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A DISCRETE TIME ECONOMIC ORDER QUANTITY MODEL FOR AMELIORATING ITEMS WITH CONSTANT DEMAND

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ABSTRACT

In this paper, we develop a discrete time Economic Order Quantity (EOQ) Model with constant demand for items that are ameliorating. In the model, the items considered are assumed to ameliorate immediately they arrive in stock. The purpose is to determine the optimal ordering quantity and replenishment cycle so as to minimize the total variable cost. It is the remodelling of Gwanda by considering time to be discrete. The model is developed using difference equation with initial and boundary conditions. Numerical examples are illustrated to see the application of the model and sensitivity analysis carried out to determine the effect of the parameter changes.

1. Introduction

Amelioration occurs when we make better the value or utility of a product in such a way that it increases over time. Young or fast-growing animals such as fish, chicken, ducks, cows, sheep and so on, are examples of ameliorating items. The livestock are being purchased when small and are reared over time. The small livestock are the items and when kept and fed they increase in weight and value over time. Hwang [1] was the first to consider the model for ameliorating items using two parameter Weibull distribution. Hwang[2] later developed economic order quantity and partial selling price model considering issuing policies of first in first out (FIFO) and last in first out (LIFO) for two parameter Weibull distribution for ameliorating and deteriorating items. Biswajit *et al.*[3] considered ameliorating items for price dependent demand with instantaneous replenishment system where shortages are not allowed.

Hwang[4] presented ameliorating and deteriorating items for storage among a discrete set of location sites to determine the minimum number of storage facilities so that the probability of each customer being covered is not below the critical value. Moon *et al.*[5] developed economic order quantity models for ameliorating and

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deteriorating items with time discounting, the ameliorating and deteriorating rates are considered as functions of time. Srichandan *et al.*[6] considered an inventory model with Weibull amelioration under the influence of inflation and time-value for money. Gobinda *et al.*[7] first proposed inventory models for ameliorating items with time dependent second order demand rate. They developed two models: one is an economic order quantity model for items whose utility is ameliorating in accordance with Weibull distribution and the other is a partial selling quantity model (PSQ) developed for selling the surplus inventory accumulated by amelioration activation with linear demand. Han-Wen *et al.*[8] developed an improvement for amelioration inventory model with Weibull distribution. The improvement is in deriving the optimal solution of the problem.

Inventory problems involving ameliorating and deteriorating items have received little attention from researchers. Some of which are Moon *et al.*[9] developed an EOQ model of ameliorating and deteriorating items with zero ending inventory for fixed order interval over a finite planning horizon. The authors considered linear trend in demand, shortages, effects of inflation and time value for money. Law and Wee [10] considered an integrated production inventory for ameliorating and deteriorating items taking account of time discounting. Hui-Ming *et al.*[11] considered an inventory model for ameliorating and deteriorating items taking account of time value for money and finite planning horizon. Valliathal and Uthayakumar [12] formulated a production inventory problem for ameliorating and deteriorating items with non-linear shortage cost under inflation and time discounting. Valliathal and Uthayakumar [13] formulated a study of inflation effects on an EOQ model for Weibull deteriorating and ameliorating items with Ramp-type demand and shortages. Gothi and Parmar [14] presented an integrated inventory model with exponential amelioration and two parameter Weibull deterioration. Gothi and Bhojak [15] formulated two inventory models for ameliorating and deteriorating items with time dependent demand. Vandana and Srivastava [16] presented an inventory model for ameliorating and deteriorating items with trapezoidal demand and complete backlogging under inflation and time discounting. Gwanda and Sani [17] considered an economic order quantity model for items that are both ameliorating and deteriorating with constant demand rate. Karthikeyan and Santhi[18] presented an EOQ model for Weibull ameliorating items with constant deteriorating items, time dependent demand rate and price discount on backorders. Gwanda [19] developed an economic order quantity model for both ameliorating and deteriorating items with exponentially increasing demand and linear time dependent holding cost.

Products whose demand are periodic and variable where the amount of order each time placed is equal to the net requirements for the product for that duration are said to exhibit discrete time demand pattern. Time duration is counted in terms of complete days, months or even years. Several authors studied the discrete time inventory models among them are: Dave and Jaiswal [20] who studied a discrete in time probabilistic inventory model for deteriorating items with stationary uniform demand, constant deterioration rate with no shortages. Dave [21] developed a discrete in time deteriorating inventory model with demand rate as a linear function of time, constant deterioration, finite planning horizon where shortages are not allowed. Ferhan *et al*. [22] considered an inventory model on deteriorating items of non -periodic discrete time domains where time points may not be necessarily evenly spaced over a time interval. Aliyu and Boukas [23] developed discrete time inventory models with deterministic demand. Zhaotong and Liming [24] developed a discrete time model for perishable inventory system with geometric inter-demand times and batch demands. Yakubu and Sani [25] proposed an EOQ model for deteriorating items that exhibit delay in deterioration with discrete time.

In this paper, a model for ameliorating items with discrete time and constant demand is presented. Shortages are not allowed. Our aim is to remodel the work of Gwanda [26]where the time was taken to be continuous.

2. Model Description and Formulation

The proposed model is described under the following notation and assumptions

2.1 Notation and Assumptions

Notation

$I_A(t)$ Inventory level at any time t

C_0 The ordering cost per order

$I_A(0)$ The order quantity

T Circle length

CA Cost of amelioration per unit

C_H The inventory holding cost per unit

D_T Total demand in a cycle, T.

i The inventory carrying charge

C The unit cost of the item

D The demand rate per unit time

TVC Total variable cost

A Ameliorated amount

Assumptions

(i) The lead time is zero.

(ii) An instantaneous ameliorating item is considered. (Amelioration occurs immediately the items arrive in stock)

(iii) The instantaneous rate of amelioration of the on-hand inventory at any time t is A, which is a constant.

(iv) Shortages are not allowed

(v) It is assumed that the item is not kept beyond the fixed time T.

2.2 Model Formulation

In this study, the items are purchased from an outside supplier. When properly taken care of, amelioration occurs when the items are kept in the stock. At the beginning of the inventory cycle, demand and amelioration take place up until the inventory reaches zero level at t = T. $I_A(0)$ is the initial order quantity. The rate of demand is constant, while the time is taken to be discrete. The behaviour of the model is described in Figure 1 below.

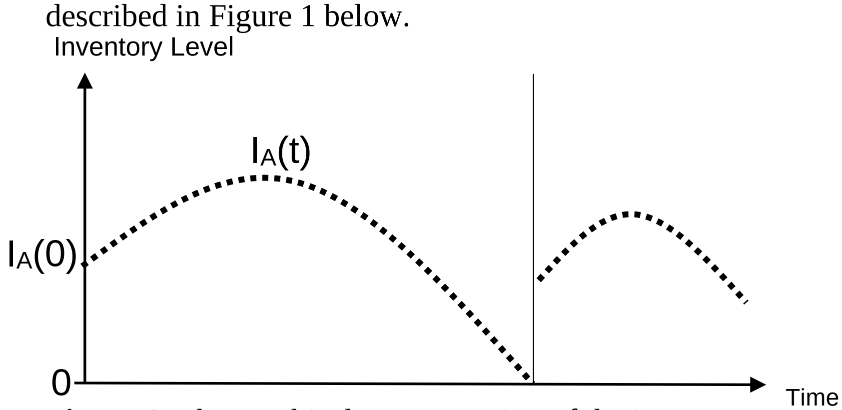

Figure 1: The graphical representation of the inventory system

$I_A(0)$ is the initial inventory, $I_A(t)$ is the inventory level at any time t. During the time interval $(0 \leq t \leq T)$ amelioration occurs at a constant rate A and the demand rate is a constant, D per unit time.

The difference equation describing the state of the inventory level I(t) is given by

$$\Delta I_A(t) = AI_A(t) - D \qquad 0 \leq t \leq T \qquad (1)$$

With the initial and boundary conditions $I_A(0) = \mathrm{I}(0)$, $t = 0$ and $I_A(T) = 0$ at $t = T$.
This can be solved as follows:
Since $\Delta f(x) = f(x+h) - f(x)$, where h is the step length, then
$\Delta I_A(t) = I_A(t+1) - I_A(t)$ with step length of 1.
This implies
$I_A(t+1) - I_A(t) = AI_A(t) - D$ from equation (1)
$\Rightarrow I_A(t+1) = (1+A)I_A(t) - \mathrm{D}$ for t = 0, 1, 2, 3, …, (m-1), m = T
for $t = 0$

$$I_A(1) = (1+A)I_A(0) - D$$

for $t = 1$

$$I_A(2) = (1+A)I_A(1) - \mathrm{D}$$

for $t = 2$

$$I_A(3) = (1+A)I_A(2) - D$$

for $t = 3$

$$I_A(4) = (1+A)I_A(3) - \mathrm{D}$$

$$= (1+A)\left[(1+A)^3 I_A(0) - \frac{D}{A}[(1+A)^3 - 1]\right] - \mathrm{D}$$

$$= (1+A)^4 I_A(0) - \left[\frac{(1+A)(1+A)^3}{A}\mathrm{D} - \frac{(1+A)}{A}\mathrm{D}\right] - D$$

$$= (1+A)^4 I_A(0) - \left[\frac{(1+A)^4}{A}\mathrm{D} - \frac{(1+A)}{A}\mathrm{D}\right] - D$$

$$= (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - (1+A)] - D$$

$$= (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - (1+A) + A]$$

$$= (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - 1]$$

$$\therefore I_A(4) = (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - 1]$$

So that in general we obtain

$$I_A(t) = (1+A)^t I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^t - 1] \quad (2)$$

Continuing up to T, yields

$$I_A(T) = (1+A)^T I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^T - 1] \quad (3)$$

Also for $t = (T-1)$, we have

$$I_A(T-1) = (1+A)^{T-1} I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^{T-1} - 1]$$

For $t = (T-2)$

$$I_A(T-2) = (1+A)^{T-2} I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^{T-2} - 1]$$

Using the boundary condition at $t = T$, $I_A(T) = 0$ from equation (3) we have

$$0 = (1+A)^T I_A(0) - \frac{D}{A}[(1+A)^{\mathrm{T}} - 1]$$

$$\Rightarrow (1+A)^T I_A(0) = \frac{\mathrm{D}}{A}[(1+A)^T - 1]$$

$$\Rightarrow I_A(0) = \frac{\frac{D}{A}[(1+A)^T - 1]}{(1+A)^T}$$

$= \frac{D}{A}[(1+A)^T - 1](1+A)^{-T}$

$\therefore I_A(0) = \frac{D}{A}[1 - (1+A)^{-T}]$ (4)

Substituting equation (4) into equation (2) yields

$I_A(t) = (1+A)^t \frac{D}{A}[1 - (1+A)^{-T}] - \frac{D}{A}[(1+A)^t - 1]$ (5)

$= \frac{D}{A}(1+A)^t - \frac{D}{A}(1+A)^t(1+A)^{-T} - \frac{D}{A}(1+A)^t + \frac{D}{A}$

$$= -\frac{D}{A}(1+A)^t(1+A)^{-T} + \frac{D}{A}$$

$\therefore I_A(t) = -\frac{D}{A}[(1+A)^t(1+A)^{-T} - 1]$ (6)

The total demand within the interval $(0 \leq t \leq T)$ is given by

$D_T =$ demand rate x time period

$$= DT$$

The ameliorated amount A is given by

A $= D_T - I_A(0)$ i.e total demand – order quantity

$$= DT - \frac{D}{A}[1 - (1+A)^{-T}]$$

The holding cost, C_H in the cycle is calculated as

$C_H = i\% \times$ cost per unit $\times \sum_{t=0}^{T} I_A(t)$

From equation (6)

$$I_A(1) = -\frac{D}{A}[(1+A)(1+A)^{-T} - 1]$$

$$I_A(2) = -\frac{D}{A}[(1+A)^2(1+A)^{-T} - 1]$$

$$I_A(3) = -\frac{D}{A}[(1+A)^3(1+A)^{-T} - 1]$$

$$I_A(4) = -\frac{D}{A}[(1+A)^4(1+A)^{-T} - 1]$$

$$I_A(5) = -\frac{D}{A}[(1+A)^5(1+A)^{-T} - 1]$$

At t = (T-1), we have

$$I_A(T-1) = -\frac{D}{A}[(1+A)^{T-1}(1+A)^{-T} - 1]$$

and at t = (T), we get

$$I_A(T) = -\frac{D}{A}[(1+A)^T(1+A)^{-T} - 1]$$

$\therefore \sum_{t=0}^{T} I_A(t) = -\frac{D}{A}(1+A)^{-T}[q^0 + q^1 + q^2 + q^3 + \cdots + q^{T-1} + q^T] + \frac{D}{A}(T+1)$, Where $q = (1+A)$. However,

$$\sum_{i=0}^{T} q^i = \left[\frac{(1+A)^{T+1}}{A} - \frac{1}{A}\right]$$

$$\therefore \sum_{t=0}^{T} I_A(t) = \frac{D}{A^2}[(1+A)^{-T} - (1+A)] + \frac{D}{A}(T+1)$$

Hence Holding cost

$$H_C = iC\sum_{t=0}^{T} I_A(t) = iC\left\{\frac{D}{A^2}[(1+A)^{-T} - (1+A)] + \frac{D}{A}(T+1)\right\}$$

Since the holding cost and cost of amelioration is between the interval $0 \le t \le T$ we have T+1 terms.
The total variable cost in the cycle is given by
T V C = Ordering cost + holding cost - cost of ameliorated amount

$$= C_0 + C_H - CA$$

$$= C_0 + iC\left\{\frac{D}{A^2}[(1+A)^{-T} - (1+A)] + \frac{D}{A}(T+1)\right\} - C\left\{DT - \frac{D}{A}[1-(1+A)^{-T}]\right\}$$

. The total variable cost per unit time is then given by,

$$TVC\ (T) = \left[\frac{C_0}{T} + \frac{iC}{(T+1)}\left\{\frac{D}{A^2}[(1+A)^{-T} - (1+A) + A(T+1)]\right\} - \frac{C}{(T+1)}\left\{DT - \frac{D}{A}[1-(1+A)^{-T}]\right\}\right] \quad (7)$$

Therefore $TVC\ (T)$

$$= \left[\frac{C_0}{T} + iC\left\{\frac{D}{(T+1)A^2}[(1+A)^{-T} - (1+A) + A(T+1)]\right\} - \frac{C}{(T+1)}\left\{DT - \frac{D}{A}[1-(1+A)^{-T}]\right\}\right]$$

In a similar way, for TVC (T-1) , let T-1 = s, so that TVC (T-1) = TVC (s)

$$= \left[\frac{C_0}{s} + iC\left\{\frac{D}{(T+1)A^2}[(1+A)^{-s} - (1+A) + A(s+1)]\right\} - \frac{C}{(T+1)}\left\{Ds - \frac{D}{A}[1-(1+A)^{-s}]\right\}\right]$$

Also, for TVC (T+1), let T+1 = e, so that TVC (T+1) = TVC (e)

$$= \left[\frac{C_0}{e} + iC\left\{\frac{D}{(T+1)A^2}[(1+A)^{-e} - (1+A) + A(e+1)]\right\} - \frac{C}{(T+1)}\left\{De - \frac{D}{A}[1-(1+A)^{-e}]\right\}\right]$$

$$TVC\ (T) - TVC\ (s)$$

$$= \frac{C_0(s-T)}{Ts} + \frac{iCD[(1+A)^{-T} - (1+A)^{-s}]}{(T+1)A^2} + \frac{iCDA[(T+1)-(s+1)]}{(T+1)A^2} + \frac{CD[s-T]}{(T+1)} + \frac{CD[(1+A)^{-s} - (1+A)^{-T}]}{A(T+1)} \quad (8)$$

Similarly,

$$TVC\ (e) - TVC\ (T)$$

$$= \frac{C_0[T-e]}{eT} + \frac{iCD[(1+A)^{-e} - (1+A)^{-T}]}{(T+1)A^2} + \frac{iCDA[(e+1)-(T+1`)]}{(T+1)A^2} + \frac{CD[T-e]}{(T+1)} + \frac{CD[(1+A)^{-T} - (1+A)^{-e}]}{A(T+1)} \quad (9)$$

Optimality Condition

An optimal solution is a feasible solution that results in the largest possible objective function value when maximizing (smallest when minimizing). The optimality conditions for the value of T to minimize TVC (T) are

$TVC\ (T^*) \le TVC$ (s) and $TVC(T^*) \le TVC$ (e)($T = T^* \ \ge 0$)
$\Rightarrow TVC\ (T^*) - TVC$ (s) ≤ 0 and TVC (e) $- TVC\ (T^*) \ge 0$
$\Rightarrow \Delta TVC$ (s) ≤ 0 and $\Delta TVC\ (T^*) \ge 0$

Thus

$$\Delta TVC\ (\mathrm{s}) \leq 0 \leq \Delta TVC\ (T^{*})$$

Therefore, for the optimal T we must have

$$\frac{C_0(s-T)}{Ts}+\frac{iCD[(1+A)^{-T}-(1+A)^{-s}]}{(T+1)A^2}+\frac{iCDA[(T+1)-(s+1)]}{(T+1)A^2}+\frac{CD[s-T]}{(T+1)}+\frac{CD[(1+A)^{-s}-(1+A)^{-T}]}{A(T+1)} \leq 0$$
$$\leq \frac{C_0[T-e]}{eT}+\frac{iCD[(1+A)^{-e}-(1+A)^{-T}]}{(T+1)A^2}+\frac{iCDA[(e+1)-(T+1`)]}{(T+1)A^2}+\frac{CD[T-e]}{(T+1)}$$
$$+\frac{CD[(1+A)^{-T}-(1+A)^{-e}]}{A(T+1)} \qquad (10)$$

Determination of the EOQ

The economic order quantity is given by total demand in a circle – ameliorated amount within the circle, i.e.

EOQ = DT - A

EOQ = $DT-\left\{DT-\frac{D}{A}[1-(1+A)^{-T}]\right\}$

$$= DT - DT + \frac{D}{A}[1-(1+A)^{-T}]$$

$$= \frac{D}{A}[1-(1+A)^{-T}] \qquad (11)$$

Using equations (7), (8), (9) and (11), the optimal values of T, EOQ and the total variable cost are calculated for the following examples.

NUMERICAL EXAMPLES

Table 1: Tabulation of the solution of ten different numerical examples for different values of the parameters as indicated.

S/NO	C	C_0	i	A	D	T^*/Days	TVC(T^*)	EOQ*	
1	N300	N1300	0.520	0.41	15000	30	31743.96	1018.73	
2	N300	N1000	0.500	0.40	20000	24	30761.11	1094.06	
3	N300	N1500	0.400	0.20	30000	16	69000.00	1194.05	
4	N330	N1400	0.130	0.10	21000	44	23322.70	2398.97	
5	N300	N2500	0.259	0.25	30000	100	18473.06	7116.48	
6	N400	N2000	0.150	0.04	25000	22	65448.84	1475.75	
7	N100	N50000	0.450	0.40	900000	62	592313.78	124991.02	
8	N300	N10000	0.560	0.40	65000	56	130602.04	8175.91	
9	N300	N1000	0.100	0.01	30000	18	40114.98	1471.74	
10	N500	N20000	0.500	0.47	18000	170	88508.49	6290.79	

Table 2: Comparing the results in Gwanda [26] and the results in this study we get the following:

S/NO	**Gwanda's T^*/DAYS**	**This study T^*/DAYS**	**Gwanda's TVC(T^*) in (N)**	**This study TVC(T^*) in (N)**	**Gwanda's EOQ* (Units)**	**This study EOQ* (units)**
1	27	30	35698.59	31743.97	1092.93	1018.73
2	21	24	34509.58	30761.11	1137.55	1094.06
3	15	16	73385.18	69000.00	1227.82	1194.05
4	43	44	24081.93	23322.70	2459.46	2398.97
5	93	100	19915.35	18473.06	7407.47	7116.48
6	22	22	66305.27	65448.84	1505.12	1475.75

7	56	62	664143.63	59231.78	133930.51	124991.02
8	50	56	146446.92	130602.04	8664.56	8175.91
9	18	18	40247.10	40114.98	1479.09	1471.74
10	150	170	100740.18	88508.49	6726.73	6290.79

The following are our findings:

In some cases, our cycle length is greater than that obtained by Gwanda [26], this will increase the amelioration since the items will stay longer in stock. Our TVC is less than that of Gwanda [1] because the longer the time it takes to ameliorate the less the total variable cost. The EOQ in this study is less than that of Gwanda. This reduces the holding cost. Since the smaller the EOQ, the less the holding cost.

Sensitivity Analysis

Due to uncertainties in decision making, a sensitivity analysis is carried out to ascertain the behaviour of the decision variables as a result of changes in the parameters, C_0, i, C, A, and D. This is done by taking one variable at a time while the remaining are kept at their original values. This has been done for the first example.

Table: 3

The Effect of Parameter Changes and the Corresponding Changes in T, TVC (T) and EOQ.

Parameter	**% Change in Parameter Value**	**Change in Results**		
		T^*(DAYS)	**TVC(T^*)**	**EOQ^***
C_0	50	37	38838.54	1252.32
	25	34	35473.92	1152.39
	10	32	33289.76	1085.63
	5	31	32524.99	1052.19
	0	**30**	**31743.97**	**1018.73**
	-5	29	30945.02	985.23
	-10	28	30126.28	951.70
	-25	26	27507.9	884.55
	-50	21	22477.21	716.12
I	50	16	58350.52	546.90
	25	20	46962.37	682.34
	10	25	38556.80	850.93
	5	27	35312.94	918.14
	0	**30**	**31743.97**	**1018.73**
	-5	34	27723.5	1152.40
	-10		23012.21	1418.23
	-25	No Solution		
	-50			
C	50	24	38918.41	817.28
	25	27	35508.59	918.14
	10	29	33303.23	985.23
	5	29	32533.18	985.23
	0	**30**	**31743.97**	**1018.73**
	-5	31	30937.01	1052.19
	-10	32	30109.07	1085.63
	-25	35	27472.43	1185.74
	-50	43	22404.60	1451.32
	50	24	38918.41	1225.92
	25	27	35508.59	1147.68
	10	29	33303.23	1083.75

D	5	29	32533.18	1034.50
	0	**30**	**31743.97**	**1018.73**
	-5	31	30937.01	999.58
	-10	32	30109.07	977.06
	-25	35	27472.4	889.30
	-50	43	22404.60	725.66
A	50	No solution		
	25	122	7947.60	3780.35
	10	38	24846.8	1264.31
	5	34	28470.37	1147.86
	0	**30**	**31743.97**	**1018.73**
	-5	27	34777.68	925.77
	-10	25	37621.66	865.15
	-25	21	45395.39	746.71
	-50	17	56924.31	632.76

Discussion of Results

From the results of the tables above, it can be deduced that

(i) With increase in C_0 the ordering cost, the EOQ, the TVC and Tall increase. This is expected because EOQ increases in order to avoid frequent ordering and vice versa. Also, since C_0 has direct cost effect on TVC, an increase in C_0 will also increase the TVC. As for T, it also increases since EOQ increases.

(ii) With an increase in the carrying charge, i, the TVC increases, EOQ decrease and T decreases. This is expected as the carrying cost has direct cost effect on the TVC. EOQ decreases so as not to incur much holding cost since the more the carrying charge, the more the holding cost. The cycle length T decrease since EOQ decreases.

(iii) With increase in item's cost, C, the order quantity EOQ decreases. This happens in order to avoid high holding cost. The TVC increases due to increase in C, while T decreases due to decrease in EOQ

(iv) With increase in the demand, D, there is a corresponding increase in the EOQ. This is obvious, because in order to cater for the demand more is expected to be ordered. The more the items are demanded for, the more the EOQ. The TVC also increases with increase in D, because of holding cost. T decreases since the demand increases.

(v) With increase in amelioration rate, the TVC decreases because amelioration reduces cost. The EOQ increases because of increase in amelioration so as to get more profit and T increases due to increase in EOQ.

Conclusion

In this paper, we develop a discrete time EOQ model for ameliorating items in which the demand rate is constant. The inventory starts with items bought from outside and put in stock. As the items ameliorate to maximum level, the stock reduces due to demand only. Items that exhibit this behaviour are fish, chicken, ducks, cows, sheep and so on. Our objective is to find the best replenishment cycle that will minimize the total variable cost. Numerical examples are given to illustrate the application of the model and sensitivity analysis carried out to see the effect of the parameter changes. As we observe in comparison with Gwanda's results our T is larger which allows for increase in amelioration and by implication, reducing the TVC. Our EOQ is also less than that of Gwanda which helps in minimizing the holding cost.

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