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ODD GENERALIZED EXPONENTIAL LAPLACE DISTRIBUTION: PROPERTIES AND APPLICATION TO FINANCIAL AND SURVIVAL DATA

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ABSTRACT

The Laplace distribution is a major distribution used in statistics to model different processes because of its flexibility. In this study, we derived a four parameter Odd Generalized Exponential Laplace Distribution (OGELAD) which unlike other variants of the Laplace distribution has a curve peak and can assume different shapes. While deriving the moments generating function, characteristic function, quantile function, order statistics and entropy, we have obtained the explicit form of the density function and distribution function of the proposed distribution. Maximum likelihood estimation has been used to determine the parameters of the suggested OGELAD, and a simulation study has been used to evaluate the performance of the estimation technique. By using two actual data sets, the flexibility of the OGELAD is further assessed. The results show that the proposed distribution outperforms other competing distributions for both the financial and survival data used.

1. Introduction

The developments in generalizing classical distributions for flexible distributions that could model the characteristics possessed by different kind of data is a developing area in the field of mathematical statistics that is gaining wider acceptability [1]. Some of the methods that have been adopted recently involve combining existing distributions into new distributions or adding parameters to existing distributions. This among other things help to provide a model with a strong empirical fit to the data [2]. One of several generators that have been developed to generate these distributions is the Odd Generalized Exponential-G (OGE-G) family of distributions by Tahir et al. [2]. The OGE-G has two parameters – a shape and a scale parameter that improves the fit of the distribution when compared to some families Generalized Exponential Power Function Distribution [4], Odd Generalized Exponential Inverse Lomax Distribution [5], Odd Generalized Exponential Gumbel Distribution [6], and Odds Generalized Exponential – Exponential Distribution [7].

Laplace distribution (LD) otherwise known as the double exponential distribution is the distribution of two independent, identically distributed exponential random variables. When the shape parameter = 1, it is a

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Simon Laplace, the LD has been applied in several fields which include but not limited to finance, economics, biology, quality control and engineering. For instance, the LD has been used to model financial returns because it has heavier tails [8,9]. Furthermore, it is found to provide a better fit to asset returns and also provide a more realistic expectation of daily returns for professionals in the stock market [10].

Skew variants of the LD considered as the generalization of the symmetric LD have also been found to be applicable for error innovations in navigation, inventory management and quality control, among others [11]. Also, [12] studied a two-parameter asymmetric Laplace distribution with significant peak and heavier tails than the classical distribution. The distribution's parameters were determined using maximum likelihood estimation, and the model's appropriateness is evaluated using actual data. On the contrary, [13] proposed a three-parameter asymmetric Laplace distribution. They showed unlike the classical LD, the asymmetric Laplace distribution has two exponential distributions of unequal scale and rate parameters. The distribution's properties are derived, and its application to a flood data illustrated.

As an alternative to the classical LD, [14] developed the beta Laplace distribution. In their work, they determined the mathematical and statistical properties of the distribution, and estimated the model parameters using maximum likelihood estimation. The performance of the model is evaluated with a real data set. Another important alternative is "a robust estimation procedure for mixture linear regression models by assuming that the error term follows a Laplace distribution" [15]. 3

Furthermore [8] proposed two-parameters modified classical Laplace distribution. The flat segment in the distribution's centre substitutes the abrupt peak of the traditional Laplace distribution. The modified classical Laplace distribution's mode is an interval as opposed to the classical Laplace distribution's. With actual data sets, the usefulness of the model is assessed. The study indicates that the classical distribution be further improved as a necessary alternative for the classical LD.

[16] proposed the weighted Laplace distribution after the method of obtaining weighted distributions by [17]. Similar to the classical Laplace distribution, the weighted LD has a sharp peak. Nevertheless, the weighted LD outperforms the classical LD in fitting of real data set.

[18] proposed a three-parameters distribution known as the modified Laplace distribution using the exponentiated family of distributions. The parameters of the distribution were determined using maximum likelihood estimation. For actual data sets, it was discovered that the distribution suited the data better than the traditional Laplace distribution and GED.

In this study, we proposed a four-parameter distribution called Odd Generalized Exponential Laplace Distribution (OGELAD). The Odd Generalized Exponential generator has a shape and scale parameter and a hazard rate which could be increasing, decreasing, J, reversed-J, bathtub and upside-down bathtub [2]. These featuresmade the generator suitable for fitting data set with differentshapes and heavier tails. "The use of four-parameter distributions should be sufficient for most practical purposes as at least three parameters are needed; there is hardly any noticeable improvement arising from including a fifth or sixth parameter" [19]. The remaining portions of study are organized as follows. We develop the proposed distribution, demonstrate that it is a density function, and illustrate the density plot. We derive the properties and parameter estimation for the proposed distribution in section 3. A simulation investigation is conducted in section 4, and section 5 provides an application of the proposed distribution to real data sets. The last remarks are then provided in section 6.

2. The Odd Generalized Exponential Laplace Distribution (OGELAD)

The cumulative distribution function (cdf) and probability density function (pdf) of a random variable Y which follows a Laplace distribution are given in (1) and (2) respectively;

$$F(y;\lambda,\tau)=\frac{1}{2}+\frac{1}{2}\operatorname{sgn}(y-\lambda)\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right]=\frac{1}{2}+\frac{1}{2}\frac{|y-\lambda|}{(y-\lambda)}\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right] \quad (1)$$

and

$$f(y;\lambda,\tau)=\frac{1}{2\tau}\exp\left(-\frac{|y-\lambda|}{\tau}\right) \quad (2)$$

where, $\operatorname{sgn}(y-\lambda)=\frac{|y-\lambda|}{(y-\lambda)}$

Alternatively, the distribution function and density function of the Laplace distribution are given as

$$F(x)=\begin{cases}\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right) & ; y\leq\lambda \\ 1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right) & ; y>\lambda\end{cases} \tag{3}$$

and

$$f(y)=\begin{cases}\frac{1}{2\tau}\exp\left(\frac{y-\lambda}{\tau}\right) & ; y\leq\lambda \\ \frac{1}{2\tau}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right) & ; y>\lambda\end{cases} \tag{4}$$

respectively. The cdf and pdf of Odd Generalized Exponential (OGE) family of distribution as defined by [2] are respectively given as:

$$G(y)=\left[1-\exp\left\{-m\left(\frac{F(y;\phi)}{1-F(y;\phi)}\right)\right\}\right]^{\theta} \tag{5}$$

$$g(x)=\frac{m\theta f(y;\phi)}{(1-F(y;\phi))^2}\exp\left\{-m\left(\frac{F(y;\phi)}{1-F(y;\phi)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{F(y;\phi)}{1-F(y;\phi)}\right)\right\}\right]^{\theta-1} \tag{6}$$

where, $F(y;\phi)$ and $f(y;\phi)$ are respectively the baseline cdf and pdf of a random variable Y with parameter vector ϕ, and m, n are shape parameters.

Given that the baseline distribution is the Laplace distribution with cdf and pdf given in equations (3) and (4), then, the cdf and pdf of the new distribution (known as Odd Generalized Exponential Laplace Distribution [OGELAD]) are respectively:

$$G(y)=\begin{cases}\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta} & ; y\leq\lambda \\ \left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta} & ; y>\lambda\end{cases} \tag{5}$$

and

$$g(y)=\begin{cases}\frac{m\theta\exp\left(\frac{y-\lambda}{\tau}\right)}{2\tau\left[1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)\right]^2}\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta-1} & ; y\leq\lambda \\ \frac{m\theta\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)}{2\tau\left[\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)\right]^2}\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta-1} & ; y>\lambda\end{cases} \tag{6}$$

where $m, \theta, \tau > 0, -\infty < \lambda, y > \infty$.

The pdf of the OGELAD has four parameters; m and θ are shape parameters, τ is a scale parameter and λ is a location parameter.

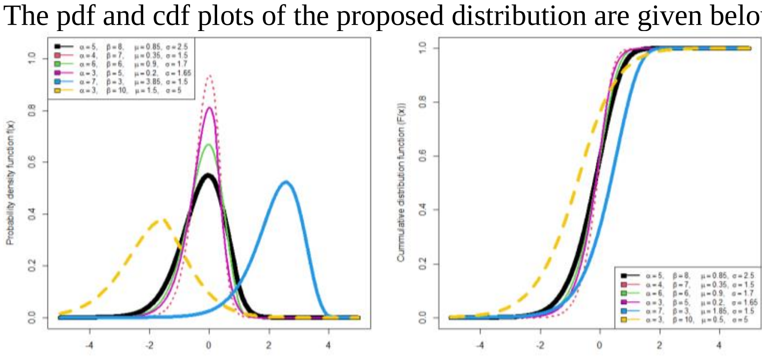
If $Y \sim OGELAD(\varphi)$, the survival and hazard functions are respectively;

$$S(y;\varphi)=\begin{cases} 1-\left[1-\exp\left\{-m\left(\dfrac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta} & ; y\leq\lambda \\ 1-\left[1-\exp\left\{-m\left(\dfrac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta} & ; y>\lambda \end{cases} \quad (7)$$

and

$$H(y;\varphi)=\begin{cases} \dfrac{m\theta\exp\left(\frac{y-\lambda}{\tau}\right)\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta-1}}{2\tau\left[1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)\right]^{2}\left\{1-\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta}\right\}} & ; y\leq\lambda \\ \dfrac{m\theta\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta-1}}{2\tau\left[1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)\right]^{2}\left\{1-\left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta}\right\}} & ; y>\lambda \end{cases} \quad (8)$$

The pdf and cdf plots of the proposed distribution are given below



(a) Pdf Plot (b) Cdf Plot

Figure 1: Density and cdf plots of the Proposed Distribution

The density plot (a) in Figure 1 is indicative of the fact that the OGELAD can be right skewed, symmetric, or left skewed depending on the values of the parameters considered as such its suitability for data sets with different shapes. Furthermore, the cdf plot (b) shows with increase in *x*, the distribution converges to 1.

2.2 Validity of the OGELAD

To show the validity of the OGELAD, we prove that $\int_{-\infty}^{\infty} g(y)dy = 1$.

Let,

$$\int_{-\infty}^{\infty} g(y)dy = \int_{-\infty}^{\lambda} g_1(y)dy + \int_{\lambda}^{\infty} g_2(y)dy \tag{9}$$

where,

$$g_1(y) = \frac{m\theta \exp\left(\frac{y-\lambda}{\tau}\right)}{2\tau\left[1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)\right]^2} \exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\} \tag{10}$$

$$\times\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta-1} \quad ; y \le \lambda$$

$$g_2(x) = \frac{m\theta \exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)}{2\tau\left[1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)\right]^2} \exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\} \tag{11}$$

$$\times\left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta-1} \quad ; y > \lambda$$

then,

$$\int_{-\infty}^{\lambda} g_1(y)dy = \int_{0}^{(1-e^{-m})^{\theta}} \frac{m\theta \exp\left(m\left(\frac{y-\lambda}{\tau}\right)\right)}{2\tau\left[1-\left(\frac{1}{2}\exp\left[\frac{y-\lambda}{\tau}\right]\right)\right]^2} \exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left[\frac{y-\lambda}{\tau}\right]\right)}{1-\left(\frac{1}{2}\exp\left[\frac{y-\lambda}{\tau}\right]\right)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left[\frac{y-\lambda}{\tau}\right]\right)}{1-\left(\frac{1}{2}\exp\left[\frac{y-\lambda}{\tau}\right]\right)}\right)\right\}\right]^{\theta-1} dy$$

$$= \left(1-e^{-m}\right)^{\theta} \tag{12}$$

also,

$$\int_{\lambda}^{\infty} g_2(y)dy = \int_{(1-e^{-m})^{\theta}}^{1} \frac{m\theta \exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)^{m-1}}{2\tau\left[\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right]^2} \exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta-1} dy$$

$$= 1-\left(1-e^{-m}\right)^{\theta} \tag{13}$$

Therefore, substituting (12) and (13) into (9) gives

$$\int_{-\infty}^{\infty} g(y)dy = \int_{-\infty}^{\lambda} g_1(y)dy + \int_{\lambda}^{\infty} g_2(y)dy = \left(1-e^{-m}\right)^{\theta} + 1 - \left(1-e^{-m}\right)^{\theta} = 1$$

as required

3. Statistical Propertiesof the OGELAD

In the following section, we derived some of the statistical properties of the OGELAD

3.1 Asymptotic Behaviour of OGELAD

The limiting distribution of the OGELAD;

(i) $\lim_{y\to-\infty} G(y) = 0$ and $\lim_{y\to\infty} G(y) = 1$

(ii) $\lim_{y\to-\infty} g(y) = 0$ and $\lim_{y\to\infty} g(y) = 0$

Proof:(i)

$$\lim_{y\to-\infty} G(y) = \lim_{y\to-\infty}\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta}$$

$$= \left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{-\infty-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{-\infty-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta} = \{1-1\}^{\theta} = 0^{\theta} = 0$$

also,

$$\lim_{y\to\infty} G(y) = \lim_{y\to\infty}\left[1-\exp\left\{-m\left(\frac{-\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta}$$

$$= \left[1-\exp\left\{-m\left(\frac{-\left(1-\frac{1}{2}\exp\left(-\left(\frac{\infty-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{\infty-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta} = \{1-0\}^{\theta} = 1^{\theta} = 1$$

(ii)

$$\lim_{y\to-\infty} g(y) = \lim_{y\to-\infty}\frac{m\theta\exp\left(\frac{y-\lambda}{\tau}\right)}{2\tau\left[1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)\right]^{2}}\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}$$

$$\times\left[1-\exp\left\{-m\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}\right]^{\theta-1}$$

$$= \frac{m\theta}{2\tau}\times 0 = 0$$

Similarly,

$$\lim_{y\to\infty} g(y) = \lim_{y\to\infty} \frac{m\theta \exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)}{2\tau\left[\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)\right]^2} \exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}$$

$$\times\left[1-\exp\left\{-m\left(\frac{\left(1-\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}{\left(\frac{1}{2}\exp\left(-\left(\frac{y-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\theta-1}$$

$$= \frac{m\theta}{2\tau}\times 0 = 0$$

The result in (ii) above shows the proposed distribution is unimodal.

3.2 Quantile Function

The quantile function of a distribution is the solution of the cumulative distribution function with respect to x i.e.

$$u = G(y)$$

where, $y = G^{-1}(u)$ and $\mathrm{G}^{-1}(u) = \inf\left\{y : G(y) \ge u\right\}$.

For the OGELAD, the quantile function is;

$$y_u = \lambda - \tau \ln\left(2\left[1-\left(\frac{-\ln\left(1-u^{\frac{1}{\theta}}\right)}{m-\ln\left(1-u^{\frac{1}{\theta}}\right)}\right)\right]\right) \quad (14)$$

where$m$, θ, $\tau > 0$ and $-\infty<\lambda<\infty$

The above quantile function can be used to generate random observations from the OGELAD. In (14) if u = 0.5, we have the median of the OGELAD given by:

$$y_{0.5} = \lambda - \tau \ln\left(2\left[1-\left(\frac{-\ln\left(1-0.5^{\frac{1}{\theta}}\right)}{m-\ln\left(1-0.5^{\frac{1}{\theta}}\right)}\right)\right]\right) \quad (15)$$

3.3 Series Expansion

Using the expression in (6),

$$\left[1-\exp\left\{-m\left(\frac{F(x)}{1-F(x)}\right)\right\}\right]^{\theta-1} = \sum_{k=0}^{\infty}(-1)^k\binom{\theta-1}{k}\left[\exp\left\{-m\left(\frac{F(y)}{1-F(y)}\right)\right\}\right]^k \quad (16)$$

substituting (16) into (6) gives;

$$g(y)=\frac{m\theta f(y;\phi)}{\left(1-F(y;\phi)\right)^{2}}\exp\left(-m\frac{F(y;\phi)}{1-F(y;\phi)}\right)\sum_{k=0}^{\infty}(-1)^{k}\binom{\theta-1}{k}\left[\exp\left(-m\frac{F(y)}{1-F(y)}\right)\right]^{k}$$
$$=\frac{m\theta f(y;\phi)}{\left(1-F(y;\phi)\right)^{2}}\sum_{k=0}^{\infty}(-1)^{k}\binom{\theta-1}{k}\exp\left[-m\left(k+1\right)\left(\frac{F(y)}{1-F(y)}\right)\right] \quad (17)$$

simplifying the last expression in (17) gives

$$g(y)=\frac{m\theta f(y,\phi)}{\left(1-F(y,\phi)\right)^{2}}\sum_{k,l=0}^{\infty}\frac{(-1)^{k+l}\left[m(k+1)\right]^{l}\binom{\theta-1}{k}}{l!}\frac{\left(F(y,\phi)\right)^{l}}{\left(1-F(y,\phi)\right)^{l}} \quad (18)$$

$$g(y)=m\theta\sum_{k,l=0}^{\infty}\frac{(-1)^{k+l}\left[m(k+1)\right]^{l}\binom{\theta-1}{k}}{l!}f(y,\phi)\frac{\left(F(y,\phi)\right)^{l}}{\left(1-F(y,\phi)\right)^{l+2}} \quad (19)$$

After further simplification,

$$g(y)=\sum_{l,v=0}^{\infty}\psi_{l,v}h_{l+v}(y) \quad (20)$$

where, $h_{l+v}(y) = f(y;\phi)F(y;\phi)^{l+v}$ represents the pdf of the Exp-G family with power parameter $l + v$ and

$$\psi_{l,v}=\frac{m^{l+1}\theta(-1)^{l+v}\binom{-(l+2)}{v}}{l!}\sum_{k=0}^{\infty}(-1)^{k}(k+1)^{l}\binom{\theta-1}{k}$$

Thus the OGELAD belongs to the class of Exp-G family of distributions.

3.4 Moments Generating Function

The moments generating function of a random variable Y, $M_Y(t)$ is given as follows;

$$M_Y(t)=E\left(e^{tY}\right)=\int_{-\infty}^{\infty}e^{ty}g(y)dy$$

From (20),

$$g(y)=\sum_{l,v=0}^{\infty}\psi_{l,v}f(y;\phi)F(y;\phi)^{l+v}$$

$$M_Y(t)=E\left(e^{tY}\right)=\int_{-\infty}^{\infty}e^{ty}\sum_{l,v=0}^{\infty}\psi_{l,v}f(y;\phi)F(y;\phi)^{l+v}dy$$
$$=\sum_{l,v=0}^{\infty}\psi_{l,v}\left[\int_{-\infty}^{\lambda}e^{ty}f_1(y;\phi)F_1(y;\phi)^{l+v}dy+\int_{\lambda}^{\infty}e^{ty}f_2(y;\phi)F_2(y;\phi)^{l+v}dy\right] \quad (21)$$

Using the first part of (21),

$$\int_{-\infty}^{\lambda} e^{ty} f_1(y;\phi)F_1(y;\phi)^{l+v} dy = \int_{-\infty}^{\lambda} e^{ty} \left[\frac{1}{2\tau} e^{\left(\frac{y-\lambda}{\tau}\right)}\right]\left[\frac{1}{2} e^{\left(\frac{y-\lambda}{\tau}\right)}\right]^{l+v} dy$$

$$= \frac{1}{\tau}\int_{-\infty}^{\lambda} e^{ty} \left[\frac{1}{2} e^{\left(\frac{y-\lambda}{\tau}\right)}\right]^{l+v+1} dy$$

$$\int_{-\infty}^{\lambda} e^{ty} f_1(y;\phi)F_1(y;\phi)^{l+v} dy = \frac{1}{\tau 2^{(l+v+1)}}\int_{-\infty}^{\lambda} e^{ty} e^{(l+v+1)\left(\frac{y-\lambda}{\tau}\right)} dy$$

$$= \frac{e^{\frac{-(l+v+1)\lambda}{\tau}}}{\tau 2^{(l+v+1)}}\int_{-\infty}^{\lambda} e^{\frac{-y}{\tau}\left(-t\tau-(l+v+1)\right)} dy \qquad (22)$$

letting $x = \frac{y}{\tau}\left(-t\tau-(l+v+1)\right)$ in (22) yields

$$\int_{-\infty}^{\lambda} e^{ty} f_1(y;\phi)F_1(y;\phi)^{l+v} dy = \frac{-e^{\frac{-(l+v+1)\lambda}{\tau}}}{\tau 2^{(l+v+1)}}\int_{\infty}^{\frac{\lambda}{\tau}(-t\tau-(l+v+1))} e^{-x}\tau \frac{dx}{\left(t\tau+(l+v+1)\right)}$$

$$= \frac{e^{\frac{-(l+v+1)\lambda}{\tau}}}{2^{(j+k+1)}\left(t\tau+(j+k+1)\right)}\left[e^{-x}\right]_{\infty}^{\frac{\lambda}{\tau}(-t\tau-(l+v+1))}$$

$$= \frac{e^{\lambda t}}{2^{(l+v+1)}\left(t\tau+(l+v+1)\right)} \qquad (23)$$

Also, using the second part of (21),

$$\int_{\lambda}^{\infty} e^{ty} f_2(y;\phi)F_2(y;\phi)^{l+v} dy = \int_{\lambda}^{\infty} e^{ty} \left[\frac{1}{2\tau} e^{-\left(\frac{y-\lambda}{\tau}\right)}\right]\left[1-\frac{1}{2} e^{-\left(\frac{y-\lambda}{\tau}\right)}\right]^{l+v} dy$$

$$= \frac{1}{\tau}\int_{\lambda}^{\infty} e^{ty} \left[\frac{1}{2} e^{-\left(\frac{y-\lambda}{\tau}\right)}\right]\sum_{b=0}^{\infty}(-1)^b\binom{l+v}{b}\left[\frac{1}{2} e^{-\left(\frac{y-\lambda}{\tau}\right)}\right]^{b} dy$$

$$\int_{\lambda}^{\infty} e^{ty} f_2(y;\phi)F_2(y;\phi)^{l+v} dy = \frac{1}{2\tau}\sum_{b=0}^{\infty}\frac{(-1)^b\binom{l+v}{b}}{2^b}\int_{\lambda}^{\infty} e^{ty} e^{-(b+1)\left(\frac{y-\lambda}{\tau}\right)} dy$$

$$= \frac{1}{2\tau}\sum_{b=0}^{\infty}\frac{(-1)^b\binom{l+v}{b}}{2^b} e^{\frac{(b+1)\lambda}{\tau}}\int_{\lambda}^{\infty} e^{-\frac{y}{\tau}\left((b+1)-t\tau\right)} dy \qquad (24)$$

letting $p=\frac{y}{\tau}\left((b+1)-t\tau\right)$ in (24) yields

$$\int_{\lambda}^{\infty} e^{ty} f_2(y;\phi)F_2(y;\phi)^{l+v}dy=\frac{1}{2\tau}\sum_{b=0}^{\infty}\frac{(-1)^b\binom{l+v}{b}}{2^b}e^{\frac{(b+1)\lambda}{\tau}}\int_{\frac{\lambda}{\tau}\left[(b+1)-t\tau\right]}^{\infty} e^{-p}\tau\frac{dp}{\left[(b+1)-t\tau\right]}$$

$$=\frac{e^{\lambda t}}{2}\sum_{b=0}^{\infty}\frac{(-1)^b\binom{l+v}{b}}{2^b}\frac{1}{\left(b+1-\tau t\right)} \quad (25)$$

Substituting (23) and (25) into (21) gives the moments generating function of the OGELAD as

$$M_Y(t)=\frac{e^{\lambda t}}{2}\sum_{l,v=0}^{\infty}\psi_{l,v}\left[\frac{1}{2^{(l+v)}\left[(l+v+1)+t\tau\right]}+\sum_{b=0}^{\infty}\frac{(-1)^b\binom{l+v}{b}}{2^b\left(b+1-\tau t\right)}\right] \quad (26)$$

3.4 Momentsof the OGELAD

Thenon-centralmomentsoftheOGELADarederivedbydifferentiatingthemoments generatingfunctionobtainedin(26)w.r.t.*t*at*t*=0.Symbolically,

$$E\left(Y^r\right)=\mu_r^{/}=\frac{d^r}{dt^r}\left[M_Y(t)\right]_{t=0} \quad (27)$$

where, r = 1, 2, 3, …. The mean, variance, skewness and kurtosis of the OGELAD are computed numerically for some choices of its parameters using suitable statistical packages like R.

3.5 Characteristic Function

The characteristic function $\phi_Y(t)$ of the OGELAD is:

$$\phi_Y(t)=E\left(e^{itY}\right)=M_Y(it)=\frac{e^{\mu it}}{2}\sum_{l,v=0}^{\infty}\psi_{l,v}\left[\frac{1}{2^{(l+v)}\left[(l+v+1)+\tau it\right]}+\sum_{b=0}^{\infty}\frac{(-1)^b\binom{l+v}{b}}{2^b\left(b+1-\tau it\right)}\right] \quad (28)$$

3.6 Order Statistics

Let Y_1, Y_2, Y_3, …,Y_q be random samples of size q from a pdf, ($f(y)$) and cdf, ($F(y)$). Suppose $Y_{1:q}$, $Y_{2:q}$, $Y_{3:q}$, …,$Y_{q:q}$ denotes corresponding order statistic derived from the samples, then, the p^{th} order statistic is defined by:

$$g_{p:q}(y)=\frac{q!g(y)}{(p-1)!(q-p)!}G(y)^{p-1}\left[1-G(x)\right]^{q-p} \quad (29)$$

From the series expansion,

$$(1-y)^r=\sum_{t=0}^{r}\frac{(-1)^t r!}{(r-t)!t!}y^t$$

then,

$$[1-G(x)]^{m-p}=\sum_{t=0}^{q-p}\frac{(-1)^t(q-p)!}{(q-p-t)!t!}G(x)^t$$

and (29) becomes

$$g_{p:q}(y)=\sum_{t=0}^{q-p}(-1)^t\frac{q!}{(p-1)!(q-p-t)!}g(y)G(y)^{t+p-1} \quad (30)$$

where, $g(y)$ and $G(y)$ is the pdf and cdf of the OGELAD. Note that,

$$\left[G(y)\right]^{t+p-1}=\begin{cases}\sum_{s=0}^{\theta(t+p-1)}(-1)^{s}\binom{\theta(t+p-1)}{s}\exp\left\{-ms\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\} & ;y\leq\lambda\\ \sum_{w=0}^{\theta(t+p-1)}(-1)^{w}\binom{\theta(t+p-1)}{w}\exp\left\{-mw\left(\frac{\left(1-\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\} & ;y>\lambda\end{cases} \tag{31}$$

Substituting (31) into (30) gives the order statistics of the OGELAD,

$$g_{p:q}(y)=\begin{cases}\sum_{t=0}^{q-p}\sum_{s=0}^{\theta(t+p-1)}(-1)^{t+s}\frac{q!\binom{\theta(t+p-1)}{s}}{(p-1)!(q-p-t)!}\exp\left\{-ms\left(\frac{\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}g_1(y) & ;y\leq\lambda\\ \sum_{t=0}^{q-p}\sum_{w=0}^{\theta(t+p-1)}(-1)^{t+w}\frac{q!\binom{\theta(t+p-1)}{w}}{(p-1)!(q-p-t)!}\exp\left\{-mw\left(\frac{\left(1-\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}{1-\left(1-\frac{1}{2}\exp\left(\frac{y-\lambda}{\tau}\right)\right)}\right)\right\}g_2(y) & ;y>\lambda\end{cases} \tag{32}$$

where, $g_1(y)$ and $g_2(y)$ is the pdf for the of the OGELAD given in (10) and (11), respectively.

3.7 Entropy

Let Y be a random variable with pdf, $g(y)$, then the Renyi entropy is given for $t>0$ and $t\neq 1$ as:

$$I_R=\frac{1}{1-t}\ln\left[\int_{-\infty}^{\infty}\left(g(y)\right)^t dy\right] \tag{33}$$

$$\int_{-\infty}^{\infty}\left[g(y)\right]^t dy=\int_{-\infty}^{\infty}\left[\sum_{l,v=0}^{\infty}\psi_{l,v}f(y;\phi)F(y;\phi)^{l+v}\right]^t dy$$

$$=\left(\sum_{l,v=0}^{\infty}\psi_{l,v}\right)^t\left[\int_{-\infty}^{\lambda}\left[f_1(y;\phi)F_1(y;\phi)^{l+v}\right]^t dy+\int_{\lambda}^{\infty}\left[f_2(y;\phi)F_2(y;\phi)^{l+v}\right]^t dy\right] \tag{34}$$

$$\int_{-\infty}^{\lambda}\left[f_1(y;\phi)F_1(y;\phi)^{l+v}\right]^t dy=\int_{-\infty}^{\lambda}\left[\frac{1}{2\tau}e^{\left(\frac{y-\lambda}{\tau}\right)}\left(\frac{1}{2}e^{\left(\frac{y-\lambda}{\tau}\right)}\right)^{l+v}\right]^t dy=\frac{\tau^{1-t}}{t(l+v+1)2^{(l+v+1)t}} \tag{35}$$

$$\int_{\lambda}^{\infty}\left[f_2(y;\phi)F_2(y;\phi)^{l+v}\right]^t dy=\int_{\lambda}^{\infty}\left[\frac{1}{2\lambda}e^{-\left(\frac{y-\lambda}{\tau}\right)}\right]^t\left[1-\frac{1}{2}e^{-\left(\frac{y-\lambda}{\tau}\right)}\right]^{(l+v)t}dy \tag{36}$$

$$=\frac{\tau^{1-t}}{t}\sum_{r=0}^{\infty}\frac{1}{(r+1)}\left[\frac{(-1)^r\binom{l+v}{r}}{2^{r+1}}\right]^t$$

Substituting (35) and (36) into (34) gives the Renyi entropy for the OGELAD as:

$$I_R = \begin{cases} \dfrac{1}{1-t}\ln\left[\dfrac{\left(\sum\limits_{l,v=0}^{\infty}\psi_{l,v}\right)^t \tau^{1-t}}{t(l+v+1)2^{(l+v+1)t}}\right] & ; y\le\lambda \\ \dfrac{1}{1-t}\ln\left\{\dfrac{\left(\sum\limits_{l,v=0}^{\infty}\psi_{l,v}\right)^t \tau^{1-t}}{t}\sum\limits_{r=0}^{\infty}\dfrac{1}{(r+1)}\left[\dfrac{(-1)^r\binom{l+v}{r}}{2^{r+1}}\right]^t\right\} & ; y>\lambda \end{cases} \tag{37}$$

3.8. Estimation of Model Parameters

Different methods are employed in the estimation of parameters. The attractiveness of the maximum likelihood lies in the estimation being asymptotically consistent, asymptotically efficient and asymptotically unbiased among other properties. Let $Y_1, Y_2, Y_3, \ldots, Y_n$ be a random sample of size n from the OGELAD with parameter vector $\Theta = (m, \theta, \lambda, \tau)$, the likelihood function for Θ is given as:

$L(\Theta) =$

$$\begin{cases} \prod\limits_{i=1}^{n} = \dfrac{m\theta\exp\left(\dfrac{y_i-\lambda}{\tau}\right)}{2\tau\left[1-\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)\right]^2}\exp\left\{-m\left(\dfrac{\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\sigma}\right]\right)}{1-\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}\right)\right\}\left[1-\exp\left\{-m\left(\dfrac{\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}{1-\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}\right)\right\}\right]^{\theta-1} & ; y\le\lambda \\ \prod\limits_{i=1}^{n} = \dfrac{m\theta\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)}{2\tau\left[1-\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)\right]^2}\exp\left\{-m\left(\dfrac{\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}\right)\right\}\left[1-\exp\left\{-m\left(\dfrac{\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]^{\beta-1} & ; y>\lambda \end{cases} \tag{38}$$

and the log likelihood function is

$$\ln\left(L(\Theta)\right) = \begin{cases} n(\ln m+\ln\lambda-\ln\tau-\ln 2)+\dfrac{1}{\tau}\sum\limits_{i=1}^{n}(y_i-\lambda)-m\sum\limits_{i=1}^{n}\left[\dfrac{\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}{1-\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}\right] \\ +(\theta-1)+\sum\limits_{i=1}^{n}\log\left[1-\exp\left\{-m\left(\dfrac{\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}{1-\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)}\right)\right\}\right]-2\sum\limits_{i=1}^{n}\log\left[1-\left(\dfrac{1}{2}\exp\left[\dfrac{y_i-\lambda}{\tau}\right]\right)\right] & ; y\le\lambda \\ n(\ln m+\ln\theta-\ln\tau-\ln 2)-\dfrac{1}{\tau}\sum\limits_{i=1}^{n}(y_i-\lambda)-m\sum\limits_{i=1}^{n}\left[\dfrac{\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}\right] \\ +(\theta-1)\sum\limits_{i=1}^{n}\log\left[1-\exp\left\{-m\left(\dfrac{\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}{1-\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)}\right)\right\}\right]-2\sum\limits_{i=1}^{n}\ln\left[1-\left(1-\dfrac{1}{2}\exp\left(-\left(\dfrac{y_i-\lambda}{\tau}\right)\right)\right)\right] & ; y>\lambda \end{cases} \tag{39}$$

The maximum likelihood estimators of parameters $\left(\hat{m}, \hat{\theta}, \hat{\lambda}, \hat{\tau}\right)$ are the solution of the simultaneous equations obtained when (39) is differentiated partially with respect to each parameter and equated to zero. The solution of equations is obtained using numerical methods found in statistical packages such as *R*.

4. Simulation

We use the quantile function of the OGELAD to carry out simulation study. The study is carried out for four different sample sizes (n = 50, 100, 200, 300) with parameter set (m, θ, λ, τ) = (1.2, 2.5, 0.3, 0.7). The simulation process is repeated 500 times for each sample size and the average parameter estimates, biases and the mean squares error (MSE) have been determined. The biases and MSE result is presented in Table 1.

Table 1: Simulation Output

Sample Size	Parameter	Bias	MSE
50	m	-0.22657	2.49931
	θ	-6.55097	58.10643
	λ	0.29380	0.13605
	τ	-0.11011	0.05401
100	m	0.09079	0.27522
	θ	-6.26059	48.99131
	λ	0.34848	0.12886
	τ	-0.11434	0.03860
200	m	0.20286	0.10622
	θ	-5.14450	31.20987
	λ	0.35440	0.12868
	τ	-0.07998	0.01897
300	m	0.23713	0.09573
	θ	-4.67887	24.75850
	λ	0.35194	0.12589
	τ	-0.06472	0.01161

Table 1 shows decrease in the biases and MSE as the sample size increases; these support the stability of the parameters of the OGELAD using the MLE.

5. Application to Real Data Sets

In this section, we will compare the OGELAD with the classical Laplace distribution and other competing distributions by Yahaya and Terna [6], and Agu and Onwukwe [18]. The comparison of the above distributions is facilitated with the aid of two real data sets described below:

Data set 1(S&P500 Data): The data are the weekly returns of the S&P500 from May, 2013 to May, 2015. The data are: -1.15, 0.77, -1.02, -2.13, 0.87, 1.58, 2.92, 2.71, -0.03, 1.06, -1.07, -2.13, 0.46, -1.85, 1.35, 1.96, 1.29, -1.07, 0.07, 0.75, 2.40, 0.87, 0.11, 0.51, 1.55, 0.37, 0.06, -0.04, -1.66, 2.39, 1.26, -0.55, 0.60, -0.20, -2.67, -0.43, 0.81, 2.29, -0.13, 1.26, 0.99, -1.98, 1.37, -0.48, 0.40, -2.68, 2.67, -0.08, 0.95, -0.14, -0.03, 1.20, 1.20, 1.34, -0.68, 1.37, -0.10, 1.24, -0.90, 0.54, 0.01, -2.73, 0.33, 1.21, 1.69, 0.75, 0.22, -1.11, 1.24, -1.38, -0.76, -3.19, 1.02, 4.04, 2.69, 0.68, 0.39, 1.15, 0.20, 0.38, -3.58, 3.36, 0.87, -1.47, -0.65, -1.25, 1.59, -2.81, 2.99, 2.00, 0.63, -0.28, -1.59, -0.87, 2.63, -2.26, 0.29, 1.68, -1.00, 1.74, -0.44, 0.37, 0.31, 0.16, -0.25.

Data set 2(Uncensored Data): The data set is from Nichols and Padgett [20] consisting of 100 observations on breaking stress of carbon fibers (in Gba). The data are: 3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19,3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53,2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15,2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98,2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12,1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68,2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82,2.05, 3.65

5.1 Result and Discussion

In this section, we have fitted the proposed distribution (OGELAD), Odd Generalized Exponential Gumbel Distribution (OGEGD), Modified Laplace Distribution (MLD), and the Laplace Distribution (LD) using the method of maximum likelihood. The popular goodness of fit statistics such as Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), and Hannan-Quinn Information Criterion (HQIC) have been employed for model comparisons. In all cases, the model with the smallest of these statistics is often judged to be the best model (Yakubu and Doguwa [21]).

Table 2 shows the parameter estimates and the log likelihood for the different fitted models. The proposed OGELAD has the smallest of all the statistics for the S&P500 data indicating its suitability for modelling of returns data among other competing models.

Similarly, Table 3 shows the fitting of the OGELAD with other competing models such as OGEGD, MLD, and LD for the uncensored data from observations on breaking stress of carbon fibres. Furthermore, the AIC, CAIC, and HQIC for the proposed model (OGELAD) is the least indicating its adequacy in fitting the uncensored data.

Figures 2 and 3 present the estimated density of the proposed OGELAD and the Q-Q plots for the two data sets. These plots support the argument on the performance of the OGELAD over its competing models.

The result in this paper indicate that flexible models are insightful in studying the characteristics of data generated by different processes. This result is consistent with result of other studies where the addition of an extra parameter is found to improve the flexibility of the existing distributions. This has been reported in several studies including Johnson et al.[19]; Nadarajah and Kotz [22]; Rosaiah et al. [23]; Bukoye and Oyeyemi [24]; and Falgore [25].

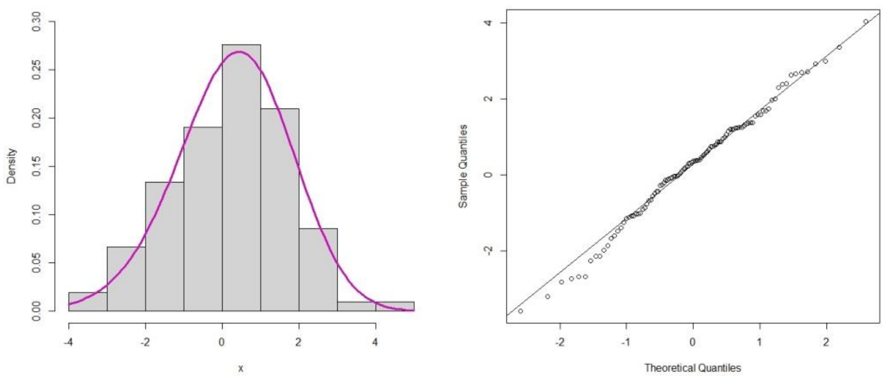
Table 2: Parameter Estimates, Log-likelihood, AIC, CAIC, and HQIC for S&P500 Data

Model	Parameter	Estimates	Log-likelihood	AIC	CAIC	HQIC
OGELAD	m	0.0662	-191.0196	390.0392	394.1240	394.3409
	θ	5.1168				
	λ	-7.8798				
	τ	2.9538				
OGEGD	m	0.0541	-191.4547	390.9094	394.9942	395.2111
	θ	5.9461				
	λ	-11.1005				
	τ	3.0688				
MLD	θ	0.9318	-194.4474	394.8948	397.9584	398.1211
	λ	0.3900				
	τ	1.1365				
LD	λ	0.3700	-194.5901	393.1802	395.2226	395.3311
	τ	1.1736				

Table 3: Parameter Estimates, Log-likelihood, AIC, CAIC, and HQIC for Uncensored Data

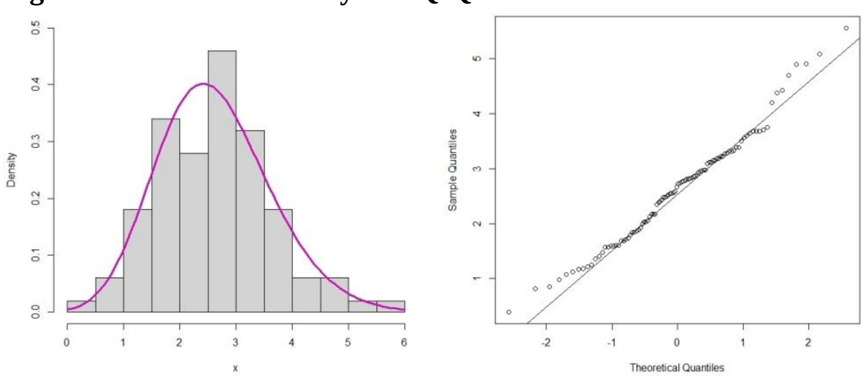
Model	Parameter	Estimates	Log-likelihood	AIC	CAIC	HQIC
OGELAD	m	1.4380	-141.5356	291.0702	295.1560	295.3729
	θ	19.4575				
	λ	0.3900				
	τ	4.2461				
OGEGD	m	5.9279	-141.5973	291.1947	295.2794	295.4963
	θ	28.3918				
	λ	2.3080				

	τ	5.0477				
MLD	θ	391.8008	-144.2340	294.4680	297.5316	297.6943
	λ	-2.6762				
	τ	0.9108				
LD	λ	2.7274	-147.1003	298.2006	300.2430	300.3515
	τ	0.8008				



(a) Density Plot (b) Q-Q Plot

Figure 2: Estimated Density and Q-Q Plots for S&P500 Data



(a) Density Plot (b) Q-Q Plot

Figure 3: Estimated Density and Q-Q Plots for Uncensored Data

6. Conclusion

In this paper, we have proposed a four parameter Odd Generalized Exponential Laplace Distribution. Plot of the proposed distribution indicate it can be symmetrical or asymmetrical depending on the chosen values of the parameters. The pdf, cdf, quantile function and other mathematical/statistical properties of the distribution has been derived. The parameters of the distribution were estimated via maximum likelihood estimation while a simulation was carried out to assess the performance of the method. The fitting of the distribution to real data sets indicate the usefulness of the model in both finance and survival analysis.

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