)]>0$. Substituting X_1 and X_2 into equation (20) yields

$$\frac{1}{T^2}\left\{-\frac{T^2}{2}X_1+X_2\right\}=0 \qquad (21)$$

which is equivalent to

$$T^2X_1-2X_2=0 \qquad (22)$$

Let

$$\Delta_1=-S_1I_e\left(\alpha\frac{M^2}{2}+\beta\frac{M^3}{3}+\gamma\frac{M^4}{4}\right)-(S_1-C)\left(\alpha t_d+\beta\frac{t_d^2}{2}+\gamma\frac{t_d^3}{3}\right)+(S_2-C)\lambda t_d$$
$$+\left[A+h\left(\frac{\alpha}{2}t_d^2+\frac{\beta}{3}t_d^3+\frac{\lambda}{4}t_d^4-\lambda t_d^2\right)\right.$$
$$\left.+cI_c\left(\frac{\alpha}{2}(t_d-M)^2+\frac{\beta}{6}(2t_d+M)(t_d-M)^2+\frac{\gamma}{12}(3t_d^2+2t_dM+M^2)(t_d-M)^2-\lambda t_d^2+M\lambda t_d\right)\right].$$

Then the following result is obtained

Lemma 1.

i If $\Delta_1\ge 0$, then the solution of $T\in[t_d,\infty)$ (say T_1^*) which satisfies equation (22) does not only exists but also unique.

ii If $\Delta_1<0$, then the solution of $T\in[t_d,\infty)$ which satisfies equation (22) does not exist.

Proof of part (i). From equation (20), a new function $F_1(T)$ is defined as follows:

$$F_1(T) = -\frac{T^2}{2}\lambda[h(t_d\theta + 1) + C\theta + cI_c(\theta(t_d - M) + 1)] + \left[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2}\right) + \frac{C\lambda\theta}{2}t_d^2 + cI_c\left(\frac{\alpha}{2}(t_d - M)^2 + \frac{\beta}{6}(2t_d + M)(t_d - M)^2 + \frac{\gamma}{12}(3t_d^2 + 2t_dM + M^2)(t_d - M)^2 - \frac{\lambda t_d^2}{2} + M\lambda t_d + \frac{\lambda\theta}{2}(t_d - M)t_d^2\right) - S_1I_e\left(\alpha\frac{M^2}{2} + \beta\frac{M^3}{3} + \gamma\frac{M^4}{4}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d\right], \quad T \in [t_d, \infty) \quad (23)$$

Taking the first-order derivative of $F_1(T)$ with respect to $T \in [t_d, \infty)$ yields

$$\frac{F_1(T)}{dT} = -TX_1 < 0$$

Hence $F_1(T)$ is a strictly decreasing of T in the interval $[t_d, \infty)$. Moreover, $\lim_{T\to\infty} F_1(T) = -\infty$ and $F_1(t_d) = \Delta_1 \geq 0$. Therefore, by applying intermediate value theorem, there exists a unique value of T say $T_1^* \in [t_d, \infty)$ such that $F_1(T_1^*) = 0$. Hence T_1^* is the unique solution of equation (22). Thus, the value of T (denoted by T_1^*) can be found from equation (22) and is given by

$$T_1^* = \sqrt{\frac{2X_2}{X_1}} \quad (24)$$

Proof of part (ii). If $\Delta_1 < 0$, then from equation (23), $F_1(T) < 0$. Since $F_1(T)$ is a strictly decreasing function of $T \in [t_d, \infty)$ and $F_1(T) < 0$ for all $T \in [t_d, \infty)$. Thus, a value of $T \in [t_d, \infty)$ such that $F_1(T) = 0$ cannot found. This completes the proof.

Theorem 1

i If $\Delta_1 \geq 0$, then the total profit $TP_1(T)$ is concave and reaches its global maximum at the point $T_1^* \in [t_d, \infty)$, where T_1^* is the point which satisfies equation (22).

ii If $\Delta_1 < 0$, then the total profit $TP_1(T)$ has a maximum value at the point $T_1^* = t_d$.

Proof of part (i). When $\Delta_1 \geq 0$, it is seen that T_1^* is the unique solution of equation (22) from Lemma l(i). Taking the second derivative of $TP_1(T)$ with respect to T, and then finding the value of the function at the point of T_1^* yields

$$\left.\frac{d^2\,TP_1(T)}{dT^2}\right|_{T_1^*} = -\frac{1}{T_1^*}\{\lambda[h(t_d\theta + 1) + C\theta + cI_c(\theta(t_d - M) + 1)]\} = -\frac{X_1}{T_1^*} < 0 \quad (25)$$

It is therefore concluded from equation (25) and Lemma 1 that $TP_1(T_1^*)$ is concave and T_1^* is the global maximum point of $TP_1(T)$. Hence the value of T in equation (24) is optimal.

Proof of part (ii). When $\Delta_1 < 0$, then $F_1(T) < 0$ for all $T \in [t_d, \infty)$. Thus, $\frac{dTP_1(T)}{dT} = \frac{F_1(T)}{T^2} < 0$ for all $T \in [t_d, \infty)$ which implies $TP_1(T)$ is a strictly decreasing function of T. Thus $TP_1(T)$ has a maximum value when T is minimum. Therefore, $TP_1(T)$ has a maximum value at the point $T = t_d$. This completes the proof.

For $(t_d < M \leq T)$

The necessary and sufficient conditions for the total profit $TP_2(T)$ to be maximum are respectively $\frac{dTP_2(T)}{dT} = 0$ and $\frac{d^2TP_2(T)}{dT^2} < 0$, which give

$$\frac{dTP_2(T)}{dT} = \frac{1}{T^2}\left\{-\frac{T^2}{2}\lambda[h(t_d\theta + 1) + C\theta + cI_c] + \left[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2}\right) + \frac{C\lambda\theta}{2}t_d^2 + cI_c\frac{\lambda}{2}M^2 - S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) - S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d\right]\right\} = 0 \tag{26}$$

And this implies that

$$\frac{1}{T^2}\left\{-\frac{T^2}{2}\lambda[h(t_d\theta + 1) + C\theta + cI_c] + \left[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2}\right) + \frac{C\lambda\theta}{2}t_d^2 + cI_c\frac{\lambda}{2}M^2 - S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) - S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d\right]\right\} = 0 \tag{27}$$

From equation (27), for notational convenience, let

$Y_1 = \lambda[h(t_d\theta + 1) + C\theta + cI_c]$ and

$$Y_2 = \left[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2}\right) + \frac{C\lambda\theta}{2}t_d^2 + cI_c\frac{\lambda}{2}M^2 - S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) - S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d\right].$$

It should be noted that $Y_1 = \lambda[h(t_d\theta + 1) + C\theta + cI_c] > 0$. Substituting Y_1 and Y_2 into equation (27) yields

$$\frac{1}{T^2}\left\{-\frac{T^2}{2}Y_1 + Y_2\right\} = 0 \tag{28}$$

which is equivalent to

$$T^2Y_1 - 2Y_2 = 0 \tag{29}$$

Let

$$\Delta_2 = -S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) - S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d + \left[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2} - \frac{\lambda t_d\theta M^2}{2} - \frac{\lambda M^2}{2}\right) + \frac{C\lambda\theta}{2}(t_d^2 - M^2)\right].$$

Then the following result is obtained

Lemma 2.

i If $\Delta_2 \geq 0$, then the solution of $T \in [M, \infty)$ (say T_2^*) which satisfies equation (29) does not only exists but also unique.

ii If $\Delta_2 < 0$, then the solution of $T \in [M, \infty)$ which satisfies equation (29) does not exist.

Proof of part (i). From equation (27), a new function $F_2(T)$ is defined as follows:

$$F_2(T) = -\frac{T^2}{2}\lambda[h(t_d\theta+1)+C\theta+cI_c] + \left[A + h\left(\frac{\alpha}{2}t_d^2 + \frac{\beta}{3}t_d^3 + \frac{\gamma}{4}t_d^4 - \frac{\lambda t_d^2}{2} + \frac{\lambda t_d^3\theta}{2}\right) + \frac{C\lambda\theta}{2}t_d^2 + cI_c\frac{\lambda}{2}M^2 - S_1I_e\left(\alpha\frac{t_d^2}{2} + \beta\frac{t_d^3}{3} + \gamma\frac{t_d^4}{4}\right) - S_2I_e\left(\frac{\lambda M^2}{2} - \frac{\lambda t_d^2}{2}\right) - (S_1 - C)\left(\alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3}\right) + (S_2 - C)\lambda t_d\right], \quad T \in [M,\infty) \quad (30)$$

The first-order derivative of $F_2(T)$with respect to $T \in [M,\infty)$ yields

$$\frac{F_2(T)}{dT} = -TY_1 < 0$$

Hence $F_2(T)$ is a strictly decreasing of T in the interval $[M,\infty)$. Moreover, $\lim_{T\to\infty} F_2(T) = -\infty$ and $F_2(M) = \Delta_2 \geq 0$. Therefore, by applying intermediate value theorem, there exists a unique value of T say $T_2^* \in [M,\infty)$ such that $F_2(T_2^*) = 0$. Hence T_2^* is the unique solution of equation (29). Thus, the value of T (denoted by T_2^*) can be found from equation (29) and is given by

$$T_2^* = \sqrt{\frac{2Y_2}{Y_1}} \quad (31)$$

Proof of part (ii). If $\Delta_2 < 0$, then from (30), $F_2(T) < 0$. Since $F_2(T)$ is a strictly decreasing function of $T \in [M,\infty)$, $F_2(T) < 0$ for all $T \in [M,\infty)$. Thus, a value of $T \in [M,\infty)$ such that $F_2(T) = 0$ cannot found. This completes the proof.

Theorem 2.

i If $\Delta_2 \geq 0$, then the total profit $TP_2(T)$ is concave and reaches its global maximum at the point $T_2^* \in [M,\infty)$, where T_2^* is the point which satisfies equation (29).

ii If $\Delta_2 < 0$, then the total profit $TP_2(T)$ has a maximum value at the point $T_2^* = M$.

Proof of part (i). When $\Delta_2 \geq 0$, it is seen that T_2^* is the unique solution of equation (29) from Lemma 2(i). Taking the second derivative of $TP_2(T)$ with respect to T and finding the value of the function at the point of T_2^* yields

$$\left.\frac{d^2TP_2(T)}{dT^2}\right|_{T_2^*} = -\frac{1}{T_2^*}\{\lambda[h(t_d\theta+1)+C\theta+cI_c]\}$$

$$= -\frac{Y_1}{T_2^*} < 0 \quad (32)$$

It is therefore concluded from equation (32) that $TP_2(T_2^*)$ is concave and T_2^* is the global maximum point of $TP_2(T)$. Hence the value of T in equation (31) is optimal.

Proof of part (ii). When $\Delta_2 < 0$, then $F_2(T) < 0$, for all $T \in [M,\infty)$.Thus, $\frac{dTP_2(T)}{dT} = \frac{F_2(T)}{T^2} < 0$ which implies $TP_2(T)$ is a strictly decreasing function of T. Thus $TP_2(T)$ has a maximum value when T is minimum. Therefore, $TP_2(T)$ has a maximum value at the point $T = M$. This completes the proof.

Thus, the EOQ corresponding to the optimal cycle length T^* is computed as follows

$$EOQ^* = \alpha t_d + \beta\frac{t_d^2}{2} + \gamma\frac{t_d^3}{3} + \frac{\lambda}{\theta}\left(e^{\theta(T^*-t_d)} - 1\right) \quad (33)$$

4 Numerical Examples

This section provides some numerical examples to illustrate the model developed.

Example 4.1 *(for $0 < M \leq t_d$)*

Consider an inventory system with the following input parameters: $A = \$250$/order, $C = \$15$/unit/year, $S_1 = \$25$/unit/year, $S_2 = \$20$/unit/year, $h = \$2$/unit/year, $\theta = 0.01$ units/year,$\alpha = 180$ units, $\beta = 30$ units, $\gamma = 15$ units,

$\lambda = 120$ units, $t_d = 0.1354$ year (49 days), $M = 0.0888$ year (32 days), $I_c = 0.1$, $I_e = 0.08$. It is seen that $M \leq t_d$ and $\Delta_1 = 81.3279 > 0$. Substituting the above values in equations (24), (17) and (33), it is obtained that the optimal cycle length $T_1^* = 0.6240$ year (228 days), the total profit $TP_1(T_1^*) = \$344.9180$, and the economic order quantity $EOQ_1^* = 83.4321$ units respectively.

Example 4.2 *(for $t_d < M \leq T$)*

The data are same as in Example 4.1 except that $M = 0.1523$ year (56 days). It is seen that $M > t_d$ and $\Delta_2 = 79.1512 > 0$. Substituting the above values into equations (31), (18) and (33), it is obtained that the optimal cycle length $T_2^* = 0.6200$ year (226 days), the total profit $TP_2(T_2^*) = \$358.1353$ and the economic order quantity $EOQ_2^* = 82.9475$ units respectively.

5. Sensitivity Analysis

The sensitivity analysis of some model parameters is performed by changing each of the parameters from $-10\%, -5\%, +5\%$ to $+10\%$ taking one parameter at a time and keeping the remaining parameters unchanged. The effects of these changes of parameters on cycle length, total profit and economic order quantity per cycle are discussed and summarised in table 2 and 3 below:

Table 2 Effect of changes of some model parameters from $-10\%, -5\%, +5\%$ to $+10\%$ on decision variables for example 4.1

Parameter	% Change in Parameter	% Change in T_1^*	% Change in EOQ_1^*	% Change in $TP_1(T_1^*)$
M	−10	0.105	0.095	−0.549
	−5	0.053	0.048	−0.275
	+5	−0.054	−0.049	0.276
	+10	−0.109	−0.099	0.553
θ	−10	0.201	0.164	0.102
	−5	0.100	0.082	0.051
	+5	−0.100	−0.081	−0.051
	+10	−0.199	−0.163	−0.102
h	−10	2.789	2.516	2.255
	−5	1.365	1.231	1.120
	+5	−1.310	−1.181	−1.106
	+10	−2.567	−2.315	−2.199
C	−10	−5.592	−5.042	59.470
	−5	−2.691	−2.427	29.704
	+5	2.506	2.260	−29.650
	+10	4.848	4.373	−59.254
S_1	−10	31.302	28.257	−24.825
	−5	16.705	15.073	−13.249
	+5	−20.125	−18.139	15.962
	+10	−47.465	−42.744	37.645
S_2	−10	−21.308	−19.204	−52.683
	−5	−10.021	−9.035	−26.843
	+5	9.104	8.213	27.570
	+10	17.505	15.796	55.698
I_c	−10	2.071	1.868	1.219
	−5	1.019	0.919	0.606
	+5	−0.988	−0.891	−0.598
	+10	−1.946	−1.755	−1.187
I_e	−10	0.084	0.076	−0.067
	−5	0.042	0.038	−0.033
	+5	−0.042	−0.038	0.033
	+10	−0.084	−0.076	0.067
A	−10	−15.912	−14.343	12.620
	−5	−7.613	−6.864	6.038
	+5	7.073	6.380	−5.610
	+10	13.707	12.367	−10.871

Table 3 Effect of changes of some model parameters from $-10\%, -5\%, +5\%$ to $+10\%$ on decision variables for example 4.2

Parameter	% Change in Parameter	% Change in T_2^*	% Change in EOQ_2^*	% Change in $TP_2(T_2^*)$
M	-10	0.016	0.014	-0.773
	-5	0.008	0.007	-0.385
	$+5$	-0.008	-0.008	0.383
	$+10$	-0.017	-0.016	0.764
θ	-10	0.200	0.163	0.097
	-5	0.100	0.081	0.048
	$+5$	-0.099	-0.081	-0.048
	$+10$	-0.198	-0.162	-0.097
h	-10	2.789	2.514	2.159
	-5	1.365	1.230	1.072
	$+5$	-1.309	-1.180	-1.059
	$+10$	-2.567	-2.313	-2.105
C	-10	-5.786	-5.213	57.046
	-5	-2.783	-2.508	28.491
	$+5$	2.589	2.334	-28.436
	$+10$	5.007	4.513	-56.825
S_1	-10	31.751	28.644	-24.090
	-5	16.958	15.292	-12.866
	$+5$	-20.497	-18.461	15.550
	$+10$	-48.604	-43.738	36.872
S_2	-10	-21.592	-19.447	-50.632
	-5	-10.145	-9.141	-25.810
	$+5$	9.207	8.300	26.522
	$+10$	17.696	15.957	53.588
I_c	-10	1.992	1.796	0.902
	-5	0.980	0.883	0.447
	$+5$	-0.950	-0.856	-0.440
	$+10$	-1.871	-1.686	-0.873
I_e	-10	0.226	0.204	-0.172
	-5	0.113	0.102	-0.086
	$+5$	-0.113	-0.102	0.086
	$+10$	-0.227	-0.205	0.172
A	-10	-16.142	-14.541	12.247
	-5	-7.718	-6.954	5.855
	$+5$	7.163	6.457	-5.435
	$+10$	13.877	12.512	-10.528

6 Results and Discussion

Based on the results shown in Table 2 and 3, the following managerial insights are obtained.

1. From table 2 and 3, it is clearly seen that as the length of credit period (M) increases, both cycle length (T^*) and economic order quantity (EOQ^*) decrease while the net profit $(TP(T^*))$ increase and vice versa. This implies that the longer the credit period is the shorter the replenishment cycle, the lower the order quantity and the higher the profit will be. From economical point of view, if the supplier provides a permissible delay in payments, the retailer will order lower quantity in order to take the benefits of the permissible delay more frequently.
2. From table 2 and 3, it clearly seen that as the rate of deterioration (θ) increases, the optimal cycle length (T^*), economic order quantity $(EOQ^*$ and the total profit $(TP(T^*))$ decrease and vice versa. This implies that the retailer can reduce the deterioration rate of items by improving the equipment in the warehouse such that the total profit will be high.

3 From table 2 and 3, it clearly seen that as the holding cost (h) increases, the optimal cycle length (T^*), economic order quantity (EOQ^*) and the total profit $(TP(T^*))$ decrease and vice versa. This result reveals that when holding cost is higher, the retailer will tend to shorten the length of cycle, and order a smaller quantity each time for keeping inventory level as low as possible. The retailer must pay more attention to storage process control to diminish the holding cost as it has a negative effect on the total profit.

4 From table 2 and 3, it clearly seen that as the unit purchasing cost (C) increases, the optimal cycle length(T^*), the economic order quantity (EOQ^*) and the total profit $(TP(T^*))$ decrease and vice versa. This result reveals that when the unit purchasing cost increases, the retailer will order smaller quantity in order to enjoy the benefits of permissible delay in payments more frequently and this will consequently shorten the cycle length.

5 From table 2 and 3, it clearly seen that as the unit selling price before deterioration set in (S_1) increases, the optimal cycle length(T^*) and the economic order quantity (EOQ^*) decrease while the total profit $(TP(T^*))$ increase and vice versa. In real market situation the higher the selling price of an item, the lower the demand. This result reveals that when the unit selling price is increasing, the retailer will order less quantity to take the benefits of the trade credit more frequently.

6 From table 2 and 3, it clearly seen that as the unit selling price after deterioration set in (S_2) increases, the optimal cycle length(T^*), the economic order quantity (EOQ^*) and the total profit $(TP(T^*))$ increase and vice versa. This result reveals that when the different between unit selling price before and after deterioration sets in is slightly small, the retailer should order large quantity of items, which increases the total profit and cycle length.

7 From table 2 and 3, it clearly seen that as the interest payable (I_c) increases, the optimal cycle length (T^*), the economic order quantity (EOQ^*) and the total profit $(TP(T^*))$ decrease. This means that when interest payable is high the retailer should order less amount of inventory.

8 From table 2 and 3, it clearly seen that as the interest earns (I_e) increases, the optimal cycle length(T^*) and the economic order quantity (EOQ^*) decrease while the total profit $(TP(T^*))$ increase. This result implies that when the interest earned is high, the optimal cycle length and the economic order quantity decrease while the total profit $(TP(T^*))$ increase.

9 From table 2 and 3, it clearly seen that as the ordering cost (A) increases, the optimal cycle length (T^*) and the economic order quantity (EOQ^*) increase while the total profit $(TP(T^*))$ decreases. This result implies that, from managerial view point, if the ordering cost per order is reduced effectively, the total profit could be increased. The retailer should order more quantity per order when the ordering cost per order is high.

7 Conclusion

This paper investigates an EOQ model for non-instantaneous deteriorating items with two-phase demand rates and two level pricing strategies under trade credit policy. Many existing models for non-instantaneous deteriorating items only assumed that the demand rate and selling price of items in the stock before and after deterioration sets in is the same, while the model in this study incorporates two-phase demand rates and two level pricing strategies under trade credit policy. The demand rate before deterioration sets in is assumed to be time-dependent quadratic and that is considered as constant after deterioration sets in. It is also assumed that the unit selling price before deterioration sets in is greater than that after deterioration sets in. Moreover, some useful theorems that prove the existent and uniqueness of the optimal solutions were provided and an easy-to-use method to determine the cycle length and economic order quantity such that total profit has a maximum value under various conditions were also presented. Some numerical examples are given to illustrate the theoretical result of model. Sensitivity analyses were also carried out to see the effect of changes of some model parameters on optimal solutions and suggestions toward maximising the total profit were also given. The results show that the retailer can maximise total profit by ordering less to shorten the cycle length when trade credit period increases, the rate of deterioration increases, holding cost increases, unit purchasing price increases, unit selling price before deterioration sets in increases, unit selling price after deterioration sets in decreases, interest charges increases and interest earn decreases respectively. The model developed could be used in inventory management and control of items such as aircrafts, computers, seasonal products, machines and their spare parts fashionable goods, android mobiles, televisions, photographic films and so on. The proposed model can be extended by taking some more realistic assumptions such as shortages, quantity discounts, linear or quadratic holding cost, variable deterioration rate, inflation rates, reliability of items, ramp type or trapezoidal type or probabilistic demand rates, finite time horizon, multi-item inventory models, two storage facilities, and so on.

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