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THE EFFECT OF THERMAL AND ZERO-POINT ENERGY OF QUANTUM VACUUM ON SPECTRA OF FINITE-SIZED NUCLEUS OF HYDROGEN-LIKE ATOMS

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ABSTRACT

This study aims to accurately model the behavior of hydrogen-like and muonic atoms with finite-sized nuclei in quantum vacuum interactions, addressing the need for precise spectroscopy and accurate interpretation of experimental data across a wide temperature spectrum. The analysis uncovers that both the quantum number, n and nuclear charge, Z have a significant impact on the vacuum field and thermal effects. Muons, due to their greater mass and proximity to the nucleus, experience more pronounced thermal and vacuum field effects than electrons. The study's findings emphasize the need to consider thermal contributions in quantum vacuum-related calculations and suggest that accounting for nuclear size differences can enhance the accuracy of muonic atom experiments. The study draws intriguing parallels between black holes and hydrogen atoms, offering exciting prospects for further exploration. The theoretical framework developed in this study holds promise for various scientific disciplines, opening new avenues for experimental investigations and deepening our understanding of fundamental physics.

1. Introduction

Understanding the intricate interplay between the thermal and zero-point energy of the quantum vacuum and its impact on the spectra of hydrogen-like atoms with finite-sized nuclei is a topic of profound interest in modern theoretical physics [1]. The vacuum, once thought to be a void devoid of any physical significance, has emerged as a dynamic entity teeming with fluctuations and quantum fields that shape the behaviour of fundamental particles and their interactions [2].

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Relic photons, also known as blackbody radiation, follow the distribution law established by Planck in 1900 and 1901 [3,4]. Stochastic Electrodynamics, an approach using classical physics in the presence of a stochastic electromagnetic field, models zero-point fluctuations of the electromagnetic field as quantum noise, introducing a stochastic component to the system's dynamics. These successfully models phenomena like blackbody radiation, harmonic oscillators, and the Casimir effect, which are due to quantum vacuum fields [5-8]. However, Planck derived blackbody radiation based on the statistical analysis of oscillators within a blackbody and then Einstein later modified the Planck formula from Bohr's atomic model, which considers discrete energies of electrons and the energy of emitted photons determined by the Bohr formula [9]. On the other hand, the field of Quantum Electrodynamics (QED) suggests that the vacuum state contains an inherent energy of $\frac{1}{2}\hbar\omega$ and linear momentum of $\frac{1}{2}\hbar k$ for each field mode, where $\hbar$ represents the reduced Planck's constant, ω is the frequency of the field mode, and k is the wave vector [10]. This energy is known as zero-point energy. It's important to note that the vacuum state possesses not only linear momentum and energy but also fluctuating electromagnetic fields and radiation pressure. These effects can be understood by considering the vibration of electrons induced by a random field with fluctuating energy per mode [11-15], where the electric and magnetic fields are considered fluctuating sources of energy. Quantum mechanics, a fundamental theory explaining physical phenomena at atomic and subatomic scales [16,17], provides a powerful framework for describing the behaviour of various physical systems: elementary particles, nuclei, atoms, and radiation. One of its key aspects is the concept of quanta, where energy is absorbed and released in discrete quantities, and at the atomic and subatomic levels, matter exhibits wave-like and particle-like properties [18,19].

The presence of temperature in the vacuum state further expands our understanding of its properties, which depend on the existence of matter and boundary conditions. Temperature influences the energy of the vacuum fields through the concept of thermal fluctuations. At non-zero temperatures, the vacuum fields are subject to thermal excitation, causing the energy to deviate from the zero-point energy. It also affects vacuum energy states differently depending on the physical system and energy scales involved. Again, the contribution of virtual photons to the zero-point energy follows Fermi-Dirac statistics, indicating a non-zero temperature associated with the vacuum state. By introducing parameters in Fermi-Dirac statistics, it becomes possible to mathematically define the non-zero temperature of the quantum vacuum state [20]. Thus, Planck's law and Fermi-Dirac statistics provide a rigorous understanding of the interrelationship between thermal and zero-point energy fluctuations [21,22]. Corrections accounting for vacuum fluctuations have a quantum nature and arise from the non-zero mean square value of fields in the vacuum [23]. Thermal fluctuations can modify the energy levels of the vacuum state even at relatively low temperatures [24]. Excitation of quasiparticles, like phonons or magnons, due to thermal fluctuations can affect the energy spectrum of the vacuum state in condensed matter systems. Ultra-cold atomic gases require temperatures on the order of microkelvin or nanokelvin to induce quantum effects and modify vacuum energy states. High-energy particle physics may require temperatures of billions of Kelvins or higher to observe significant deviations from vacuum energy states. Experimental verification of the effects of electromagnetic fluctuating fields in the vacuum includes phenomena like the Lamb shift and the Casimir effect [25-28]. It also plays a crucial role in the spontaneous emission of radiation by nuclei, as without them, nuclei would remain indefinitely in their stable state [29].

The relationship between temperature and the energy of vacuum fields is further explored in the framework of QED and other field theories. Initially, vacuum fluctuations were found to cause infinite energy shifts for free electrons [30], but the development of QED by Bethe resolved this issue by considering the vacuum field fluctuations and explains the Lamb shift in atomic energy levels [31,32]. In quantum field theories, which serve as framework for particle interactions, the concept of empty space is replaced by a vacuum state representing the lowest energy density state of quantum fields, exhibiting zero-point fluctuations everywhere, leading to significant vacuum energy density. The energy of the vacuum fields at a given temperature is described using the formalism of statistical mechanics. This involves calculating the average energy of the fields over a range of possible configurations, taking into account the temperature-dependent probabilities of these configurations [4,33-37].

The specific mathematical expressions and calculations depend on the field theory and context. Ongoing research focuses on understanding the interplay between temperature, zero-point energy, and thermal fluctuations in the vacuum state. Equations describing the energy density of the vacuum state considering both zero-point energy and thermal fluctuations are the main objectives of this study. Therefore, this work, considered the intriguing realm of hydrogen-like atoms with finite-sized nuclei to investigate the influence of both thermal and zero-point energy of the quantum vacuum on their spectral properties. The aim was to elucidate how the interplay between these two distinct sources of vacuum energy affects the energy levels of the atomic systems. Also, considered is the change in lepton energy states as caused by the interaction of the lepton with the vacuum fields at different temperatures and perturb the lepton interaction by both finite temperature and vacuum fields. Based on this model, an effective nuclear-lepton interaction that takes into account the effect of the quantum vacuum was derived. It is worth noting that the precise mathematical expressions and calculations depend on the specific field theory and the context in which it is applied. To accomplish this, a theoretical framework that combines concepts from quantum electrodynamics, statistical mechanics, and quantum field theory were employed. Perturbation theory, as an approximation method, was applied to determine the changes in the energy states of the perturbed leptons. The determination of lepton interactions at finite temperature poses a fundamental problem in Quantum Field Theory [38]. However, Stochastic Electrodynamics has successfully modelled various phenomena, including blackbody radiation, harmonic oscillators, and the Casimir effect [7,8]. The determination of potentials at finite temperature also poses a fundamental problem in Quantum Field Theory, which is a framework for describing particle interactions and predicts phenomena like zero-point vacuum fluctuations [33,34].

Contemporary models often overlook the impact of thermal effects on atomic systems, particularly hydrogen-like and muonic atoms with finite-sized nuclei. This study aims to bridge this gap by accurately modeling the behavior of these atoms within the context of quantum vacuum interactions, addressing the need for precise spectroscopy and accurate interpretation of experimental data across a wide temperature spectrum. Thus, the understanding of the interplay between the thermal and zero-point energy of the quantum vacuum and its influence on hydrogen-like atoms with finite-sized nuclei is an ongoing area of research that draws upon multiple branches of physics and continues to deepen our understanding of the fundamental nature of the universe [39]. The work shed light on the intricate interplay and reveals its consequences for the observed spectral lines of hydrogen-like atoms. Additionally, the study has potential implications for precision spectroscopy and the interpretation of experimental data, as it provides a theoretical framework that incorporates the effects of the quantum vacuum. Therefore, this research will contribute to the advancement of fundamental atomic physics and paves the way for new avenues of experimental investigation.

2. Materials and Methods

2.1. The effective interaction

For a δr change in lepton quantum orbit due to vacuum fields' fluctuation, the effective interaction between orbiting lepton and atomic nucleus takes the form

$$U_{eff}(r,\delta r)=\frac{1}{V}\int U(\vec{r}+\delta\vec{r})d^3\varepsilon(1)$$

where $U(\vec{r}+\delta\vec{r})$ is the coulomb interaction which depends on the position $\vec{r}$ of lepton from the nucleus and the displacement $\delta\vec{r}$ of lepton from its quantum orbit due to vacuum fields. For the finite-size nuclear model the coulomb interaction is finite at origin and thus depends on both the size of nucleus R and the distance r. Therefore, using Taylor's series around the average position $\delta\vec{r}$, the effective interaction to first order approximation takes the form [40-42]:

$$U_{eff}(R,r,\delta r)=U(R,\vec{r})+\delta\widehat{H}_{eff}$$

where

$$\delta\widehat{H}_{eff}=\frac{1}{6}\delta r^2\nabla^2U(R)\delta_{ij} \qquad (2)$$

is the perturbation caused by vacuum field's fluctuation. Thus, taking the Laplacian of the potential for the extended charge distribution [43-49],

$$U(R,r)=\frac{-Zke^2}{R}\left[\frac{3}{2}-\frac{1}{2}\left(\frac{r}{R}\right)^2\right] \qquad (3)$$

the relation (2) becomes

$$\delta\widehat{H}_{eff}=\frac{Zke^2}{6R^3}\langle\delta r^2\rangle_t \qquad (4)$$

where $\langle\delta r^2\rangle_t$ is mean square position of an orbiting lepton.

2.2. Mean square fluctuation in thermal bath

Classically, an orbiting lepton of mass m_l executes harmonic oscillation induced by a single mode of the vacuum electromagnetic field $\mathcal{E}_0$ given by the equation

$$m_l\omega^2\delta r_0=e\mathcal{E}_0$$

or the amplitude,

$$\delta r_0=\frac{e}{m_l\omega^2}\mathcal{E}_0 \qquad (5)$$

taking the square of (5) gives the mean square amplitude as

$$\left\langle \delta r_0^2 \right\rangle_t = \frac{e^2}{2 m_l^2 \omega^4} \left\langle \mathcal{E}_0^2 \right\rangle_t \quad (6)$$

and the zeroth energy level a harmonic oscillator is quantized as

$$E_\omega = \frac{\hbar\omega}{2} \quad (7)$$

where $\hbar\omega$ is the quantum of energy [50-53]. However, the energy of fluctuating electric $\mathcal{E}$ and magnetic fields H is given by

$$E_\omega = \frac{1}{8\pi} \int \left(\mathcal{E}^2 + H^2 \right) d\omega = \frac{\mathcal{E}_0^2}{4\pi} \Omega \quad (8)$$

where for a set of plane waves, $\left\langle \mathcal{E}^2 \right\rangle_t = \left\langle H^2 \right\rangle_t$[54]. By comparing (7) and (8), the fluctuating fields take the value:

$$\mathcal{E}_0^2 = \frac{2\pi}{\Omega} \hbar\omega \quad (9)$$

Now using (9), the mean square oscillation (6) becomes

$$\left\langle \delta r_0^2 \right\rangle = \frac{\pi}{\Omega} \frac{\hbar e^2}{m_l^2 \omega^3} \quad (10)$$

The mean square fluctuation is the result of non-coherent action of all components of the field, thus,

$$\left\langle \delta \vec{r}^2 \right\rangle_t = \int \left\langle \delta r_0^2 \right\rangle_t \rho_T(\omega) d\omega \, (11)$$

where $\rho_T(\omega)$, is the spectral density of the thermal radiation and is given by [55]:

$$\rho_T(\omega) = \frac{\hbar \omega^3}{2\pi^2 c^3} \left(\frac{1}{e^{\hbar\omega\beta} - 1} \right) \quad (12)$$

Therefore,

$$\rho_T(\omega) d\omega = \frac{\hbar \omega^2}{\pi^2 c^3} \left(e^{\hbar\omega\beta} - 1 \right)^{-1} d\omega \quad (13)$$

where ω is the angular frequency, $\beta = k_B T$, T is temperature and k_B = 1.380662(44) × 10^{-23} J/K is the Boltzmann constant [24,56]. Now using (13) and (12), the full mean square fluctuation in thermal vacuum fields takes the form:

$$\left\langle \delta \vec{r}^2 \right\rangle_t = \left(\frac{e^2}{m^2} \right) \left(\frac{\hbar}{\pi c^3} \right) \int_{\omega_0}^{\infty} \left(e^{\hbar\omega\beta} - 1 \right)^{-1} \frac{d\omega}{\omega}$$

$$¿\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi c^3}\right)\left[\ln\left(\frac{\omega_2}{\omega_1}\right)-\hbar(\omega_2-\omega_1)\beta\right](14)$$

with

$$\omega_1=\xi|E_1|=\xi(Z\alpha)^2\frac{m_l c^2}{2}$$

and

$$\omega_2=m_l c^2$$

where $\alpha = ke^2/\hbar c$ is the fine structure constant and $+Ze$ is the nuclear charge [57-59]. Now, the perturbation (4) reads

$$\delta\hat{H}_{eff}=\frac{Zke^4\hbar}{6R^3\pi m_l^2 c^3}\left\{\ln\left[\frac{2}{\xi(Z\alpha)^2}\right]-\hbar\beta m_l c^2\left[1-\frac{\xi(Z\alpha)^2}{2}\right]\right\} \quad (15)$$

The value of the perturbation (15) is very small compared to Z/r lepton-nuclear interaction; therefore, an approximate solution can be sought using perturbation theory.

2.3. Approximate solution of the energy shifts due to thermal contribution

The addition of the small perturbation term given in (15) to Z/r nuclear-lepton interaction gives the Hamiltonian of the form,

$$\hat{H}_{pert.}=\hat{H}_0+\lambda\delta\hat{H}_{eff}$$

where

$$\hat{H}_0=\frac{-\hbar^2}{2m_l}\nabla^2-\frac{Zke^2}{r}$$

The Schrödinger equation can be written in terms of a new Hamiltonian as

$$\hat{H}_{pert.}\psi_n'=\hat{E}_n'\psi_n' \quad (16)$$

with solution

$$\hat{E}_n'(\lambda)=\sum_{n=0}^{\infty}\lambda^n\hat{E}_n^{(N)}=\hat{E}_n^{(0)}+\lambda\hat{E}_n^{(1)}+\lambda^2\hat{E}_n^{(2)}+\ldots(17)$$

Or

$$\hat{E}_n'(\lambda)=\langle\psi_n|\hat{H}_0|\psi_n\rangle+\lambda\langle\psi_n|\delta\hat{H}_{eff}|\psi_n\rangle+\lambda^2\sum_{m\neq n}\frac{|\langle\psi_n|\delta\hat{H}_{eff}|\psi_m\rangle|^2}{\hat{E}_n-\hat{E}_m}+O(\lambda^3)(18)$$

where $\hat{E}_n^{(0)}$ is the 0th order correction to the nth eigenvalue, and ψ_n is the 0th order correction to the nth eigen functions; $\hat{E}_n^{(1)}$ is the first order corrections; $\hat{E}_n^{(2)}$ is the second order corrections λ is taken to be dimensionless small number, $\lambda \ll 1$ and $O(\lambda^3)$ is the Landau symbol [60-63]. The second and higher order corrections to lepton energy states are always insignificant and mostly ignored even for muonic atoms. Thus, this study considered applying the first order correction to obtain the isotope shifts caused by the effects of fluctuating vacuum fields. The first-order perturbation theory is the most important equation in quantum mechanics for determining the small changes due to vacuum fields' effects and is given by:

$$\hat{E}_{nlm}^{(1)} = \int \psi_{nlm}^{¿} \delta \hat{H}_{eff} \psi_{nlm} d\tau \quad (19)$$

where the ψ_{nlm} is the unperturbed normalized wave function, and the intensity of the wavefunction corresponding to s states is given by [57-58;64]:

$$|\psi_{n00}|^2 = \begin{cases} \frac{Z^3}{\pi n^3 a_0^3}; & l=0, m=0 \\ ¿0 & l \geq 1 \end{cases} \quad (20)$$

where $a_0 = \hbar^2/kme^2$, is the Bohr radius. The shift in $n00$ energy states are determined using (19) as,

$$\Delta E_{n00}^{(1)} = \frac{4\pi Zke^4\hbar}{6R^3\pi m^2c^3}\left\{\ln\left[\frac{2}{\xi(Z\alpha)^2}\right] - \hbar\beta m_l c^2\left[1 - \frac{\xi(Z\alpha)^2}{2}\right]\right\}\int_0^R |\psi_{n00}|^2 r^2 dr$$

$$¿ \frac{4}{9\pi}\frac{|E_n|}{n}(Z\alpha)^2\left\{\ln\left[\frac{2}{\xi(Z\alpha)^2}\right] - \hbar\beta m_l c^2\left[1 - \frac{\xi(Z\alpha)^2}{2}\right]\right\} \quad (21)$$

Thus,

$$\Delta E_{n00}^{(1)} = \delta E_n + \delta E_T \quad (22)$$

where

$$\delta E_n = \frac{4}{9\pi}\frac{|E_n|}{n}(Z\alpha)^2 \ln\left[\frac{1}{9(Z\alpha)^2}\right] \quad (23)$$

and the thermal contribution,

$$\delta E_T = \frac{4\beta\hbar m_l c^2}{9\pi k_B T}\frac{|E_n|}{n}(Z\alpha)^2\left[1 - \frac{\xi(Z\alpha)^2}{2}\right] \quad (24)$$

It can readily be observed that at zero-point temperature ($T = 0$), the contribution, δE_n vanishes. The analyses of equation (23) have been done by many authors (see for example, [31, 42, 62, 65-68]). However, the results obtained from equation (22) as in most articles can only be true at zero-point temperature. To justify for the thermal contribution to vacuum fields' effects, equation

(24) is used to compute the thermal contribution to the energy level shifts of electron and muonic hydrogen-like atoms at different temperature, due to fluctuating vacuum fields and the results are presented on Table 1 to Table 6.

3. Results

Tables (1 to 6) present the computed thermal contribution to the vacuum fields' effect using equation (23). These tables display the results for the thermal contribution at various energy states of hydrogen-like electron and muon atoms. The calculations were performed considering three different temperatures: cosmological temperature (T = 2.7 K), room temperature (T = 300 K), and a temperature shortly after the Big Bang ($T = 10^{10}K$). The tables provide insights into the impact of thermal fluctuations on the vacuum field effect for these atomic systems at different temperature regimes.

Table 1: The Thermal Contribution, log $\delta\mathscr{E}_T$ for hydrogen-like atoms at a minute after the big bang ($T = 10^{10}K$)

Quantum Number, *n*	Thermal Contribution, log $\delta\mathscr{E}_T$ (eV)						
	$^{1}H_{1}$	$^{6}Li_{3}$	$^{21}Na_{11}$	$^{39}K_{19}$	$^{85}Rb_{37}$	$^{137}Sc_{55}$	$^{223}Fr_{87}$
1	-19.7290	-16.8662	-13.4806	-12.0564	-10.3198	-9.2868	-8.0918
2	-20.6321	-17.7693	-14.3837	-12.9595	-11.2228	-10.1899	-8.9949
3	-21.1603	-18.2976	-14.9120	-13.4878	-11.7511	-10.7181	-9.5232
4	-21.5351	-18.6724	-15.2868	-13.8626	-12.1259	-11.0930	-9.8980
5	-21.8259	-18.9631	-15.5775	-14.1533	-12.4167	-11.3837	-10.1888
6	-22.0634	-19.2007	-15.8151	-14.3909	-12.6542	-11.6212	-10.4263
7	-22.2643	-19.4015	-16.0159	-14.5917	-12.8550	-11.8221	-10.6271
8	-22.4382	-19.5755	-16.1899	-14.7657	-13.0290	-11.9961	-10.8011
9	-22.5917	-19.7290	-16.3433	-14.9192	-13.1825	-12.1495	-10.9546
10	-22.7290	-19.8662	-16.4806	-15.0564	-13.3198	-12.2868	-11.0918
11	-22.8531	-19.9904	-16.6048	-15.1806	-13.4439	-12.4110	-11.2160
12	-22.9665	-20.1038	-16.7181	-15.2940	-13.5573	-12.5243	-11.3294
13	-23.0708	-20.2081	-16.8224	-15.3983	-13.6616	-12.6286	-11.4337
14	-23.1673	-20.3046	-16.9190	-15.4948	-13.7581	-12.7252	-11.5302
15	-23.2572	-20.3945	-17.0089	-15.5847	-13.8480	-12.8151	-11.6201

Table 2: The Thermal Contribution, log $\delta\mathscr{E}_T$ for muonic hydrogen-like atoms at a minute after the big bang ($T = 10^{10}K$)

Quantum Number, *n*	Thermal Contribution, log $\delta\mathscr{E}_T$ (eV)						
	$^{1}H_{1}$	$^{6}Li_{3}$	$^{21}Na_{11}$	$^{39}K_{19}$	$^{85}Rb_{37}$	$^{137}Sc_{55}$	$^{223}Fr_{87}$
1	-15.0970	-12.2343	-8.8487	-7.4245	-5.6878	-4.6548	-3.4599
2	-16.0001	-13.1374	-9.7518	-8.3276	-6.5909	-5.5579	-4.3630
3	-16.5284	-13.6657	-10.2800	-8.8559	-7.1192	-6.0862	-4.8913
4	-16.9032	-14.0405	-10.6548	-9.2307	-7.4940	-6.4610	-5.2661
5	-17.1939	-14.3312	-10.9456	-9.5214	-7.7847	-6.7518	-5.5568
6	-17.4315	-14.5687	-11.1831	-9.7590	-8.0223	-6.9893	-5.7944
7	-17.6323	-14.7696	-11.3840	-9.9598	-8.2231	-7.1901	-5.9952
8	-17.8063	-14.9436	-11.5579	-10.1338	-8.3971	-7.3641	-6.1692
9	-17.9597	-15.0970	-11.7114	-10.2872	-8.5505	-7.5176	-6.3226
10	-18.0970	-15.2343	-11.8487	-10.4245	-8.6878	-7.6548	-6.4599
11	-18.2212	-15.3585	-11.9728	-10.5487	-8.8120	-7.7790	-6.5841
12	-18.3346	-15.4718	-12.0862	-10.6620	-8.9254	-7.8924	-6.6974
13	-18.4388	-15.5761	-12.1905	-10.7663	-9.0296	-7.9967	-6.8017
14	-18.5354	-15.6727	-12.2870	-10.8629	-9.1262	-8.0932	-6.8983
15	-18.6253	-15.7626	-12.3769	-10.9528	-9.2161	-8.1831	-6.9882

Table 3: The Thermal Contribution, log $\delta\mathscr{E}_T$ for hydrogen-like atoms at room temperature (T = 300 K)

Quantum Number, n	Thermal Contribution, log $\delta\mathscr{E}_T$ (eV)						
	$^{1}H_{1}$	$^{6}Li_{3}$	$^{21}Na_{11}$	$^{39}K_{19}$	$^{85}Rb_{37}$	$^{137}Sc_{55}$	$^{223}Fr_{87}$
1	-13.2061	-10.3434	-6.9577	-5.5336	-3.7969	-2.7639	-1.5690
2	-14.1092	-11.2464	-7.8608	-6.4367	-4.7000	-3.6670	-2.4721
3	-14.6374	-11.7747	-8.3891	-6.9649	-5.2282	-4.1953	-3.0003
4	-15.0123	-12.1495	-8.7639	-7.3397	-5.6031	-4.5701	-3.3751
5	-15.3030	-12.4403	-9.0546	-7.6305	-5.8938	-4.8608	-3.6659
6	-15.5405	-12.6778	-9.2922	-7.8680	-6.1313	-5.0984	-3.9034
7	-15.7414	-12.8786	-9.4930	-8.0689	-6.3322	-5.2992	-4.1043
8	-15.9154	-13.0526	-9.6670	-8.2428	-6.5061	-5.4732	-4.2782
9	-16.0688	-13.2061	-9.8205	-8.3963	-6.6596	-5.6266	-4.4317
10	-16.2061	-13.3434	-9.9577	-8.5336	-6.7969	-5.7639	-4.5690
11	-16.3303	-13.4675	-10.0819	-8.6577	-6.9210	-5.8881	-4.6931
12	-16.4436	-13.5809	-10.1953	-8.7711	-7.0344	-6.0014	-4.8065
13	-16.5479	-13.6852	-10.2996	-8.8754	-7.1387	-6.1057	-4.9108
14	-16.6445	-13.7817	-10.3961	-8.9719	-7.2353	-6.2023	-5.0074
15	-16.7344	-13.8716	-10.4860	-9.0618	-7.3251	-6.2922	-5.0972

Table 4: The Thermal Contribution, log $\delta\mathscr{E}_T$ for muonic hydrogen-like atoms at room temperature (T = 300 K)

Quantum Number, n	Thermal Contribution, log $\delta\mathscr{E}_T$ (eV)						
	$^{1}H_{1}$	$^{6}Li_{3}$	$^{21}Na_{11}$	$^{39}K_{19}$	$^{85}Rb_{37}$	$^{137}Sc_{55}$	$^{223}Fr_{87}$
1	-19.7290	-16.8662	-13.4806	-12.0564	-10.3198	-9.2868	-8.0918
2	-20.6321	-17.7693	-14.3837	-12.9595	-11.2228	-10.1899	-8.9949
3	-21.1603	-18.2976	-14.9120	-13.4878	-11.7511	-10.7181	-9.5232
4	-21.5351	-18.6724	-15.2868	-13.8626	-12.1259	-11.0930	-9.8980
5	-21.8259	-18.9631	-15.5775	-14.1533	-12.4167	-11.3837	-10.1888
6	-22.0634	-19.2007	-15.8151	-14.3909	-12.6542	-11.6212	-10.4263
7	-22.2643	-19.4015	-16.0159	-14.5917	-12.8550	-11.8221	-10.6271
8	-22.4382	-19.5755	-16.1899	-14.7657	-13.0290	-11.9961	-10.8011
9	-22.5917	-19.7290	-16.3433	-14.9192	-13.1825	-12.1495	-10.9546
10	-22.7290	-19.8662	-16.4806	-15.0564	-13.3198	-12.2868	-11.0918
11	-22.8531	-19.9904	-16.6048	-15.1806	-13.4439	-12.4110	-11.2160
12	-22.9665	-20.1038	-16.7181	-15.2940	-13.5573	-12.5243	-11.3294
13	-23.0708	-20.2081	-16.8224	-15.3983	-13.6616	-12.6286	-11.4337
14	-23.1673	-20.3046	-16.9190	-15.4948	-13.7581	-12.7252	-11.5302
15	-23.2572	-20.3945	-17.0089	-15.5847	-13.8480	-12.8151	-11.6201

Table 5: The Thermal Contribution, $\delta\mathscr{E}_T$ for hydrogen-like atoms at cosmological temperature (T = 2.7 K)

Quantum Number, n	Thermal Contribution, log $\delta\mathscr{E}_T$ (eV)						
	$^{1}H_{1}$	$^{6}Li_{3}$	$^{21}Na_{11}$	$^{39}K_{19}$	$^{85}Rb_{37}$	$^{137}Sc_{55}$	$^{223}Fr_{87}$
1	-11.1603	-8.2976	-4.912	-3.4878	-1.7511	-0.7181	0.4768
2	-12.0634	-9.2007	-5.8151	-4.3909	-2.6542	-1.6212	-0.4263
3	-12.5917	-9.729	-6.3433	-4.9192	-3.1825	-2.1495	-0.9546
4	-12.9665	-10.1038	-6.7181	-5.294	-3.5573	-2.5243	-1.3294
5	-13.2572	-10.3945	-7.0089	-5.5847	-3.848	-2.8151	-1.6201
6	-13.4948	-10.6321	-7.2464	-5.8223	-4.0856	-3.0526	-1.8577
7	-13.6956	-10.8329	-7.4473	-6.0231	-4.2864	-3.2534	-2.0585
8	-13.8696	-11.0069	-7.6212	-6.1971	-4.4604	-3.4274	-2.2325

9	-14.0231	-11.1603	-7.7747	-6.3505	-4.6138	-3.5809	-2.3859
10	-14.1603	-11.2976	-7.912	-6.4878	-4.7511	-3.7181	-2.5232
11	-14.2845	-11.4218	-8.0361	-6.612	-4.8753	-3.8423	-2.6474
12	-14.3979	-11.5351	-8.1495	-6.7253	-4.9887	-3.9557	-2.7608
13	-14.5022	-11.6394	-8.2538	-6.8296	-5.0929	-4.06	-2.865
14	-14.5987	-11.736	-8.3504	-6.9262	-5.1895	-4.1565	-2.9616
15	-14.6886	-11.8259	-8.4402	-7.0161	-5.2794	-4.2464	-3.0515

Table 6: The Thermal Contribution, $\delta\mathscr{E}_T$ for muonic hydrogen-like atoms at cosmological temperature ($T = 2.7\ K$)

Quantum Number, *n*	Thermal Contribution, log $\delta\mathscr{E}_T$ (eV)						
	$^{1}H_{1}$	$^{6}Li_{3}$	$^{21}Na_{11}$	$^{39}K_{19}$	$^{85}Rb_{37}$	$^{137}Sc_{55}$	$^{223}Fr_{87}$
1	-6.5284	-3.6657	-0.2800	1.1441	2.8808	3.9138	5.1087
2	-7.4315	-4.5687	-1.1831	0.2410	1.9777	3.0107	4.2056
3	-7.9597	-5.0970	-1.7114	-0.2872	1.4495	2.4824	3.6774
4	-8.3346	-5.4718	-2.0862	-0.6620	1.0746	2.1076	3.3026
5	-8.6253	-5.7626	-2.3769	-0.9528	0.7839	1.8169	3.0118
6	-8.8628	-6.0001	-2.6145	-1.1903	0.5464	1.5793	2.7743
7	-9.0637	-6.2010	-2.8153	-1.3912	0.3455	1.3785	2.5734
8	-9.2377	-6.3749	-2.9893	-1.5651	0.1716	1.2045	2.3995
9	-9.3911	-6.5284	-3.1428	-1.7186	0.0181	1.0511	2.2460
10	-9.5284	-6.6657	-3.2800	-1.8559	-0.1192	0.9138	2.1087
11	-9.6526	-6.7898	-3.4042	-1.9800	-0.2434	0.7896	1.9846
12	-9.7659	-6.9032	-3.5176	-2.0934	-0.3567	0.6762	1.8712
13	-9.8702	-7.0075	-3.6219	-2.1977	-0.4610	0.5720	1.7669
14	-9.9668	-7.1040	-3.7184	-2.2942	-0.5576	0.4754	1.6703
15	-10.0567	-7.1939	-3.8083	-2.3841	-0.6474	0.3855	1.5805

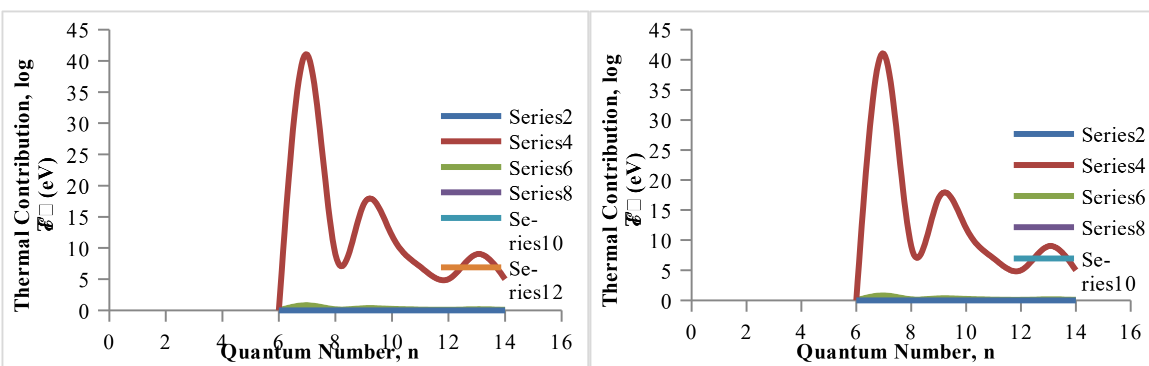

Figure 1: The Thermal Contribution, log $\delta\mathscr{E}_T$ for hydrogen-like and muonic hydrogen-like atoms at a minute after the big bang ($T = 10^{10}K$)

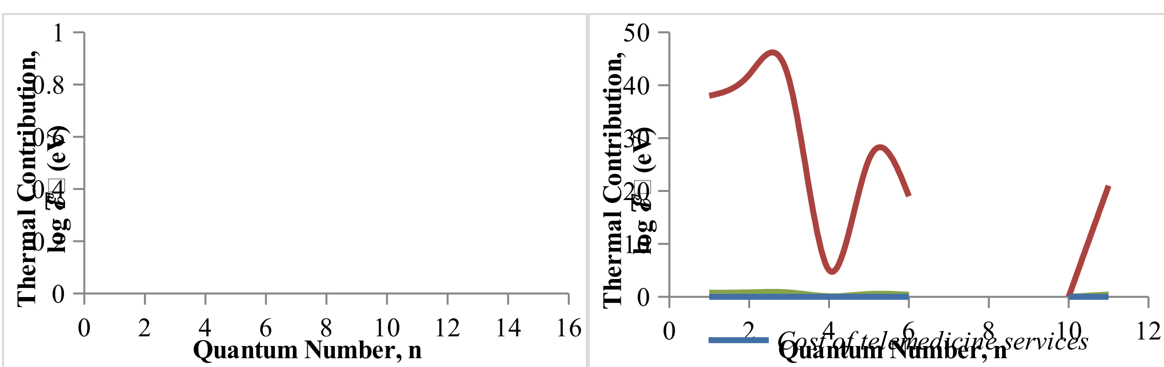

Figure 2: The Thermal Contribution, log $\delta\mathscr{E}_T$ for hydrogen-like and muonic hydrogen-like atoms at room temperature ($T = 300\ K$)

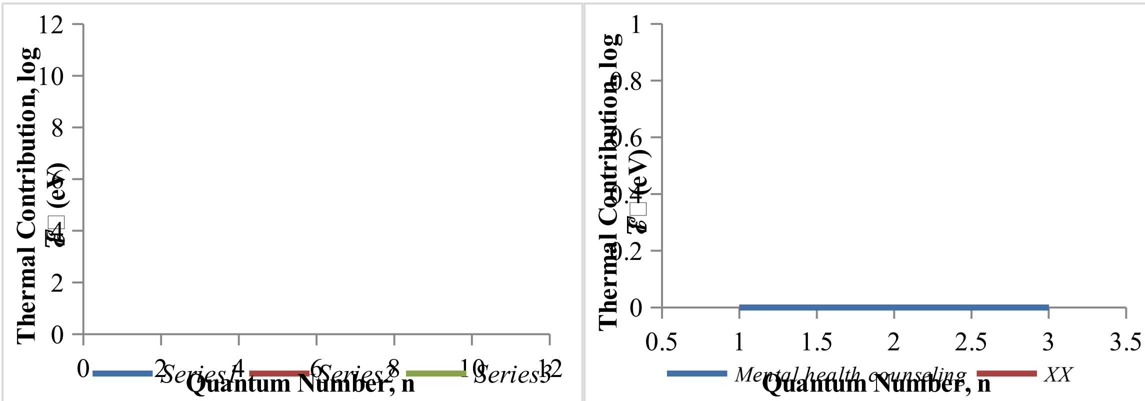

Figure 3: The Thermal Contribution, $\delta\mathscr{E}_T$ for hydrogen-like and muonic hydrogen-like atoms at cosmological temperature ($T = 2.7\ K$)

Discussion

From Table 1 to Table 6, it is evident that both the quantum number, n, and the nuclear charge, Z, play significant roles in determining the vacuum field effect and thermal effects. Due to its heavier mass and closer proximity to the nucleus, the muon experiences thermal and vacuum field effects more prominently than the electron. The influence of temperature changes on orbiting leptons under the influence of vacuum fields can be vividly seen in all the calculated values in the tables, highlighting the dependence of the fluctuating vacuum fields on temperature. The thermal contribution to the vacuum field effect increases significantly with nuclear (proton) charge. For instance, while the thermal contribution for different quantum states of the lightest nuclide (hydrogen atom) is very small ($\sim 10^{-24}$ to $10^{-9} eV$), there are substantial differences of about an order of 10^6 observed between different states of the lightest and heaviest nuclide (francium atom). To aid in visualizing the thermal contribution to the vacuum field effects for various nuclides, Figure 1 is plotted. The figure illustrates that hydrogen has the lowest thermal contribution, while the difference in thermal contribution between lithium and sodium nuclides is significant compared to other nuclides. The francium nuclide experiences the greatest thermal and vacuum fields' effect.

The exploration of temperature dependence in thermal and zero-point energy effects on atomic spectra across a wider temperature range presents an exciting opportunity to gain a comprehensive understanding of their interplay and discover new phenomena. Additionally, delving into the implications of vacuum fields and thermal contributions in cosmological contexts has the potential to deepen our comprehension of early universe dynamics, structure formation, and cosmological observables, bridging the gap between quantum field theory and cosmology. Consequently, this research has far-reaching implications, extending to theoretical frameworks, experimental techniques, and practical applications in various fields, including fundamental physics, cosmology, and quantum technologies.

Conclusion

This study explores the complex interplay between thermal and zero-point energy of the quantum vacuum and its effects on the spectra of hydrogen-like atoms with finite-sized nuclei and muonic atoms. By employing a theoretical framework that combines quantum electrodynamics, statistical mechanics, and quantum field theory, the modified energy levels of these atomic systems are calculated. The analysis reveals that the quantum number and nuclear charge play significant roles in influencing the vacuum field effects and thermal effects. The muon experiences greater thermal and vacuum field effects due to its heavier mass and proximity to the nucleus compared to the electron. Moreover, temperature plays a crucial role in affecting the fluctuating vacuum fields and the behaviour of the orbiting particles. The study emphasizes the necessity to consider the thermal contribution in calculations related to the quantum vacuum and highlights how differences in nuclear size can enhance the accuracy of muonic atom experiments. Furthermore, intriguing connections between black holes and hydrogen atoms are discovered, offering exciting prospects for further exploration of fundamental phenomena. The findings have far-reaching implications and provide valuable insights for precision spectroscopy, experimental atomic physics, and the broader field of quantum field theory. The theoretical framework proposed in this research opens up new avenues for experimental investigation in various scientific disciplines and enhances our understanding of fundamental physics. By shedding light on the intricate nature of the vacuum and its impact on atomic systems, this study contributes to advancing our knowledge in this field and paves the way for future breakthroughs in theoretical and experimental physics.

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