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PREDICTING NEUTRON STAR MAXIMUM MASS THROUGH RELATIVISTIC EQUATION OF STATE MODELLING

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ABSTRACT

The maximum mass of neutron stars (NSs) marks a fundamental limit set by general relativity and the equation of state (EOS) of ultra-dense matter. This study systematically explores NS maximum masses using relativistic models across a range of EOS types, including nucleonic, hyperonic, and quark matter. By solving the Tolman-Oppenheimer-Volkoff equations with piecewise-polytropic EOS parameterizations, we identify collapse thresholds and their sensitivity to high-density physics. Observational constraints from NICER (PSR J0740+6620: 2.08±0.07 $M_{\odot}$) and GW170817 tidal deformability are incorporated. Results show that the maximum mass (M_{max}) strongly depends on EOS stiffness above nuclear saturation, with M_{max} approaching or exceeding $3M_{\odot}$. Hybrid EOS with quark deconfinement predict distinct kinks in the mass-radius relation near M_{max}. These findings offer theoretical limits for distinguishing NSs from black holes in gravitational and electromagnetic signals, and enable stringent.

1. Introduction

Neutron star (NS) are giant ball of neutrons, this ball has a diameter of about $20\,Km$ and somewhere around 1.5 $M_{\odot}$. This exotic system is extremely dense, it is so dense that the neutrons are basically all touching one another, so it's sitting somewhere around nuclear density [1; 2]. To give an idea of how dense the nuclear matter is, a teaspoon full of NS material would weigh as much as mount Everest, and at the surface of the star the gravitational forces are very strong.

This exotic system has been used to directly detect gravitational waves (GW) emitted by objects like black holes (BH) and neutron stars (NSs), testing a number of GR predictions with previously unheard-of accuracy. GW are space-time ripples brought about by the motion of large objects, such as NSs. One would have a BH if the metric deviation in equation (22) were one, which is almost half. This indicates that NSs are on the verge of becoming BHs.

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Since the central density needed to support the star against gravity reaches infinity, the star will collapse if the metric deviation reaches 8/9. The delicate balance between the outward pressure supplied by the NS's interior structure and the inward pull of gravity is highlighted by this important threshold. Although the exact maximum mass of an NS is unknown, it is thought to be between 2 and $3M_{\odot}$. Because the EOS of NS material is still unknown, scientists are unable to pinpoint the precise maximum mass of an NS [3; 4]. In all of astrophysics, this is one of the most significant unknowns.

The EOS describes how matter behaves at the extreme densities found in NS interiors, where pressures can exceed 10^{34} pascals. At such densities, matter is no longer composed of ordinary atoms but rather of degenerate neutron matter, a state where neutrons are packed so tightly that they are on the verge of collapsing into a BH. NSs are made of this degenerate neutron matter, which represents matter at the limits of Quantum Mechanics. One cannot confine a neutron to too small a volume due to the Pauli exclusion principle, which states that no two neutrons (or other fermions) can occupy the same quantum state simultaneously. Since NS material is so dense, we have essentially reached the Quantum Mechanical limit where neutrons cannot be compressed any further without violating this principle [5].

The uncertainty in the EOS arises because the behaviour of matter at such extreme densities is governed by the strong nuclear force, which is not fully understood in these conditions. Theoretical models of the EOS must account for not only neutrons but also potentially exotic particles such as hyperons, pion or kaon condensates, and even deconfined quark matter. Each of these possibilities leads to different predictions for the maximum mass and radius of a NS. For example, the presence of hyperons or quark matter could soften the EOS, resulting in a lower maximum mass, while a stiffer EOS (e.g., due to repulsive nuclear interactions) could allow for more massive NSs.

2.0 Neutron Star

Observational data from NSs provide critical constraints on the EOS. For instance, the detection of massive NSs, such as PSR J0740+6620 with a mass of approximately $2.08\ M_{\odot}$, rules out softer EOS models that cannot support such high masses. Similarly, GW signals from NS mergers, like GW170817, offer insights into the tidal deformability of NSs, which is directly related to the EOS. These observations are helping scientists narrow down the range of possible EOS models and refine our understanding of NS structure.

The study of NSs and their EOS is not only important for understanding these exotic objects but also for probing the fundamental physics of dense matter. NSs serve as natural laboratories for testing theories of Quantum Chromodynamics (QCD) under conditions that cannot be replicated on Earth. Additionally, the maximum mass of a NS has profound implications for astrophysics, as it determines the boundary between NSs and BHs. If a NS exceeds its maximum mass, it will collapse into a BH, releasing a tremendous amount of energy in the form of GWs and possibly gamma-ray bursts.

The metric deviation and the EOS are central to understanding the stability and structure of NSs. While NSs are on the brink of becoming BHs, the exact maximum mass remains uncertain due to the unknown nature of matter at extreme densities. By combining theoretical models with observational data, scientists are gradually unravelling the mysteries of NSs and the fundamental physics that govern them [3; 4; 5].

Neutron stars are produced when a massive star runs out of fuel and falls. When the star's core collapses, every proton and electron inside is crushed together to create neutrons [6]. If a star's core has a mass of one to three solar masses, these newly created neutrons can stop the star from collapsing and leave behind a neutron star (stars with bigger masses will still fall into black holes with a stellar mass) [7].

The sun-sized object with the mass of a metropolis is the densest known object as a result of this collapse. These star remnants have a diameter of around twenty kilometres, or twelve miles. One trillion kilogrammes, or one billion tonnes, is the approximate weight of a single sugar cube produced of material from NSs on Earth [8].

A NS with a strong magnetic field (field lines in blue shown in Figure 1) and a light beam travelling along the magnetic axis are depicted in this pulsar diagram. The magnetic field spins along with the NS, moving that beam throughout space. We perceive the beam as a regular pulse of light if it sweeps across Earth. Since NSs originated as stars, they can be found sporadically in the same locations as stars throughout the galaxy. They can also be discovered alone or in binary systems with a companion, just like stars. Due to their insufficient radiation output, a large number of NSs are probably undetected [9]. However, they are easily observable in specific situations.

2.1 Structure of Neutron Star

A neutron star's cross-section can be loosely classified into four areas (refer to Figure 9): There are only a few centimetres of atmosphere, a Fermi liquid of relativistic degenerate electrons and a lattice of atomic nuclei make up the outer crust. In essence, this is stuff from white dwarfs. The inner crust, which stretches from the neutron drip density to a transition density ($Q_{tr} \simeq 1.7 \times 10^{14}\ gcm^{-3}$) is surrounded by the outer crust.

Concerning the matter's compressibility, three distinct groups can be applied when organising the corresponding EOSs: soft, mild, and stiff. According to research by [10], with varying EOSs one can generate a variety of stellar models, especially concerning maximum masses, which range from $M_{max} \sim 1.4\ M_{\odot}$ for the softest EOSs to $M_{max} \sim 2.5\ M_{\odot}$ for the stiffest. The EOSs can also be separated based on the matter's composition; only nucleon matter can be responsible for extremely stiff EOSs. [11].

Neutron star models can be computed in the framework of GR after obtaining the EOS. The central density ρ_C is used to parameterize a family of NS models, where the gravitational mass $M = M(\rho_C)$ and the circumferential radius $R = R(\rho_C)$ are derived and the proper length of the NS equator is represented by $2\pi R$ [12]. The EOS is a critical input for constructing NS models, as it describes how pressure varies with density in the star's interior. It encapsulates the microphysics of matter at extreme densities, including nuclear interactions, phase transitions, and the possible presence of exotic particles like hyperons or deconfined quarks. Once the EOS is determined, it can be integrated into the TOV equations, which are the general relativistic equations governing the structure of spherically symmetric, static stars. These equations balance the inward pull of gravity against the outward pressure gradient, ensuring hydrostatic equilibrium.

The central density ρ_C serves as a key parameter in these models and by varying ρ_C, one can generate a sequence of NS models, each corresponding to a different mass and radius. This sequence is often represented as a mass-radius (M-R) curve, which is a fundamental tool for comparing theoretical predictions with observational data. For example, observations of NS masses from binary systems and radii from X-ray timing or GW events provide critical constraints on the EOS.

The circumferential radius R is particularly significant because it defines the star's size as measured by an observer at infinity. It is related to the star's gravitational mass M through the curvature of space-time, as described by GR. The proper length of the NS equator, $2\pi R$ is a measure of the star's surface geometry in its own reference frame. This quantity is important for understanding phenomena such as surface emission, rotational effects, and the star's interaction with its environment.

Recent advances in astrophysical observations and theoretical modelling have significantly improved our understanding of NS structure. For instance, the detection of GWs from NS mergers (e.g., GW170817) has provided unprecedented constraints on the EOS by limiting the maximum mass and radius of NSs. Similarly, X-ray observations from missions like Neutron Star Interior Composition Explorer (NICER) have allowed for precise measurements of NS, further refining our models.

The computation of NS models within the framework of GR, parameterized by the central density ρ_C, provides a powerful tool for exploring the properties of these extreme objects. By combining theoretical EOS models with observational data, scientists can probe the nature of dense matter, test the limits of GR, and uncover the secrets of NS interiors [12].

2.2 Binary Neutron Star

By monitoring GW radiation, BNSs can be used as new, highly accurate observatories to track their final years of inspiral. GR plays a major role in determining the eventual inspiral, which results in the well-predicted "chirping" kind of evolution. As a result, little disturbances from their backdrop do not affect the chirping inspiral [13].

This not only enables precision cosmology in conjunction with optical equivalents but also makes it possible to detect binary GWs as small as 10^{-21} fractional oscillations of the metric [14]. BNS as illustrated in figure 10, two NSs going around a common central mass; these exotic systems are used to test the various predictions of the theory of GR to unprecedented accuracy. Most of the stars one sees in the sky are binary systems, most of them are very white binaries with orbital periods of years or centuries, and some of them are close binaries, close enough to interact with one another [15].

This abundance of diverse binary systems improves our knowledge of GW emissions and star evolution. In particular, processes like mass transfer and tidal interactions can occur in close binaries, resulting in a dynamic development that may eventually cause the NSs to fuse or cause common envelope events. Not only do these mergers result in measurable GWs, but they are also essential for nucleosynthesis, especially for the synthesis of heavy elements like platinum and gold. Astronomers can test the predictions of GR with amazing accuracy and learn more about the ultimate destiny of binary systems in the universe by examining these interactions [16].

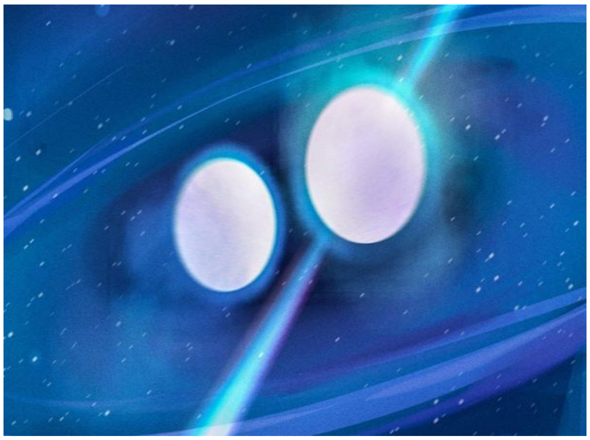

Figure 1 Binary neutron stars going around a common central mass. Credit: NICER

3.0 Relativistic Framework

The metric, which represents the most significant general relativity concept relevant to stellar applications, was studied. A generalized Pythagorean theorem that incorporates the time coordinate uses the metric as a geometric tool to relate distances in space-time. Since the underlying physics is more important than the particular coordinate system, tensors, which are multi-indexed objects, are the invariant language used to represent all equations. The Einstein summation convention assumes an inferred sum across repeated indices, which simplifies the notation. Keeping this in mind, and starting with the general relativity Einstein-Hilbert action, which is provided by:

$$S = \int \frac{1}{2k} R\sqrt{-g}d^4x \tag{1}$$

where d^4x is the space-time volume element, k is the gravitational constant, g is the determinant of the metric tensor, S is the action, and R is the Ricci scalar, and note that the minus sign under the square root arises because we are in a Lorentzian space-time.

The action about the metric tensor $g_{\alpha\beta}$ was altered in order to derive the field equations which resulted in

$$\delta S = \int \frac{1}{2k} \delta(R\sqrt{-g})d^4x \tag{2}$$

By expanding the variation $\delta\left(R\sqrt{-g}\right)$, using product rule and applying the variation to each term gives:

$$\delta S = \int \frac{1}{2k}(\delta R\sqrt{-g} + R\delta\sqrt{-g})d^4x \tag{3}$$

The variation of the determinant can be simplified using the relation:

$$\delta\left(\sqrt{-g}\right) = \frac{1}{2\sqrt{-g}} g^{\alpha\beta}\delta g_{\alpha\beta} \tag{4}$$

Where $g^{\alpha\beta}$ is the inverse metric tensor. The contracted Bianchi identity states that:

$$\nabla_\alpha G^{\alpha\beta} = 0 \tag{5}$$

Where $G^{\alpha\beta}$ is the Einstein Tensor. By using this identity, we obtain:

$$\delta R = -\frac{1}{2} g^{\alpha\beta}\delta g_{\alpha\beta} \tag{6}$$

Substituting the variations into the expression from (43) and simplifying, one obtains:

$$\delta S = -\frac{1}{2k}\int \sqrt{-g}\ g^{\alpha\beta}\delta g_{\alpha\beta}d^4x + \frac{1}{2k}\int R\sqrt{-g}\ g^{\alpha\beta}\delta g_{\alpha\beta}d^4x \tag{7}$$

When setting the variation δS to zero and simplifying, one is able to obtain the following field equations:

$$\frac{1}{2k}\int\left(\sqrt{-g}\ G^{\alpha\beta} - R\sqrt{-g}\ g^{\alpha\beta}\right)\delta g_{\alpha\beta}d^4x = 0 \tag{8}$$

Since the variation $\delta g_{\alpha\beta}$ is arbitrary, one can equate the integrand to zero:

$$G^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta} = 0 \tag{9}$$

The total energy momentum tensor of gravity and of matter are combined to obtain the gravitational field equations in the presence of matter and radiation kT^α_β so that the general field equation can now be written in the form that is most familiar in the literature:

$$G^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta} = kT^\alpha_\beta \tag{10}$$

This equation represents the Einstein field equations. it connects the space-time curvature defined by the Einstein tensor $G^{\alpha\beta}$ to the way in which matter and energy are distributed as stated by the Ricci scalar R and the metric tensor $g^{\alpha\beta}$. Einstein calculates the constant $k = 8\pi G$ (when the speed of light (c) is set to 1 in units).

$$G^\alpha_\beta = \frac{8\pi G}{C^4}T^\alpha_\beta \tag{11}$$

The stress energy tensor is on the right side, and the metric space time geometry is on the left. Taking the stress energy tensor to be a perfect fluid, hence

$$T_0^0 = \rho_o c^2(1 + c),\ T_i^i = -P, \text{and } T_{\beta\neq\alpha}^{\alpha} = 0 \qquad (12)$$

$$\text{Continuity of } T_\beta^\alpha \rightarrow \nabla_\alpha T_\beta^\alpha = 0$$

and

$$\nabla_\alpha - \text{covariant derivative } = \partial_\alpha T_\beta^\alpha + \Gamma_{\alpha\sigma}^{\alpha} T_\beta^\sigma - \Gamma_{\alpha\beta}^{\sigma} T_\sigma^\alpha$$

Solving for the G_0^0 component:

$$\frac{1}{r^2}.[1 - \frac{d}{dr}\left(re^{-\lambda}\right)] = \frac{8\pi G}{c^2}\rho_o(1+\epsilon) \qquad (13)$$

$$\rightarrow \frac{d}{dr}\left(re^{-\lambda}\right) = 1 - \frac{8\pi G}{c^2}r^2\rho_o(1+\epsilon) \ \rightarrow \int_0^r dr$$

$$re^{-\lambda} = r - \frac{2G}{c^2}\int_0^r 4\pi r^2\rho_o(1+\epsilon)dr \ \Longrightarrow M(r) \qquad (14)$$

$$e^\lambda = \left(1 - \frac{2GM(r)}{rc^2}\right)^{-1} \qquad (15)$$

and solving for G_1^1 component:

$$\frac{1}{r^2}\ .\left[1 - re^{-\lambda}\left(1 + r\frac{d\nu}{dr}\right)\right] = -\frac{8\pi G}{c^4}P \qquad (16)$$

$$\frac{d\nu}{dr} = \frac{1}{r}\left(\frac{2GM(r)}{rc^2} + \frac{8\pi G}{c^4}Pr^2\right)\left(1 - \frac{2GM(r)}{rc^2}\right)^{-1} \qquad (17)$$

Using the equation of continuity of stress energy tensor,

$$\nabla_\alpha T_\beta^\alpha = 0 \ \rightarrow \text{static} + \text{symmetry only } \partial r \neq 0$$

$$T_{\beta\neq\alpha}^{\alpha} = 0$$

$$\Longrightarrow \ \partial_\alpha T_1^\alpha + \Gamma_{\alpha\beta}^{\alpha} T_{\alpha 1}^{\beta} - \Gamma_{\alpha 1}^{\beta} T_\beta^\alpha = 0 \qquad (18)$$

Recall that,

$$\Gamma_{\alpha\beta}^{\alpha} = \frac{1}{2}g^{\sigma\rho}\left(\partial_\alpha g_{\rho\beta} + \partial_\beta g_{\alpha\rho} - \partial_\rho g_{\alpha\beta}\right) \qquad (19)$$

$$\Longrightarrow \ \partial_r T_1^1 + \Gamma_{\alpha 1}^{\alpha} T_1^1 - \Gamma_{\alpha 1}^{\alpha} T_\alpha^\alpha = 0$$

$$\Longrightarrow \ \Gamma_{01}^{0} = \frac{1}{2}\frac{d\nu}{dr},\ \Gamma_{21}^{2} = \Gamma_{31}^{3} = \frac{1}{r} \ \text{ and,}$$

$$T_0^0 = \rho_0 c^2(1+\epsilon),\ T_i^i = -P \quad (i = 1,2,3)$$

$$\Longrightarrow \ \frac{d(-P)}{dr} - \frac{1}{2}\frac{d\nu}{dr}\ \left[\rho_o c^2(1+\epsilon) + P\right] = 0 \qquad (20)$$

$$\Longrightarrow \frac{d\nu}{dr} = \frac{1}{r}\left(\frac{2Gm_{(r)}}{rc^2} + \frac{8\pi G}{c^4}Pr^2\right)\ \left(1 - \frac{2Gm_{(r)}}{rc^2}\right)^{-1} \qquad (21)$$

By simplifying equation (21), we arrived at the TOV equation, as shown in equation (22)

$$\Longrightarrow \ \frac{dp}{dr} = -\frac{Gm_{(r)}}{r^2}.\left(1 + \epsilon + \frac{P}{\rho_o c^2}\right)\left(1 + \frac{4\pi r^3 P}{m_{(r)}c^2}\right)\left(1 - \frac{2Gm_{(r)}}{rc^2}\right)^{-1} \qquad (22)$$

Equation (22) above is the TOV equation, and a non-relativistic star with $P \ll \rho c^2,\ Pr^3 \ll mc^2$, and $r \gg 2m$ creates a weak space time curvature. So, the relativistic equation gives us the Newtonian equation of stellar equilibrium; then the parenthesis reduces to one leaving the terms:

$$\Longrightarrow \ \frac{dp}{dr} = -\ \frac{Gm_{(r)}}{r^2} \qquad (23)$$

	$\frac{dm}{dr} = -4\pi r^2$	(24)

4.0 Neutron Star Maximum Mass Using TOV

From equation (62) where p = pressure, ρ_o = rest mass density, $m_{(r)}$ = enclosed mass, and $\left(1 - \frac{2Gm_{(r)}}{rc^2}\right)^{-1}$ is the metric deviation; For a NS one assume zero temperature limit and a constant density, now NS actually have a high temperature, so the zero-temperature limit does not actually mean the temperature is zero, it means that it is low compared to the chemical potential. Constant density can be interpreted in two ways, either one takes equation (25) constant or equation (26) constant.

	$\rightarrow\ \rho_o, (1+\epsilon) = \text{constant}$	(25)
	$\rightarrow\ \rho_o = \text{constant and } \epsilon = 0$	(26)

Defining:

	$x = \frac{P}{\rho c^2},\quad \beta_{(r)} \equiv \frac{2Gm_{(r)}}{rc^2}$	(27)

Here β is metric deviation, plug this (27) into equation (21) gives

	$\rightarrow\ \frac{d(\rho c^2 x)}{dr} = -\frac{\rho}{2r}.(1+x)(\beta c^2 + 8\pi G r^2 \rho x)(1-\beta)^{-1}$	(28)

Since the density is constant, one have mass is just the volume times the density ρ. We can plug this into β to find that β is proportional r^2.

	$m_{(r)} = \frac{4\pi r^3}{3}\rho$	(29)

and

	$\beta_{(r)} = \frac{8\pi G \rho r^2}{3c^2}$	(30)

Note, $8\pi G r^2 \rho = 3\beta c^2$

Then,

	$\rightarrow\ \beta_{(r)} = \frac{8\pi G\rho r^2}{3c^2} \rightarrow \frac{d\beta}{dr} = \frac{2\beta}{r}$	(31)

Using the chain rule,

	$\rightarrow\ \frac{d}{dr} = \frac{d\beta}{dr}, \frac{d}{d\beta} = \frac{2\beta}{r}.\frac{d}{d\beta},$	(32)

Then plug (32) into equation (22)

	$\Rightarrow\ \rho c^2.\frac{2\beta}{r}\frac{dx}{d\beta} = -\frac{\rho}{2r}.(1+x)(\beta c^2 + 3\beta c^2 x)(1-\beta)^{-1}$	(33)

Some term cancelling out and rearranging,

	$\Rightarrow\ \frac{dx}{(1+x)(1+3x)} = -\frac{1}{4}.\frac{d\beta}{1-\beta}$	(34)

Then integrate equation (34) gives

	$\rightarrow\ ln\left(\frac{1+3x}{1+x}\right) = \frac{1}{2}.\ln(1-\beta) + C$	(35)

Find the constant, assume the $\rho_s = 0$, means that $x(r=R) = 0$.

	$\Rightarrow\ ln\left(\frac{1+3x}{1+x}\right) = \frac{1}{2}.\ln(1-\beta) + C$	(36)

$$\rho_s = 0\ \rightarrow x(r=R) = 0,$$

Then one defines another quantity, $\bar{\beta} \equiv \frac{2Gm}{Rc^2} \rightarrow x(\bar{\beta}) = 0$

Then plug in $c = -\frac{1}{2}\ln(1-\bar{\beta})$ in equation (34) and re write

$$\Rightarrow \ ln\left(\frac{1+3x}{1+x}\right) = ln\sqrt{\frac{1-\beta}{1-\bar{\beta}}} \tag{37}$$

Solving for $x(\beta) \rightarrow P(\beta) = \rho c^2 . x(\beta)$

$$\Longrightarrow \quad P(\beta) = \rho c^2 . \frac{\sqrt{1-\beta}-\sqrt{1-\bar{\beta}}}{3\sqrt{1-\bar{\beta}}-\sqrt{1-\beta}} \tag{38}$$

$$\beta = \frac{2Gm_{(r)}}{rc^2} = \frac{8\pi G\rho r^2}{3c^2}, \ \ \bar{\beta} = \frac{2Gm}{Rc^2}$$

and central pressure:

$$\rho_o \equiv p_{(o)} = \rho c^2 . \frac{1-\sqrt{1-\bar{\beta}}}{3\sqrt{1-\bar{\beta}}-1} \tag{39}$$

Note the denominator if it → 0; $p_{(o)} \rightarrow \alpha \ and \ if \ \bar{\beta} \rightarrow \frac{8}{9}$, the star will collapse to black hole.

The maximum mass of NS can be, when $\frac{2Gm}{Rc^2} = \frac{8}{9}$, rewriting the radius gives;

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3} \tag{40}$$

assume the nuclear density:

$$\rho \approx 2 . \frac{2M_n}{4\pi r_n^3} \tag{41}$$

here r_n is nuclear radius $\approx 10^{-15}\, m$. Neutron has two spin state, spin up/down, hence the 2. substituting the density (41) into the radius (40)

$$\Longrightarrow \ \frac{2Gm}{Rc^2} = \frac{2G}{r_n c^2} \ . (m^2 \cdot M_n)^{\frac{1}{3}} = \frac{8}{9} \tag{42}$$

Now solve for the mass thus as;

$$M_{max} \approx \frac{1}{\sqrt{2M_n}} . \left(\frac{r_n C^2 . \frac{8}{9}}{2G}\right)^{\frac{3}{2}} \gg 3\, M_{\odot} \tag{43}$$

Results and Discussion

Some detailed models put the maximum mass of NS around $2 - 2.6\, M_{\odot}$ and the largest NS throughout the literatures reviewed in this research is around $1.9 - 2.95\, M_{\odot}$ and for a while it was thought one could not have NS over $2\, M_{\odot}$ and there was quite a debate on to whether the measurements of this particular NS were accurate; but today it is obvious that astronomers are convinced that it is over $2\, M_{\odot}$ and the calculated results confirmed that M_{max} of NS theoritically $\gg 3\, M_{\odot}$.

Considering causality, the speed of sound in the material has to be less than c. if one assumes a relativistic gas with the model we have chosen, that will bring down the maximum mass to just over $3\, M_{\odot}$. There is other thing that might cause instability, a self-gravitating object will go unstable when the sound crossing time is approximately equal to the force fall time.

One enters the core above the transition density when all atomic nuclei have broken down into their component parts-protons and neutrons. The core may also contain hyperons, more massive baryon resonances, and possibly a gas of free up, down, and strange quarks due to the high Fermi pressure [16]. Lastly, there might also be $\pi -$ and $K -$ meson condensates there. This transition marks a profound shift in the nature of matter, as the extreme densities and pressures in the core of a NS overcome the nuclear forces that bind protons and neutrons into atomic nuclei.

At these densities, which can exceed several times the nuclear saturation density approximately $2.8 \times 10^{14}\ gcm^{-3}$, the distinction between individual nucleons blurs, and the core becomes a soup of exotic particles. Hyperons, which are baryons containing one or more strange quarks, may appear as the Fermi energy of the neutrons increases, providing a new degree of freedom for the system. Additionally, the high Fermi pressure can lead to the deconfinement of quarks, resulting in a phase transition from hadronic matter to a quark-gluon plasma. This state of matter, where quarks and gluons are no longer confined within protons and neutrons, is reminiscent of the conditions in the early universe, just moments after the Big Bang.

The presence of $\pi-$ and $K-$ meson condensates further complicate the picture, these mesons, which are normally short-lived particles, can form a Bose-Einstein condensate in the dense environment of a NS core. Such condensates can significantly alter the EOS of the matter, affecting the star's overall structure and stability. For instance, the softening of the EOS due to meson condensates or quark matter can lead to a smaller maximum mass for NSs, which has implications for the observed population of these objects.

Understanding the composition and behaviour of matter at these extreme densities is one of the greatest challenges in modern astrophysics and nuclear physics. Observations of NSs, particularly through GW signals from mergers (such as those detected by LIGO and Virgo) and precise measurements of their masses and radii (e.g., from NICER missions), provide critical constraints on the EOS. These observations help scientists test theoretical models and explore the possible existence of exotic phases of matter, such as colour superconductivity in quark matter or the formation of strange stars [17].

The core of a NS is a natural laboratory for studying matter under conditions that cannot be replicated on Earth. By probing the properties of this ultra-dense matter, researchers can gain insights into the fundamental forces of nature, the behaviour of quantum chromodynamics (QCD) at high densities, and the evolution of compact objects in the universe.

Conclusion

In conclusion, the maximum mass of a neutron star is a critical probe into the nature of ultra-dense matter and the restrictions imposed by strong gravity. By applying relativistic equation of state (EOS) models, we may confine the feasible mass range while accounting for fundamental nuclear interactions and general relativistic effects. Although there is still opportunity for improvement due to uncertainties in the EOS at supranuclear densities, current theoretical predictions, which are backed by astronomical data like gravity waves and X-ray pulse patterns, indicate that neutron stars most likely have a maximum mass between $\sim$2.2 and 2.6 $M_{\odot}$, but this research has calculated M_{max} of NS theoritically to be $\gg 3\ M_{\odot}$. Future developments in high-energy experiments and multi-messenger astronomy will be crucial in reducing these limitations, which will ultimately improve our comprehension of matter in harsh environments and the viability of alternative gravity theories.

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