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MODIFICATION OF HESTON'S MODEL FOR STOCK MARKET

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Abstract

This research work is centred on the return on investment that informs investors and entrepreneurs on general performance of a company. The research problem was a modification of an existing model. This was done by incorporating rate of reinvestment of return and return on investment that follow random process. In addition to the modification, the existing model was extended from a two to four compartment stochastic differential equation. Formulation of a four- compartment stock market model was based on a four dimensional geometric Brownian motion. The compartments were that of stock price, stock price volatility, return on investment and return on investment volatility compartment. The solution of a four-compartment stock market model was proved to exist and also unique. The Octave programming language was used to implement the simplified stochastic Runge-Kutta (SSR-K) scheme. Data were collected from Nigerian Stock Exchange (NSE) for the period of 2007 -2014 to validate the model formulated. It was found that, the stock price, stock price volatility, return on investment and return on investment volatility follow random (stochastic) process. The correlation between the empirical stock price on the other hand and the corresponding computed values on the other hand from the model formulation was found to be significant as the significant value was zero (0).The absolute error of the empirical return on investment and the corresponding computed values from the model formulation was found to be zero(0). Implying that, the computed results agreed very well with the empirical values. In addition, results showed that stock price increases when return on investment decreases. We concluded that return on investment was considered to be the best index to use to study the general performance of companies rather than stock price.

1.0 INTRODUCTION

The idea of Stochastic Differential Equation (SDE) was started by Einstein in 1905. It is a differential equation with at least one stochastic process. It is given in an overall structure as,

$$dX(t) = f(X(t),t)dt + g(X(t),t)dW(t) \quad (1)$$

Several researches had been carried out on investment modelling. According to [1] developed a one-compartment stochastic differential equation model that describes the evolution of interest rate movement which is driven by only one source of market risk. The Heston model, named after Steven Heston, is a two-compartment stochastic differential equation model which described the evolution of the volatility of an asset. It is a stochastic volatility model which assumes that the volatility of the asset is neither constant, nor deterministic, but follows a random process [2]. According to [3] proposed a mathematical model that describes the evolution of interest rate. It was an extension of the models of [1] and [2] from a one and two-compartment stochastic differential equation model to three compartments stochastic differential equation model. Interest rate was describes as movement driven by three sources of market risk and particularly on instantaneous interest rate [3]. In this study, we set out to modify Heston model by incorporating rate of return on reinvestment, transition probability of stock market interaction and return on investment that follow stochastic process. Several scholars had carried

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out research in this area of stochastic investment and stochastic volatility model, such as [1], [2] and [3]. Their models informed investors and entrepreneur on stochastic nature of stock price, interest rate and instantaneous interest rate on money loaned out. The limitation of their models is that, their formulations concentrate on stochastic nature of interest rate and stock price of the organisations. This research therefore set out to modify existing model by using fundamental such as rate of return on reinvestment, transition probability of stock market interaction and return on investment that follows stochastic process.

The stock market is very vital to any economy because it encourages savings and real investment in any healthy economic environment [4]. This is achieved through aggregate savings that are channelled into real investment via stock market exchange which increases the stock market ^{growth} and invariably economic growth of the country. In other words, through the secondary market, the stock market converts long – term or perpetual investment enlarged which in turn accelerate economic growth. It has been posited, that without high levels of domestic savings, broadly based human capital, good macro-economic management and limited price distortions, there would be no basis for economic growth. Investment is one of the driving forces of economic growth. It is widely believed that savings and investment must go hand in hand for sustained economic growth. Thus policies to assist the financial sector, especially banks whose traditional business is financial intermediation which captures non-financial savings, to increase household and corporate savings are considered central. In the developed world, stock market is the crucial tool that drives any economy on it path to growth and development. The capital market of which stock market is subset aids economic growth in the following ways: promotion of commodities exchanges to facilitate liquidity for agricultural products in an organized market, Internationalization of capital market by cross-border listings, cross listing on other stock exchange and provision of investment information on all securities listed on the Nigeria Stock Exchange to the international community. This encourages foreign inflows of capital through equity; Changes in ownership of businesses take place through the purchase and/or sale of stock and Promotion of small and medium sized industries through the second – tier Securities Market. However, some problems still affect the development of Nigerian Stock Market. The Nigerian Stock Market is still very small in relation to other emerging markets. As at 2003, South Africa, Brazil, Egypt and Malaysia had equity listings of 450, 399, 1148 and 865 respectively, while Nigeria had only 214 equity listings. Not only that, the proportion of the adult population that own ordinary shares is still rather small: Nigeria 4%,United Kingdom 16%,France 18% ,Japan 18%,Germany19% United States21%,Sweden 22%,Hong Kong 38% . Aside the afore mentioned, in the Nigerian Stock Market there is high concentration with the top ten companies controlling 47% of market capitalization between 1999 – 2004 . This makes the market vulnerable to shocks and price instabilities from the dominating stocks of banking sectors. It is therefore important that any consideration of a stock market in an emerging economy must be closely linked with its expected essential purpose [4]. The Nigerian Stock Exchange (NSE) was founded in 1960 as the Lagos Stock Exchange. NSE started operations in Lagos in 1961 with 19 securities listed for trading. In December 1977 it became known as The Nigerian Stock Exchange, with branches established in some of the major commercial cities of the country. As of December 31, 2013, it had about 200 listed companies with a total market capitalization of about N12.88 trillion ($80.8billion). The Exchange maintains an All-Share Index formulated in January 1984 (January 3, 1984 = 100). Only common stocks (ordinary shares) are included in the computation of the index. The index is value-weighted and is computed daily. The highest value of 66,371.20 was recorded on March 3, 2008. Also, The Exchange has introduced the NSE-30 Index, which is a sample-based capitalization-weighted index plus four sectorial indices. Similarly, five sectoral indices have been introduced to complement existing indices. These are NSE-Food/Beverages Index (later renamed NSE – Consumer Goods Index), NSE Banking Index, NSE Insurance Index, NSE Industrial Index and NSE Oil/Gas Index. The Nigerian Exchange is a member of the World Federation of Exchanges (FIBV). It is also an observer at meetings of International Organization of Securities Commissions (IOSCO), and a foundation member of the African Stock Exchanges Association (ASEA). On 31 October 2013, it joined the United Nation's Sustainable Stock Exchanges (SSE) initiative. The NSE is regulated by the Securities and Exchange Commission, which has the mandate of Surveillance over the exchange to forestall breaches of market rules and to deter and detect unfair manipulations and trading practices. The exchange has an automated trading System. Data on listed companies' performances are published daily, weekly, monthly, quarterly and annually.

The ability of financial system and markets to play their roles hinges on the stability in the system particularly the stock prices. The major component of instability in stock prices is exhibited by the varying conditional variance (volatility) of the stock prices. Hence, to be able to establish and maintain a viable stock market that could enhance the growth of the economy, there must be an in depth and comprehensive understanding of the volatility of stock price [5]. Volatility is a measure of the range of an asset price about its mean level over a fixed time [6]. It follows that volatility is linked to the variance of an asset price. If stock is labelled volatile, then the price will vary greatly over time. Conversely, a less volatile stock will have a price that will deviate relatively little over time [6]. Several studies such as [7], [8],[9] and [10] reveal the evidence of market volatility in the emerging stock markets. Theoretical works including [11] consistently assert that stock

market volatility should be negatively correlated with stock returns. The earlier study of [12] found a positive and significant relationship. However, studies such as [13]; [14] reported a positive but insignificant relationship between stock market volatility and stock returns. Consistent with the asymmetric volatility argument, researchers report negative and often significant relationship between the volatility and return [15] and [16]).Volatility is a measure for variation of price of a stock over time. Stochastic volatility is described as processes in which the return variation dynamics include an unobservable shock that cannot be predicted using current available information. Stochastic volatility models, which let the volatility follow Brownian motion, make the option price much better adapted to the realities of the market [17]. According to [18] found the parameter estimates of GARCH models close to unity and suggested a high level of persistence in the Ghana stock exchange. Nonlinear models are considered to be the dominant models than the linear class of models. According to [19] investigated a class of models and found that the nonlinear GARCH models dominate the other class of models in predicting stock market volatility in [20] used one compartment of geometric Brownian motion model to study the behaviour of stock market prices and stock price volatility. Were interest rate was considered constant. Also, [17] used a two compartment Heston model to study the dynamic nature of stock prices where stock price volatility was considered randomly distributed. The stock price solution of the model is assumed to be the same as the Black-Scholes solution and interest rate was considered as constant. Again,[17] used Euler's maruyama scheme to solve the SDEs. In Nigeria, studies on modelling volatility of stock returns provide different perspectives. Where [21] considered asymmetric GARCH methodology to observe the result of stock market liberalization on stock returns volatility of fifteen emerging markets, including Nigeria, for the period December 1984 to March 2000. The study reports, among others, that positive (negative) change in prices have been followed by negative (positive) changes indicating a cyclical type behaviour in stock price changes rather than volatility clustering in Nigeria. Also,[22] investigated the emerging market volatility using Nigeria and Kenya stock return series. Their results of the exponential GARCH model indicate that asymmetric volatility found in the U.S. and other developed markets is also present in Nigerian, but Kenya shows evidence of significant and positive asymmetric volatility, suggesting that positive shocks increase volatility more than negative shocks of an equal magnitude. Whereas [23] investigated the relation between stock returns and volatility in Nigeria using E-GARCH-in-mean model in the light of banking reforms, insurance reform, stock market crash and the global financial crisis. His findings showed little evidence on the relationship between stock returns and risk as measured by its own volatility and show that banking reform and stock market crash negatively impacts on stock return while insurance reform and the global financial crisis have no impact on stock return. According to [24], modelling and forecasting the volatility of returns in stock markets has become a fertile field of empirical research in financial markets. This is simply because volatility is considered as an important concept in many economic and financial applications like asset pricing, risk management and portfolio allocation. In their paper, they attempt to explore the comparative ability of different statistical and econometric volatility forecasting models in the context of Sudanese stock market known as the Khartoum Stock Exchange (KSE). A total of five different models were considered in their study. The volatility of the KSE index returns have been modelled by using a univariate Generalized Autoregressive Conditional Heteroscedastic (GARCH) models including both symmetric and asymmetric models that captures most common stylized facts about index returns such as volatility clustering and leverage effect. Although there have already been many practical and successful applications of multivariate GARCH models, the theoretical literature on Multivariate Stochastic Volatility (MSV) models has developed significantly over the last few years. In GARCH-type models the conditional variance of returns is assumed to be a deterministic function of past returns, whereas in SV models the volatility process is random. The introduction of the additional error term makes the SV model more flexible than the GARCH type models and also more directly linked to continuous time models that are often used in asset pricing in finance. However, a comprehensive review of the important aspects of existing discrete time MSV models in the literature does not yet seem to exist. There are both economic and econometric reasons why multivariate volatility models are important. The knowledge of correlation structures is vital in many financial applications, such as asset pricing, optimal portfolio risk management, and asset allocation, so that multivariate volatility models are useful for making financial decisions. Moreover, as financial volatility moves together across different assets and markets, modelling volatility in a multivariate framework can lead to greater statistical efficiency.

Meanwhile, [25] compared the variance structure of high (daily) and low (weekly, monthly) frequencies of data of the Pakistani stock market. By employing ARCH (1) and GARCH (1, 1) models, the study found that statistical properties of the three data series of returns were substantially different from one another. The presence of persistency was more in the daily stock returns as compared to other data sets, which showed that the volatility models were sensitive to the frequencies of data series. It has been empirically demonstrated that the relationship between return and volatility depends on the specification of the conditional volatility. In particular, using a parametric GARCH-M model, [26] found out that a positive but statistically insignificant relationship exists. In contrast, using a flexible semi-parametric GARCH-M model, the study observed that a negative relationship prevails instead. While the volatility for the stock market as a whole has been

remarkably stable over time, the volatility of individual stocks appears to have increased. Also,[26] examined the relationship between expected stock returns and volatility in the twelve largest international stock markets. As [27] studied the volatility of the Pakistani stock market and found persistence, which signified inefficiency in the stock market. They found that lagged returns in the GARCH model were significant in explaining current returns.

Focusing on the forces that determine volatility, [9] found out that in fully integrated markets, volatility is strongly influenced by the local and the world factors while in segmented capital markets, volatility is more likely to be influenced by the local factors. They argue that political risk measured by low credit rating and unstable macroeconomic policies might be translated into high stock market returns volatility. As [28] investigate which events causes volatility of emerging stock market returns by examining the global and local events (social, political, and economic) during the period of increased volatility and find that most events tend to be local. Persistence of Egypt and Israel stock markets was found and concluded that long run component converges slowly. The volatility varies from time to time at different frequency. Scholars started to look at country-specific risk in addition to the world risk in order to explain the local factors that cause stock market returns volatility, observe that differences in return volatility are because of market thinness and share turnover. Emerging markets are characterized by high risk and return, highly unpredictable and high volatility compared to the developed markets ([29], [9]).

Stock returns volatility differs dramatically across international markets and have received a great attention from both academicians and practitioners over the last two decades because it can be used as a measure of risk in financial markets. Volatility of stock returns has long been an issue of interest in financial literature. A wide variety of research has been conducted on stock returns volatility in developed and emerging markets since 1970s. The nature of volatility in different markets at different times are discovered, which are indeed of great interest for financial economists. Financial economists are also interested in the causes and variables behind the existence and nature as well as the anomalies relating to market volatility, (see [30], [28], and [31]). Where [32] provides evidence that volatility in emerging equity markets is less than in developed equity markets.

The model of [2] is one of the well known stochastic volatility models that is widely used. In this research, a review of Heston model is carried out. We also modified and extended it from a two to four compartment stochastic differential equation model for stock market. The four compartments are stock price, stock price volatility, return on investment and return on investment volatility. Existence and uniqueness of solution of the modified and extended model equation would be established .The stochastic Runge-Kutta scheme will be used to solve the modified and extended model.

2.0 METHODOLOGY

We present the [2] model as the existing model and the modified model equation. The following assumptions were made by [2]:

(i) The interest rate μ is a constant.

(ii)The stock price S_t follows a Black-Scholes type of stochastic process but with a stochastic variance V_t that follows a [1] (CIR) process. The model is given as:

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{V_t}\, S_t dW_t, \\ dV_t &= X(\theta - V_t)dt + \sigma\sqrt{V_t}\, dZ_t, \\ dW_t dZ_t &= \rho dt . \end{aligned} \qquad (2)$$

The parameters and variables of the model (Equation 2) above are defined below:

μ is the drift coefficient of the stock price,

θ is the long-term mean of variance,

x is the rate of mean reversion,

σ is the rate of volatility,

S_t and V_t are stock price and volatility respectively

To take into account the leverage effect, stock returns and implied volatility are negatively correlated. W_t and Z_t are correlated Wiener processes, with correlation coefficients ρ (Yuan,2013)[18].

We will drop the time index and write $S = S_t, V = V_t$

The formulation of a four compartment SDE will be presented. It is a modification of an existing model (Heston's model) in the sense that transition probability, rate of return reinvestment and return of investment will be used in obtaining the SDEs model. The four compartments SDE model consists of stock price, return on investment and its volatility. Our formulation will follow Geometric Brownian Motion model. To achieve the formulation, we shall assume that stock price and return on investment follow random process and as well Markov process which is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future.

Definition of variables and parameters of a four-compartment stock market model

Here, we shall define the variables and parameters that will use in Figure 1;

S_t	Stock price,
R_t	Return on investment,
V_t	Stock price volatility,
U_t	Return on investment volatility,
ρ	Correlation coefficient between dW_1 and dW_3,
β	Rate of return of reinvestment,
η	Correction coefficient between dW_2 and dW_4,
τ	Mean reversion of V_t,
ϕ	Rate of V_t ,
γ	Mean reversion of U_t ,
λ	Rate of return without reinvestment.

Having given the above definitions, we describe the stock market interaction as shown in Figure 1.

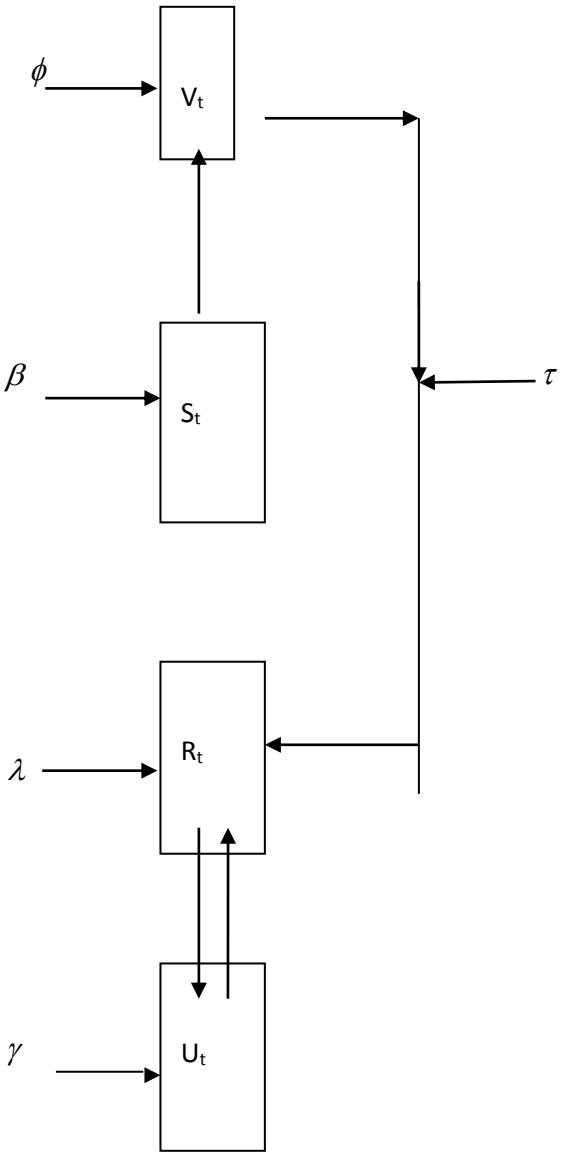

Figure 1: Schematic diagram of stock market interaction of a four-compartment stock market model

The stock market model transition matrix is given as

T=

	S	*R*	*V*	*U*	*TOTAL*
S	$\frac{1}{2}$	0	$\frac{1}{2}$	0	1
R	0	$\frac{1}{2}$	0	$\frac{1}{2}$	1
V	0	$\frac{1}{2}$	$\frac{1}{2}$	0	1
U	0	$\frac{1}{2}$	0	$\frac{1}{2}$	1
TOTAL					

$$= \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \quad (3)$$

Using Figure 1 and Equation (3), a four- compartment stock market model is formulated as follows:
The stochastic differential equation model with respect to the stock price compartment is given as,

$$dS_t = \frac{1}{2} S_t \beta dt + \frac{1}{2} S_t \sqrt{V_t}\, dW_{1t} \quad (4)$$

Next, the stochastic differential equation model with respect to the return on investment is given as,

$$dR_t = \frac{1}{2} R_t \lambda dt + \frac{1}{2} R_t \sqrt{U_t}\, dW_2 t \quad , \ \lambda > 0 \quad (5)$$

Also, the stochastic differential equation model with respect to stock price volatility is given as

$$dV_t = \frac{1}{2} V_t R_t \pi dt + \frac{1}{2} \phi \sqrt{V_t}\, dW_3 t \quad \phi, \tau > 0 \quad (6)$$

Finally, the stochastic differential equation model with respect to return on investment volatility is given as therefore

$$dU_t = \frac{1}{2} \gamma U_t dt + \frac{1}{2} R_t \sqrt{U_t}\, dW_4 t \quad \gamma > 0 \quad (7)$$

where,

$$dW_1 t dW_2 t = \rho dt \ , \ dW_3 t dW_4 t = M dt \ ,$$

$$R_0 = \frac{V_f - V_i}{V_i} , V_0 = \sum_{i=1}^{T} \frac{(S_i - \bar{S})^2}{T} or \sqrt{\sum_{i=1}^{T} \frac{(S_i - \bar{S})^2}{T}} , \beta = (1+R)^{\frac{1}{T}} - 1 \ U_0 = \sum_{i=1}^{N} \frac{(R_i - \bar{R})^2}{T} or \sqrt{\sum_{i=1}^{N} \frac{(R_i - \bar{R})^2}{T}} , N = \frac{R}{T}. \quad (8)$$

Presenting (4), (5), (6) and (7) in a matrix form,we have

$$\begin{bmatrix} dS_t \\ dR_t \\ dV_t \\ dU_t \end{bmatrix} = \begin{bmatrix} \frac{1}{2} S_t \beta \\ \frac{1}{2} R_t \lambda \\ \frac{1}{2} \pi R_t V_t \\ \frac{1}{2} \gamma U_t \end{bmatrix} dt + \begin{bmatrix} \frac{1}{2} S_t \sqrt{V_t} \\ \frac{1}{2} R_t \sqrt{U_t} \\ \frac{1}{2} \phi \sqrt{V_t} \\ \frac{1}{2} R_t \sqrt{U_t} \end{bmatrix} \begin{bmatrix} dW_1 t \\ dW_2 t \\ dW_3 t \\ dW_4 t \end{bmatrix} \quad (9)$$

where

$$\begin{bmatrix} dW_1 t & dW_2 t & dW_3 t & dW_4 t \end{bmatrix}^T = dW \ .$$

Equation (9) is the required 4-compartment model equation.
For easy representation, equation (9) would be expressed as follows;

$$dZ_t = G_t dt + H_t dW_t \quad (10)$$

wher $dZ_t = \begin{bmatrix} dS_t \\ dR_t \\ dV_t \\ dU_t \end{bmatrix}$, $G_t = \begin{bmatrix} \frac{1}{2} S_t \beta \\ \frac{1}{2} R_t \lambda \\ \frac{1}{2} \pi R_t V_t \\ \frac{1}{2} \gamma U_t \end{bmatrix}$, $H_t = \begin{bmatrix} \frac{1}{2} S_t \sqrt{V_t} \\ \frac{1}{2} R_t \sqrt{U_t} \\ \frac{1}{2} \phi \sqrt{V_t} \\ \frac{1}{2} R_t \sqrt{U_t} \end{bmatrix}$, $dW_t = \begin{bmatrix} dW_1 t \\ dW_2 t \\ dW_3 t \\ dW_4 t \end{bmatrix}$.

Equation (10) now appears like a one-dimensional SDE. This would enable us present the subsequent results in a more compact form.

2.1 Numerical Method

Below is the description of 4-dimensional simplified stochastic Runge-Kutta (SSR-K) scheme for a four-compartment stock market model equations.

$$S_{i+1} = S_i + \frac{1}{6}(K_{0S} + 2K_{1S} + 2K_{2S} + K_{3S})\Delta t_i + \frac{1}{6}(G_{0S} + 2G_{1S} + 2G_{2S} + G_{3S})\Delta w_i \quad (12)$$

$$R_{i+1} = R_i + \frac{1}{6}(K_{0R} + 2K_{1R} + 2K_{2R} + K_{3R})\Delta t_i + \frac{1}{6}(G_{0R} + 2G_{1R} + 2G_{2R} + G_{3R})\Delta w_i \quad (13)$$

$$V_{i+1} = S_i + \frac{1}{6}(K_{0V} + 2K_{1V} + 2K_{2V} + K_{3V})\Delta t_i + \frac{1}{6}(G_{0V} + 2G_{1V} + 2G_{2V} + G_{3V})\Delta w_i \quad (14)$$

$$U_{i+1} = U_i + \frac{1}{6}(K_{0U} + 2K_{1U} + 2K_{2U} + K_{3U})\Delta t_i + \frac{1}{6}(G_{0U} + 2G_{1U} + 2G_{2U} + G_{3U})\Delta w_i \quad (15)$$

3.0 RESULTS

3.1 Validation of a Four-Compartment Stock Market Model

We shall use published data from Nigeria Stock Exchange (appendix V), Table1, 2, 5, 6, 7, Appendix I and III to validate a four-compartment stock market model formulated model equation (8). From the data the initial value of S_0 and parameter values of, ρ, η, τ, ϕ and γ were obtained while other initials and parameter were computed using equation (9) We used stochastic Runge-Kutta Schemes (12), (13), (14), (15) solve the stochastic differential equations (4), (5), (6) and (7) respectively in a simultaneous process and implement the scheme in Octave programming language. We shall present daily, weekly, monthly and yearly validation of the model results. We shall compute return on investment of Seven Up Bottling Company Plc as below using Appendix III. Define

Table 1: Agricultural, Consumer goods, bank Index data for January 2014.

Date	OKOMU OIL PALM PLC	LIVESTOCK FEED	7UP	HONEYFLOUR	GTB
01/07/2014	40.61	4.10	82.64	4.00	27.40
01/08/2014	42.20	3.90	71.40	3.90	27.50
01/09/2014	40.61	4.09	74.97	3.95	27.05
01/10/2014	42.45	4.19	82.64	3.70	24.73
1/13/2014	42.20	4.09	82.64	3.72	26.03
1/15/2014	38.40	3.95	82.64	3.52	24.62
1/16/2014	38.50	4.00	82.64	3.50	25.91
1/17/2014	38.50	3.85	82.64	3.74	23.67
1/20/2014	39.00	3.88	82.64	3.70	25.39
1/21/2014	41.00	4.08	82.64	3.80	25.01
1/22/2014	40.47	3.90	82.64	3.64	25.40
1/23/2014	40.50	3.95	82.64	3.82	24.30
1/24/2014	41.00	3.92	82.64	3.91	24.02
01/25/2014	38.55	3.95	82.64	3.80	24.90
01/26/2014	38.40	3.99	82.64	3.73	24.75

Source: www.nse.com.ng(2014)

Table 2: Default parameters for a four-compartment stock market model.

Parameter	**Value**
Mean reversion	2
Correlation of $dw_1(t)$ $dw_3(t)$ and $dw_2(t)$ $dw_4(t)$	-0.5,0.5
Volatility of volatility parameter	0.1

Source: Heston(1993, p.336)

Table3: Computer program output of stock price, stock price volatility, return on investment and return on investment volatility of Table 4.1 for 1=33

TIME IN DAYS	STOCK PRICE IN NAIRA(S)	RETURN ON INVESTMENT (R)	STOCK PRICE VOLATILITY(V)	RETURN ON INVESTMENT VOLATILITY(U)
1	40.6100	0.0392	2.1480	0.0015
2	40.5885	0.0392	2.1342	0.0014
3	40.6013	0.0392	2.1434	0.0015
4	40.5747	0.0392	2.1261	0.0013
5	40.5655	0.0392	2.1206	0.0012
6	40.5609	0.0392	2.1181	0.0012
7	40.5629	0.0391	2.1201	0.0012
8	40.5623	0.0391	2.1203	0.0012
9	40.5434	0.0391	2.1083	0.0011
10	40.5667	0.0391	2.1245	0.0013
11	40.5488	0.0391	2.1131	0.0012
12	40.5667	0.0391	2.1258	0.0013
13	40.5888	0.0391	2.1413	0.0014
14	40.6187	0.0391	2.1620	0.0016
15	40.6332	0.0391	2.1724	0.0018
16	40.6728	0.0391	2.1997	0.0021
17	40.7069	0.0392	2.2233	0.0024
18	40.7525	0.0392	2.2546	0.0028
19	40.7108	0.0391	2.2273	0.0024
20	40.7130	0.0391	2.2294	0.0024
21	40.7254	0.0391	2.2383	0.0026
22	40.7343	0.0391	2.2450	0.0027
23	40.7178	0.0391	2.2345	0.0025
24	40.7122	0.0391	2.2315	0.0025
25	40.7037	0.0391	2.2264	0.0024
26	40.6980	0.0391	2.2232	0.0024
27	40.7178	0.0391	2.2372	0.0026
28	40.7172	0.0391	2.2375	0.0026
29	40.7183	0.0391	2.2388	0.0026
30	40.7002	0.0391	2.2274	0.0025
31	40.6964	0.0391	2.2255	0.0024
32	40.6627	0.0391	2.2035	0.0022
33	40.6634	0.0391	2.2046	0.0022

Table 4: Empirical data for 2008 Weekly and monthly stock price of Seven Up Bottling Company Plc. For $t \in [1,50]$, $t \in [1,12]$

Week	Stock Price	Week	Stock Price	Week	Stock Price	Week	Stock Price	Month	Stock Price
1	48.38	14	53.01	27	54.46	40	48.99	1	47.35
2	47.65	15	52.182	28	53.916	41	44.458	2	50.36
3	46.46	16	50.93	29	49.676	42	43.89	3	50.36
4	46.64	17	49.92	30	49.672	43	43.436	4	51.33
5	48.23	18	49.662	31	49.992	44	40.85	5	53.33
6	49.52	19	55.144	32	50.49	45	38.054	6	52.46
7	50.77	20	56.116	33	47.062	46	36.134	7	52.44
8	51.63	21	50.892	34	51.104	47	35.924	8	49.7
9	51.522	22	51.08	35	51.196	48	38.63	9	51.14
10	51.862	23	51.08	36	52.296	49	39.236	10	47.74
11	56.492	24	50.98	37	51.194	50	37.99	11	40.94
12	56.12	25	52.906	38	50.99	51		12	37.57
13	55.86	26	54.46	39	50.49	52			

Table 5: Empirical data for yearly turnover of Seven Up Bottling Company Plc from 2007-2014. $t \in [1,8]$

S/No.	Year	Turnover in Naira	Stock Price in Naira
1	2007	27,309,000,000	47.61
2	2008	30,572,000,000	49.17
3	2009	34,864,000,000	33.81
4	2010	41,069,000,000	40.92
5	2011	51,098,000,000	44.96
6	2012	59,864,385,000	41.65
7	2013	64,088,879,000	61.70
8	2014	77,889,000,000	114.96

Source: www.google.com/ Seven Up Bottling Company Plc financial statement/2007-2014.

Table 6: Daily validation of a four-compartment stock market model
at t=1 to 43 , S_0=46.30,R_0=0.00047,U_0=0.0036,V_0=5.26
$\beta = 0.00044452, \lambda = 0.00047$ $t \in [1,255]$

TIME IN DAY	STOCK PRICE IN NAIRA(S)	STOCK PRICE VOLATILITY(V)	RETURN ON INVESTMENT IN NAIRA(R)*10^{-3}	RETURN ON INVESTMENT VOLATILITY(U)
1	46.3000	0.0000	0.4700	0.0036
2	46.3005	0.0000	0.4700	0.0036
3	46.3163	0.0000	0.4700	0.0036
4	46.3118	0.0000	0.4700	0.0036
5	46.3167	0.0000	0.4700	0.0036
6	46.2842	0.0000	0.4700	0.0036
7	46.2858	0.0000	0.4700	0.0036
8	46.2850	0.0000	0.4700	0.0036
9	46.2950	0.0000	0.4700	0.0036
10	46.2852	0.0000	0.4700	0.0037
11	46.2665	0.0000	0.4700	0.0037
12	46.2708	0.0000	0.4700	0.0037
13	46.2682	0.0000	0.4700	0.0037
14	46.2766	0.0000	0.4700	0.0037
15	46.2654	0.0000	0.4700	0.0037
16	46.2796	0.0000	0.4700	0.0037
17	46.2574	0.0000	0.4700	0.0037
18	46.2492	0.0000	0.4700	0.0037
19	46.2258	0.0000	0.4700	0.0037
20	46.2240	0.0000	0.4700	0.0037
21	46.2500	0.0000	0.4700	0.0037
22	46.2252	0.0000	0.4700	0.0037
23	46.2441	0.0000	0.4700	0.0038
24	46.2751	0.0000	0.4700	0.0038
25	46.2696	0.0000	0.4700	0.0038
26	46.2860	0.0000	0.4700	0.0038
27	46.2740	0.0000	0.4700	0.0038
28	46.2941	0.0000	0.4700	0.0038
29	46.2950	0.0000	0.4700	0.0038
30	46.2867	0.0000	0.4700	0.0038
31	46.2726	0.0000	0.4700	0.0038
32	46.2390	0.0000	0.4700	0.0038
33	46.2800	0.0000	0.4700	0.0038
34	46.2251	0.0000	0.4700	0.0038
35	46.1762	0.0000	0.4700	0.0038
36	46.1772	0.0000	0.4700	0.0039
37	46.2687	0.0000	0.4700	0.0039
38	46.2706	0.0000	0.4700	0.0039
39	46.2975	0.0000	0.4700	0.0039
40	46.2935	0.0000	0.4700	0.0039
41	46.2861	0.0000	0.4700	0.0039
42	46.3499	0.0000	0.4700	0.0039
43	46.2203	0.0000	0.4700	0.0039

Table7: Weekly validation of a four-compartment stock market model $t \in [1,51], S_0 = 48.38, R_0 = 0.00231, U_0 0.0036, V_0 = 5.32, \beta = 0.0022, \lambda = 0.00231$

TIME IN DAY	STOCK PRICE IN NAIRA(S)	STOCK PRICE VOLATILITY(V)*10^6	RETURN ON INVESTMENT (R)*10^{-3}	RETURN ON INVESTMENT VOLATILITY(U)
1	48.3800	0.0000	0.0023	0.0026
2	48.3888	0.0000	0.0023	0.0025
3	48.3878	0.0000	0.0023	0.0026
4	48.4043	0.0000	0.0023	0.0026
5	48.4124	0.0000	0.0023	0.0026
6	48.4171	0.0001	0.0023	0.0026
7	48.4113	0.0001	0.0023	0.0026
8	48.4358	0.0001	0.0023	0.0026
9	48.4536	0.0002	0.0023	0.0026
10	48.4529	0.0003	0.0023	0.0026
11	48.4506	0.0005	0.0023	0.0026
12	48.4797	0.0009	0.0023	0.0026
13	48.4811	0.0013	0.0023	0.0026
14	48.4923	0.0021	0.0023	0.0027
15	48.5106	0.0034	0.0023	0.0026
16	48.4945	0.0055	0.0023	0.0027
17	48.5207	0.0088	0.0023	0.0027
18	48.4889	0.0142	0.0023	0.0027
19	48.5031	0.0217	0.0023	0.0027
20	48.4941	0.0318	0.0023	0.0027
21	48.4923	0.0483	0.0023	0.0027
22	48.5363	0.0767	0.0023	0.0027
23	48.5263	0.1375	0.0023	0.0027
24	48.5633	0.2718	0.0023	0.0027
25	48.5856	0.4450	0.0023	0.0027
26	48.5650	0.7895	0.0023	0.0027
27	48.5351	1.3751	0.0023	0.0027
28	48.5299	1.6992	0.0023	0.0027
29	48.5593	1.9152	0.0023	0.0027
30	48.6085	2.1290	0.0023	0.0027
31	48.5566	2.1528	0.0023	0.0027
32	48.5013	2.1542	0.0023	0.0027
33	48.4518	2.1563	0.0023	0.0027
34	48.4902	2.1589	0.0023	0.0027
35	48.4890	2.1601	0.0023	0.0027
36	48.5269	2.1608	0.0023	0.0027
37	48.5301	2.1617	0.0023	0.0027
38	48.5141	2.1619	0.0023	0.0027
39	48.5237	2.1621	0.0023	0.0027
40	48.5146	2.1622	0.0023	0.0027
41	48.5648	2.1622	0.0023	0.0027
42	48.4689	2.1623	0.0023	0.0027
43	48.4629	2.1625	0.0023	0.0027
44	48.4850	2.1627	0.0023	0.0027
45	48.4301	2.1628	0.0023	0.0027
46	48.3685	2.1628	0.0023	0.0027
47	48.2330	2.1629	0.0023	0.0028
48	48.2761	2.1629	0.0023	0.0028
49	48.3600	2.1631	0.0023	0.0028
50	48.4563	2.1630		0.0028
51	48.2586	2.1630		0.0028

Correlation Significant coefficient value=0.00

Table 8: Monthly validation of the modified and extended model at

$$t \in [1,12], S_0 = 47.35, R_0 = 0.01, U_0 = 0.0036, V_0 = 4.82, \beta = 0.0095, \lambda = 0.01$$

TIME IN MONTH	STOCK PRICE IN NAIRA(S)	STOCK PRICE VOLATILITY(V)*10^4	RETURN ON INVESTMENT (R)	RETURN ON INVESTMENT VOLATILITY(U)
1	47.3500	0.0005	0.010000	0.0036
2	47.3418	0.0026	0.010000	0.0036
3	47.3547	0.0135	0.010000	0.0036
4	47.3785	0.0689	0.010000	0.0037
5	47.3863	0.3508	0.010000	0.0037
6	47.3831	1.3639	0.010000	0.0037
7	47.3923	0.7725	0.010000	0.0038
8	47.4004	0.7282	0.010000	0.0037
9	47.3761	0.6935	0.010000	0.0037
10	47.3887	0.6729	0.010001	0.0038
11	47.3905	0.6473	0.010001	0.0039
12	47.4143	0.6262	0.010001	0.0038

Table 9: Yearly validation of the modified and extended model at

$$t \in [1,12], S_0 = 49.17, R_0 = 0.159143, U_0 = 0.061, V_0 = 25.78, \beta = 0.0213214, \lambda = 0.159143$$

TIME IN YEAR	STOCK PRICE IN NAIRA(S)	STOCK PRICE VOLATILITY(V)*10^4	RETURN ON INVESTMENT (R)	RETURN ON INVESTMENT VOLATILITY(U)
1	49.1700	0.0003	0.1591	0.0610
2	49.1526	0.0014	0.1596	0.0659
3	49.1769	0.0070	0.1604	0.0739
4	49.1607	0.0380	0.1610	0.0904
5	49.1365	0.1899	0.1615	0.0955
6	49.1808	0.4271	0.1617	0.1047
7	49.1401	0.6903	0.1622	0.1404
8	49.1744	0.9429	0.1621	0.1556
9	49.1536	0.8740	0.1627	0.1492
10	49.1437	0.9030	0.1644	0.1956
11	49.2025	0.9565	0.1657	0.1987
12	49.2213	1.0363	0.1669	0.2104
13	49.2623	1.0460	0.1676	0.2251

Table 10: Empirical, model data and absolute error for return on investment of seven bottling company plc from 2008-2014

S/No.	Year	Empirical Return on Investment(ER)	Model Return on Investment(MR)	Absolute Error of ER and MR
1	2008	0.120	0.1591	0.0391
2	2009	0.114	0.1593	0.0453
3	2010	0.178	0.1600	0.018
4	2011	0.244	0.1601	0.0839
5	2012	0.172	0.1605	0.0115
6	2013	0.071	0.1610	0.09
7	2014	0.215	0.1616	0.0534

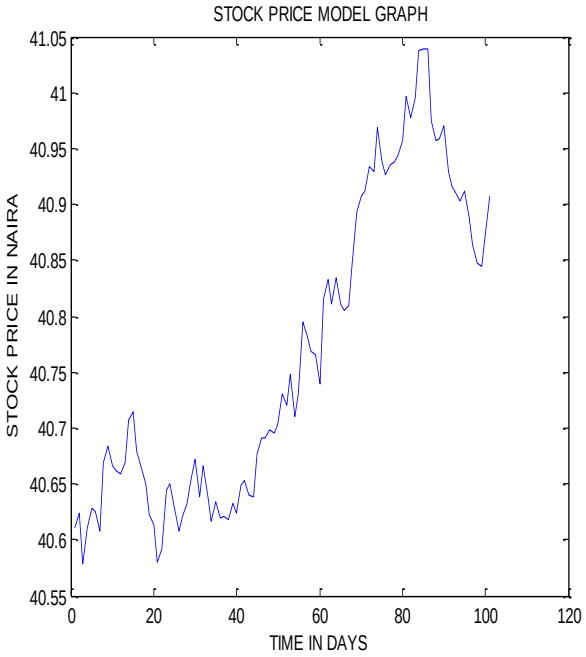

Figure 2: Stock price model graph for Okomu Oil Palm Plc

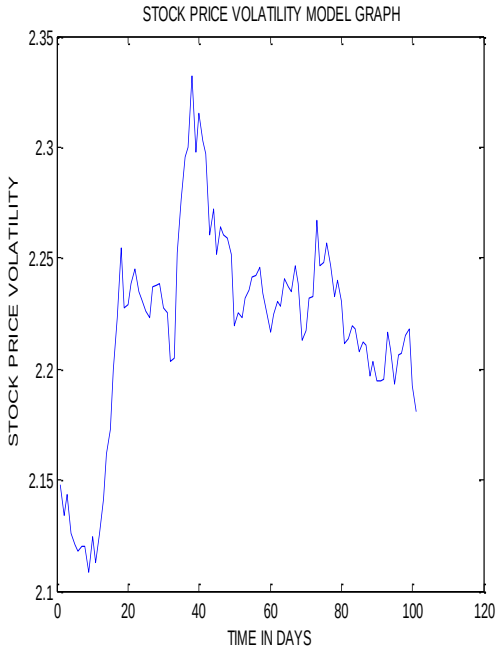

Figure 3: Stock price volatility model graph for Okomu Oil Palm Plc

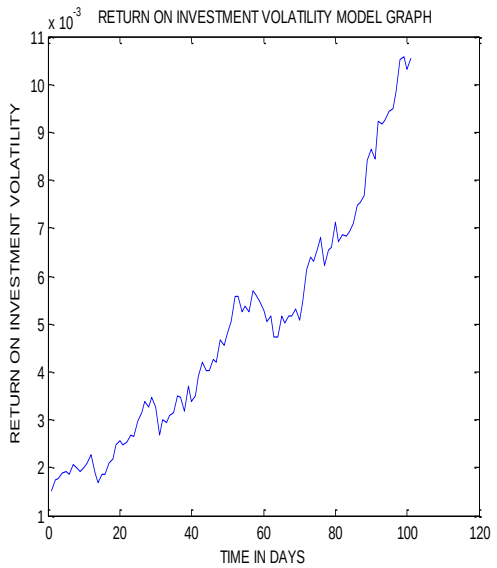

Figure 4: Return on investment volatility model graph for Okomu Oil Palm Plc

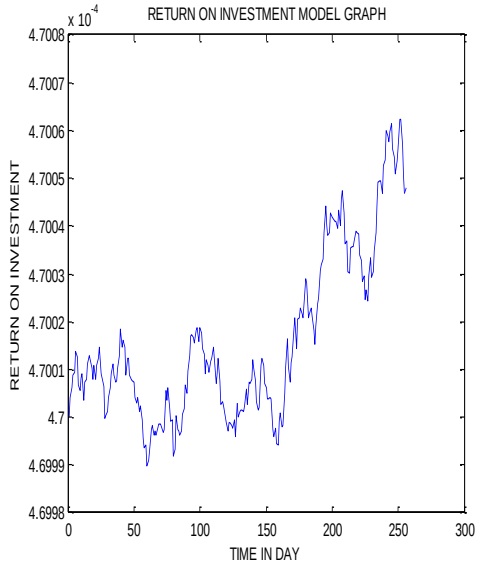

Figure 5: Daily return on investment model graph for Seven Up Bottling Company Plc

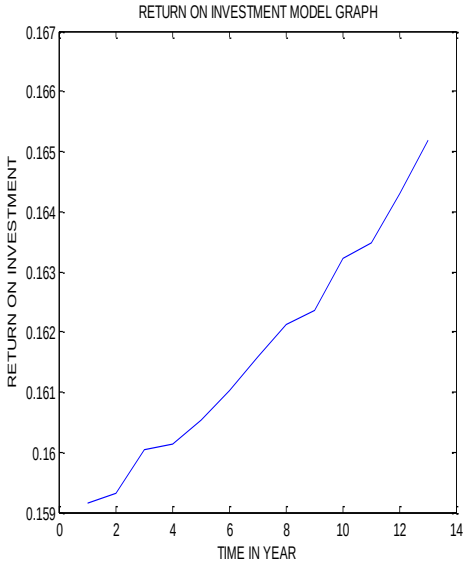

Figure 6: Yearly return on investment model graph for Seven Up Bottling Company Plc

4.0 CONCLUSION

We conclude that, total performance of a company cannot be comprehended through stock price and interest rate but a four-compartment stock market model achieved this through return on investment. Also a four-compartment stock market model can fit a real situation since coefficient of correlation significance and absolute error of empirical and model value is equal to zero (0). In addition, stock price, stock price volatility, return on investment, return on investment volatility follows random (stochastic) process. Furthermore, return on investment is the best index to use to validate performance of company not stock price because even if stock price might increase it doesn't mean the company is viable. For instance, Seven-Up Bottling Company Plc at 2013 the stock price is high but return on investment is poor. Finally, every quoted developed company are stable in return on investment than emerging company.

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