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A BAYESIAN STATISTICAL FRAMEWORK FOR RELIABILITY, AND MAINTENANCE COST MODELING OF NATURAL GAS COMPRESSOR

T. E. ABE.; P.E. AMIOLEMHEN; AND E. IKPOZA

Department of Production Engineering, University of Benin, Benin City, Nigeria.

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ABSTRACT

Predicting failure occurrences is vital for ensuring operational efficiency and minimizing downtime in production systems. This study characterizes the reliability parameters of a High-Pressure Compressor (HPC-2) using a Weibull Bayesian framework and estimates its annual maintenance cost. Operational data including shutdown and start times were obtained from a major crude oil and gas company in the Niger Delta, from which relevant secondary data were extracted. The developed Bayesian model under the Weibull distribution estimated an expected failure rate of 0.008749 failures per hour (standard error, SE = 3.74×10^{-5}) with a 95% two-tail Bayesian prediction interval of [0.00866, 0.00893]. At 36 hours of operation, reliability was 72.98% (SE = 9.91×10^{-4}; 95% prediction interval, PI [0.725, 0.732]). The non-informative prior produced a mean time to failure (MTTF) of 105.89 hours (95% PI [68.42, 161.20]), while the Gamma prior estimated 150.95 hours. The annual maintenance cost was estimated at 41,954.65 USD.

1. INTRODUCTION

As the world transitions toward cleaner energy, natural gas has emerged as a crucial bridging fuel, offering significantly lower carbon emissions than coal and oil. With growing demand for supporting infrastructure, the reliability, maintainability, and economic viability of natural gas equipment are now more vital than ever. Equipment failures not only pose serious safety risks but also threaten environmental integrity and can result in substantial financial consequences [1]. Traditional reliability assessments often rely on static models that assume constant failure and repair rates. However, real-world systems are dynamic, influenced by aging, operational variability, and environmental conditions.

*Corresponding author: P.E. AMIOLEMHEN

E-mail address: **patrick.amiolemhen@uniben.edu**

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This complexity has led to the adoption of Bayesian statistical frameworks, which allow for probabilistic modeling and continuous updating of system knowledge as new data becomes available [2].

Bayesian networks (BNs) and hierarchical Bayesian models have proven effective in modeling complex systems like natural gas pipelines and purification plants. For example, Guo et al. [2] developed a discrete-time Bayesian network to analyze dynamic systems with common cause failures, demonstrating its relevance to safety-critical systems like nuclear power and gas systems. Similarly, Gong et al. [3] integrated interpretive structural modeling with Bayesian networks to identify key risk factors in natural gas purification plants, emphasizing the method's ability to capture interdependencies among variables.

Bayesian statistical analysis techniques are useful for a wide range of applications [4]. Singh et al. [5] presented an algorithm for reliability prediction that permits the system engineer to analyze system reliability before it is built while considering the estimates of component reliability and their expected use. This method enables the identification of crucial components and the influence of their replacement on the system when they are replaced. The method was incorporated on a unified modelling language, UML.

Although Bayes theorem that was introduced in the 1770s is still somewhat complex, it is continually attracting attention and is being applied in different fields of researches. "Bayesian statistics" is a mathematical method that applies probabilities to statistical problems [6]. This methodology makes use of well known, statistically accurate, and logically sensible techniques to combine different types of data. Paul and Bani [7] stated that Bayesian statistics has become increasingly popular in engineering, and one reason for its increased application is that it allows researchers to input expert opinion as a key input in the analysis (through the prior distribution). Most "reliability assessment algorithms", employs these estimates as "prior probabilities" [4].

In this study, the systems considered for analysis are repairable. For a repairable system, the focus is not on the time to first failure. Instead, the primary interest is on the probability of system failure as a function of system age. Exact reliability analyses for complex, repairable systems are often difficult because of the complicated failure process that may result from the replacement or repair policy [4]. A common procedure in practice is to approximate the complicated stochastic process by a simpler stochastic process, which although not exact, still yield useful practical results.
Therefore, this study is the application of a Bayesian statistical framework for reliability, maintainability, and maintenance cost modeling of natural gas compressors.

2. METHODOLOGY

This study employs a structural approach in solving the research problem. The equipment operational information is acquired from a major crude oil and gas company in the Niger delta area of Nigeria, from which the failure and repair time data are extracted. The data collected is a reflection of the operational performance of the equipment at the customer's use end. Consequently, the research design proceeded in the following order:

1) First, equipment failure and repair time data, along with cost information related to preventive and corrective maintenance actions, are collected.
2) The data are presented in the required format for analysis, including the TTF, TTR, spare parts and labor cost per maintenance interval.
3) Verification of the independence of the data.

4) A quantitative research design is adopted to analyze system behavior – implemented using Bayesian modeling framework.
5) Annual maintenance cost – preventive, corrective and overall cost estimation

The Bayesian and cost estimation analysis was performed using a combination of Excel formulas and functions: This process facilitated the analysis efficiently and reduced the tedious nature of manual computation, and it is time-saving. The required formula has presented in Equation (1.1) through (1.40) was imputed in Excel and used for all the Bayesian and cost estimation analysis performed in this study

Bayesian Estimation in Weibull Distribution

In the case were scale parameter, λ is random it is easily only shown that, if the failure times, T has a Weibull distribution, $w \sim (\lambda, \beta)$, then (Martz and Waller, 2020): $\boldsymbol{T^{\beta}}$ is exponentially distributed, $\boldsymbol{\varepsilon(\lambda)}$.

Considering a life test of n items in which s items have failed at ordered times $t_1, \ldots, t_s$ and $n - s$ items have operated for times $t^*_{s+1,\ldots,}t^*_n$ without failing; thus $T_{s+1} > t^*_{s+1}, \ldots, T_n > t^*_n$. The times $t^*_{s+1,\ldots,}t^*_n$ are the withdrawer times of the non-failed items. Then this statistic [8]:

$$\boldsymbol{\omega = \sum_{i=1}^{s} T_i^{\beta} + \sum_{i=s+1}^{n} T_i^{*\beta}} \tag{1.1}$$

is for estimating λ (or θ) if s is fixed. This seen by the examination of the likelihood corresponding to the above sampling scheme which is expressed as [8]:

$$\boldsymbol{L(\lambda|\underline{Z}) \propto \lambda^s \beta^s \left(\prod_{i=1}^{s} t_i\right)^{\beta-1} exp\left[-\lambda\left(\sum_{i=1}^{s} t_i^{\beta} + \sum_{i=s+1}^{n} t_i^{*\beta}\right)\right]} \tag{1.2}$$

If there are withdrawals prior to test termination, we find that equation (1.1) becomes

$$\boldsymbol{\omega = \sum_{i=1}^{s} T_i^{\beta} + (n-s)\, T_s^{\beta}} \tag{1.3}$$

The posterior distribution of the failure rate, λ is expressed as [8]:

$$\boldsymbol{g(\lambda|\omega; s, \alpha_0, \beta_0) = \frac{\omega^{s+1}\lambda^s \exp\,(-\lambda\omega)}{\Gamma(s+1, \beta_0\omega) - \Gamma(s+1, \alpha_0\omega)}, \qquad \alpha_0 < \beta_0} \tag{1.4}$$

The posterior mean of equation ( ) is computed to be

$$\mathbf{E}\boldsymbol{(\lambda|\omega; s, \alpha_0, \beta_0) = \frac{\Gamma(s+2, \beta_0\omega) - \Gamma(s+2, \alpha_0\omega)}{\omega[\Gamma(s+1, \beta_0\omega - \Gamma(s+1, \alpha_0\omega)]}} \tag{1.5}$$

and the posterior variance can be calculated using the relationship

$$\boldsymbol{Var(\lambda|\omega; s, \alpha_0, \beta_0) = E(\lambda^2|\omega; s, \alpha_0, \beta_0) - E^2(\lambda|\omega;\ \alpha_0, \beta_0)} \tag{1.6}$$

A symmetric 100(1-γ)% TBPI estimate for λ is expressed as:

$$\mathbf{Pr}\boldsymbol{((\lambda \le \lambda_*|\omega;\ s, \alpha_0, \beta_0) = \frac{\Gamma(s+1,\ \lambda_*\omega) - \Gamma(s+1,\ \alpha_0 w)}{\Gamma(s+1,\ \beta_0\omega) - \Gamma(s+1,\ \alpha_0 w)} = \frac{\gamma}{2}} \tag{1.7}$$

and

$$\mathbf{Pr}\boldsymbol{((\lambda \ge \lambda^*|\omega;\ s, \alpha_0, \beta_0) = \frac{\Gamma(s+1,\ \beta_0\omega) - \Gamma(s+1,\ \lambda^* w)}{\Gamma(s+1,\ \beta_0\omega) - \Gamma(s+1,\ \alpha_0 w)} = \frac{\gamma}{2}} \tag{1.8}$$

where,

α_0 and β_0 = uniform prior distribution parameters.

β = specified Weibull shape parameter obtained from a preliminary analysis of data.

s = number of failures.

γ = specified significant level (5%).

λ = failure rate

Weighted Square – Error Loss Function: Minimizing the posterior expected loss yields the Bayesian point estimator for λ given by [8]:

$$\boldsymbol{\hat{\lambda} = \frac{E[\lambda h(\lambda)|\omega]}{E[h(\lambda)|\omega]}} \tag{1.9}$$

If $h(\lambda) = \lambda^{-2}$, then there is a special case for $\hat{\lambda}$ given by [8]:

$$\boldsymbol{\hat{\lambda} = \frac{s+\alpha_0-2}{\beta_0+\omega}} \tag{1.10}$$

for a Gamma prior distribution on λ, [$G_1(\alpha_0, \beta_0)$].

Reliability Estimation: The Bayesian estimation of the Weibull reliability function is expressed as [8]:

$$r(t_0;\ \lambda, \beta) = \exp\left(-\lambda t_0^{\beta}\right), \tag{1.11}$$

in the case of Weibull distribution with parameters λ and β, [$W_1(\lambda, \beta)$] or

$$r(t_0;\ \theta, \beta) = \exp\left(-\frac{t_0^{\beta}}{\theta}\right), \tag{1.12}$$

in the case of Weibull distribution with parameter θ and β, [W1(θ, β)].

Letting $g_\lambda(.)$ *and* $g_\theta(.)$ represent either the prior or posterior distribution of λ and θ respectively, the corresponding prior or posterior distribution of R, denoted by $g_r(.)$ may be obtained as

$$g_r(r) = g_\lambda\left(-\frac{\ln r}{t_0^{\beta}}\right)\left(\frac{1}{t_0^{\beta}}\right) \tag{1.13}$$

or

$$g_r(r) = g_\theta\left(-\frac{t_0^{\beta}}{\ln r}\right)\left(\frac{t_0^{\beta}}{r\, ln^2 r}\right) \tag{1.14}$$

The induced prior and posterior distribution on R for a uniform prior distribution on λ is expressed as [8]:

Prior distribution is

$$g_r(r;\ \alpha_0, \beta_0) = \frac{1}{(\beta_0 - \alpha_0) r (t_0^{\beta})}, \qquad e^{\beta_0 t_0^{\beta}} < r < e^{\alpha_0 t_0^{\beta}} \tag{1.15}$$

and

Posterior distribution is

$$g_r(r|w\!: \alpha_0, \beta_0) = \frac{w^{s+1}\left[-\ln r / t_0^{\beta}\right] e^{w \ln r / t_0^{\beta}}}{[\Gamma(s+1, \beta_0 w) - \Gamma(s+1, \alpha_0 w)]\, r\, t_0^{\beta}}, \quad e^{\beta_0 t_0^{\beta}} < r < e^{\alpha_0 t_0^{\beta}} \tag{1.16}$$

The Bayesian point estimate of $r(t_0)$ is expressed as [8]:

The posterior mean of R given t_0 is

$$E(R|w; s, t_0, \alpha_0, \beta_0) = \left(\frac{w}{w+t_0}\right)^{s+1} \times \frac{\Gamma[s+1, \beta_0(w+t_0)] - \Gamma[s+1, \alpha_0(w+t_0)]}{\Gamma(s+1, \beta_0 w) - \Gamma(s+1, \alpha_0 w)} \tag{1.17}$$

The second moment of R given t_0 is

$$E(R^2|w; s, t_0, \alpha_0, \beta_0) = \left(\frac{w}{w+t2_0}\right)^{s+1} \times \frac{\Gamma[s+1, \beta_0(w+2t_0)] - \Gamma[s+1, \alpha_0(w+2t_0)]}{\Gamma(s+1, \beta_0 w) - \Gamma(s+1, \alpha_0 w)} \tag{1.18}$$

The posterior variance (risk) of R given t_0 is

$$Var(R|w; s, t_0, \alpha_0, \beta_0) = E(R^2|w; s, t_0, \alpha_0, \beta_0) - E^2(R|w; s, t_0, \alpha_0, \beta_0) \tag{1.19}$$

The 95% LBPI (lower Bayesian prediction interval) and the 95% UBPI (upper Bayesian interval) is expressed as follows (Martz and Walker, 2020):

$$R(t_0, \lambda^*) = \exp\left(-\lambda^* t_0\right) \tag{1.20}$$

and,

$$R(t_0, \lambda_*) = \exp\left(-\lambda_* t_0\right) \tag{1.21}$$

MTTF Estimation: A Bayesian estimation of the Weibull [W_1 ~(λ, β)] MTTF is expressed as (Martz and Walker, 2020):

$$\text{MTTF} = \text{E(T; λ, β)} = \lambda^{-\frac{1}{\beta}}\, \Gamma\left(1 + \frac{1}{\beta}\right) \tag{1.22}$$

Uniform Prior Distribution on λ: If the failure rate, λ has uniform prior distribution [U~(α_0, β_0)] then the posterior distribution of λ is given by [8]:

$$g(\lambda|w;\ \alpha_0, \beta_0) = \frac{w^{s+1}\lambda^{s} \exp(-\lambda w)}{\Gamma(s+1, \beta_0 w) - \Gamma(s+1, \alpha_0 w)}, \quad \alpha_0 < \lambda < \beta_0 \tag{1.23}$$

The posterior expectation of MTTF under the squared – error loss function is expressed as (Martz and Walker, 2020):

$$\mathbf{E\{MTTF \mid w;\ \alpha_0, \beta_0\}} = \frac{w^{1/\beta}\Gamma(1+1/\beta)\left[\Gamma\left(s+1-1/\beta, \beta_0\right)-\Gamma(s+1-1/\beta,\ \alpha_0 w\right]}{\Gamma(s+1,\ \beta_0 w)-\Gamma(s+1, \alpha_0 w)} \quad (1.24)$$

The posterior risk is proportional to the posterior variance of MTTF and is expressed as (Martz and Walker, 2020):

Var[MTTF | w; α_0, β_0] =

$$\frac{w^{2/\beta}\Gamma^2(1+1/\beta)\Gamma(s+1-2/\beta,\ \beta_0 w)-\Gamma(s+1-2/\beta,\ \alpha_0 w)}{\Gamma(s+1, \beta_0 w)-\Gamma(s+1, \alpha_0 w)} - E^2[MTTF \mid w;\ \alpha_0,\ \beta_0] \quad (1.25)$$

A symmetric 100(1-γ) % two tail Bayesian prediction interval (TBPI) is expressed as [8]:

The upper limit of the 100(1-γ) % TBPI estimate for MTTF is

$$MTTF^* = \Gamma(1 + 1/\beta)(\lambda_*)^{-1/\beta} \quad (1.26)$$

Similarly, the lower limit of the desired TBPI estimate is

$$MTTF_* = \Gamma(1 + 1/\beta)(\lambda^*)^{-1/\beta} \quad (1.27)$$

where,

γ = significant level (normally set at 5%).

λ^* = upper limit of failure rate.

λ_* = lower limit of failure rate.

Non-informative Prior Distribution on λ: If the probability distribution of λ, g(λ) is non-informative, then the posterior distribution of λ given ω is given by [8]:

$$g(\lambda) = \frac{\omega^s}{\Gamma(s)}\lambda^{s-1}\exp(-\lambda\omega) \quad (1.28)$$

And this is a gamma distribution with parameter s and w $[G_1(s, \omega)]$. The posterior expected MTTF is:

$$E[MTTF|\omega] = \frac{\omega^{1/\beta}\Gamma(1+1/\beta)\Gamma(s-1/\beta)}{\Gamma(s)} \quad (1.29)$$

The posterior variance is expressed as:

$$Var[MTTF|\omega] = \frac{\omega^{2/\beta}(1+1/\beta)\Gamma(s-2/\beta)}{\Gamma(s)} - E^2[MTTF|\omega] \quad (1.30)$$

Also, the symmetric 100(1-γ) % TBPI estimate for MTTF is:

LBPI for MTTF: $$MTTF_* = \Gamma\left(1 + \frac{1}{\beta}\right)\left[\frac{\chi^2_{\gamma/2}(2s)}{2\omega}\right]^{-1/\beta} \quad (1.31)$$

and

UBPI for MTTF: $$MTTF^* = \Gamma\left(1 + \frac{1}{\beta}\right)\left[\frac{\chi^2_{1-\gamma/2}(2s)}{2\omega}\right]^{-1/\beta} \quad (1.32)$$

Gamma Prior Distribution on λ: If has a gamma distribution [G(α_0, β_0)] prior distribution, then the posterior distribution of λ given ω is $G[\alpha_0 + s, \beta_0/(\beta_0\omega + 1)]$ [8]:

Therefore, the posterior MTTF is:

$$E[MTTF|\omega;\ \alpha_0, \beta_0] = \frac{\Gamma(\alpha_0+s-1/\beta)\Gamma(1+1/\beta)}{\Gamma(\alpha_0+s)[\beta_0/(\beta_0\omega+1)]^{1/\beta}} \quad (1.33)$$

And the posterior variance of MTTF is

$$Var[MTTF|\omega;\ \alpha_0, \beta_0] = \frac{\Gamma(\alpha_0+s-2/\beta)\Gamma(1+1/\beta)}{\Gamma(\alpha_0+s)[\beta_0/(\beta_0\omega+1)]^{2/\beta}} - E^2[MTTF|\omega;\ \alpha_0, \beta_0] \quad (1.34)$$

The Two tail Bayesian prediction interval (TBPI) for MTTF is:

LBPI for MTTF: $$MTTF_* = \Gamma(1 + 1/\beta)\left[\frac{\beta_0\chi^2_{\gamma/2}(2s+2\alpha_0)}{2\beta_0\omega+2}\right]^{-1/\beta} \quad (1.35)$$

and

UBPI for MTTF: $\boldsymbol{MTTF^{*}} = \boldsymbol{\Gamma}(\boldsymbol{1} + \boldsymbol{1}/\boldsymbol{\beta}) \left[\frac{\beta_0 \chi^2_{1-\gamma/2}(2s+2\alpha_0)}{2\beta_0\omega+2}\right]^{-1/\beta}$ (1.36)

Standard Error (S.E): The standard error of a computed sample metric is expressed as follow [9]:

$$\boldsymbol{S.E} = \sqrt{\frac{\boldsymbol{Sample\ Variance}}{\boldsymbol{Sample\ Size}}} \quad (1.37)$$

Maintenance Cost Estimation Models

The cost of preventive maintenance is expressed as [10]:

Preventive maintenance cost, $\boldsymbol{PMC} = \frac{(\boldsymbol{ST_{pm}}+\boldsymbol{TT_{pm}})\times \boldsymbol{SOH}\times \boldsymbol{LC}}{\boldsymbol{SI_{pm}}}$ (1.38)

where,

ST_{pm} = scheduled time for preventive maintenance (PM)

TT_{pm} = expected travel time for PM

SOH = equipment usage hours per time period

SI_{pm} = scheduled time interval for PM

LC = cost of labour

The corrective maintenance cost (CMC) model is expressed as [10]:

$$\text{Model 1: } CMC_1 = \frac{\boldsymbol{SOH}\times\boldsymbol{LC}\times\boldsymbol{MTTR}}{\boldsymbol{MTTF}} \quad (1.39)$$

where,

MTTF = mean time to failure

MTTR = mean time to repair

$$\text{Model 2: } CMC_1 = \frac{(\boldsymbol{TT_{cm}}+\boldsymbol{MTTR})\times\boldsymbol{SOH}\times\boldsymbol{LC}}{\boldsymbol{MTTF}} \quad (1.39)$$

where,

TT_{cm} = expected travel time for CM

and,

Total Annual Cost per year = $\boldsymbol{PMC} + \mathbf{max}\,[\boldsymbol{CMC_1}; \boldsymbol{CMC_2}]$ (1.40)

RESULTS AND DISCUSSION

In the Bayesian analysis of the HPC-2 TTF data using the Weibull distribution: scale parameter (η) is a random variable and shape parameter (β) is fixed (refer to Appendix 1, sample size: 42), the procedure is initiated by a preliminary fit of the data to the Weibull model. This fitting informs the use of a shape parameter, β = 0.71, which characterizes the observed failure times. The manufacturer's failure rate specifications for this equipment with a range of 3.0E-3 to 9.0E-3 failures per hour as obtained were translated into a uniform prior distribution on failure rate (η) by taking the minimum and maximum failure rate as the bound of the uniform prior distribution: η ~ uniform (0.003, 0.009) . The computed posterior distribution of the failure rate (λ) is presented in Table 1.1 (an extraction of Microsoft spreadsheet).

Table 1.1: HPC-2 Posterior Distribution Failure Rate.

Parameter: s = n = 42, w = 954.26, α_0 = 0.003, β_0 = 0.009, β = 0.71, 0.003<η<0.009		
Prior Distribution λ	**g(η/w; α_0, β_0)**	**Pdf**
0.00300	1.078848E-14	2.401729E-18
0.00350	4.340209E-12	9.662160E-16
0.00400	7.344420E-10	1.635013E-13
0.00450	6.414289E-08	1.427947E-11
0.00500	3.324667E-06	7.401364E-10

0.00550	1.129864E-04	2.515299E-08
0.00600	2.709759E-03	6.032457E-07
0.00650	4.849804E-02	1.079662E-05
0.00700	6.764971E-01	1.506016E-04
0.00750	7.612134E+00	1.694611E-03
0.00800	7.103917E+01	1.581472E-02
0.00850	5.624687E+02	1.252166E-01
0.00900	3.850117E+03	8.571120E-01
0.07800	**4491.965351**	**1.000000**

As seen in Table 1.1, the failure rate posterior probability density function, PDF is an increasing function of the prior distribution. Presented in Figure 1.1 is the graph of HPC-2 failure rate prior and posterior distribution.

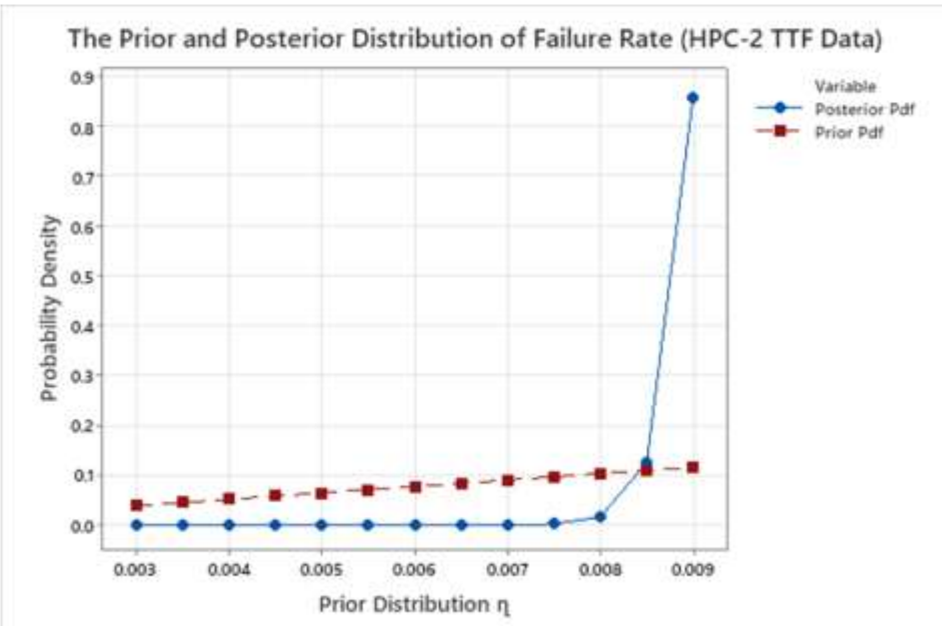

Figure 1.1: HPC-2 failure rate prior and posterior distribution

As shown in Figure 1.1, the failure rate posterior pdf of HPC-2 is an increasing function of the prior distribution. Present in Table 1.2 is an estimate of mean of failure rate, variance and 95% TBPI of HPC-2 (an extraction of Microsoft spreadsheet).

Table 1.2: Estimate of mean of failure rate (η), variance and 95% TBPI of HPC-2

Weibull Prior Distribution on λ Estimation	
Parameter: $s = n = 42$, $w = 954.26$, $\alpha_0 = 0.003$, $\beta_0 = 0.009$, $0.003<\lambda<0.009$	
	Bayesian Estimation
Posterior Mean , $E(\lambda/w; s, \alpha_0, \beta_0)$	0.008749128
Posterior Variance , $Var(\lambda/w; s, \alpha_0, \beta_0)$	5.8830355E-08
Standard Error (S.E) of η	0.00003743
95% LBPI, $Pr(\lambda \leq \lambda_*/w; s, \alpha_0, \beta_0)$	0.008657802

95% UBPI, $Pr(\lambda \geq \lambda^*/w; s, \alpha_0, \beta_0)$	0.008925668

LBPI: Lower Bayesian Probability Interval and UBPI: Upper Bayesian Probability Interval

As shown in Table 1.2, HPC-2 has an estimated posterior mean of failure rate of 0.008749128 with a standard error of 3.743E-5. Also, the failure rate has a 95% lower and upper Bayesian probability interval of 0.008657802 and 0.008925668 failures per hour respectively. Presented in Table 1.3 is the prior and posterior distribution of weibull reliability (an extraction of Microsoft spreadsheet).

Table 1.3: HPC2 Posterior Reliability Distribution

Parameter: $s = n = 42$, $w = 954.26$, $\alpha_0 = 0.003$, $\beta_0 = 0.009$, $\beta = 0.7100$, $t_0 = 36$, $0.891715<r<0.962518$				
r	**Prior, $g_r(r; \alpha_0, \beta_0)$**	**Prior Pdf**	**Posterior, $g_r(r/w; \alpha_0, \beta_0)$**	**Posterior Pdf**
0.8917150	14.6773585	0.0493712	3.3906560E+02	4.2489424E-159
0.8952552	14.6193191	0.0491759	1.0369460E+02	1.2994311E-159
0.8987953	14.5617369	0.0489822	3.0199164E+01	3.7843563E-160
0.9023355	14.5046066	0.0487901	8.3467656E+00	1.0459606E-160
0.9058756	14.4479228	0.0485994	2.1811206E+00	2.7332339E-161
0.9094158	14.3916803	0.0484102	5.3659070E-01	6.7241943E-162
0.9129559	14.3358740	0.0482225	1.2369505E-01	1.5500633E-162
0.9164961	14.2804988	0.0480362	2.6576287E-02	3.3303618E-163
0.9200362	14.2255497	0.0478514	5.2899924E-03	6.6290632E-164
0.9235764	14.1710219	0.0476680	9.6886842E-04	1.2141208E-164
0.9271165	14.1169106	0.0474860	1.6200580E-04	2.0301478E-165
0.9306567	14.0632109	0.0473053	2.4510477E-05	3.0714883E-166
0.9341968	14.0099182	0.0471261	3.3205786E-06	4.1611261E-167
0.9377370	13.9570278	0.0469482	3.9797151E-07	4.9871116E-168
0.9412771	13.9045354	0.0467716	4.1598339E-08	5.2128245E-169
0.9448173	13.8524363	0.0465963	3.7285428E-09	4.6723593E-170
0.9483574	13.8007261	0.0464224	2.8082216E-10	3.5190747E-171
0.9518976	13.7494006	0.0462497	1.7340562E-11	2.1730027E-172
0.9554377	13.6984554	0.0460784	8.5170589E-13	1.0673006E-173
0.9589779	13.6478864	0.0459083	3.2040279E-14	4.0150726E-175
0.9625180	13.5976893	0.0457394	8.7986450E-16	1.1025871E-176
19.4694465	0.6722338	0.0022612	7.9780050E+160	9.9975000E-01
	297.2859994	**1.0000000**	**7.98E+160**	**0.9997500**

As seen in Table 1.3, the prior and posterior distribution of reliability in the interval of $0.891715 < r < 0.962518$ is both a decreasing and increasing function. Presented in Figure 1.2 is the graph of prior and posterior of Weibull reliability.

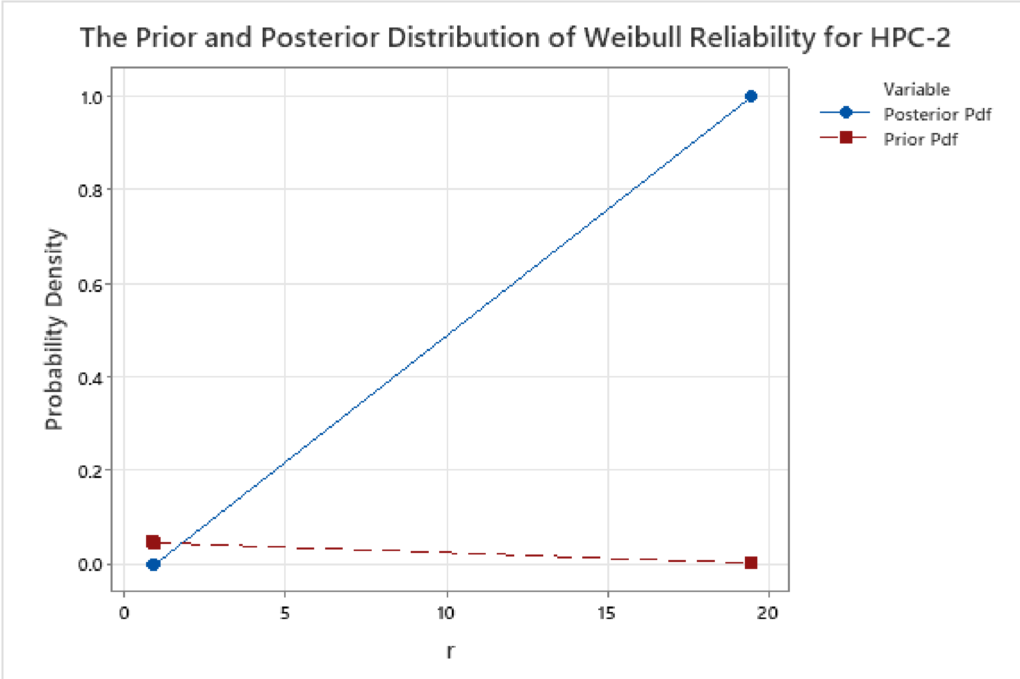

Figure 1.2: The Prior and Posterior of Weibull Reliability for HPC-2

In Figure 1.2, HPC-2 has an increasing reliability pdf in the interval of 0.891715 < r < 0.962518. Presented in Table 1.4 is the posterior estimate for HPC-2 expected reliability and 95% two tail Bayesian prediction interval (TBPI) at 36 hours (an extraction of Microsoft spreadsheet).

Table 1.4: Posterior Estimate for HPC-2 Expected Reliability in Weibull Data Fitting

Parameter: $s = n = 42$, $w = 954.26$, $\alpha_0 = 0.003$, $\beta_0 = 0.009$, $\beta = 0.7100$, $t_0 = 36$, $0.891715<r<0.962518$	
	Bayesian Estimation
Posterior Relability , $E[R(36)/w; \alpha_0, \beta_0]$	0.7298398
Posterior Variance , $Var(R(36)/w; s, \alpha0, \beta0)$	4.1264215E-05
Standard Error (S.E) of R(36 hrs)	0.00099120
95% LBPI estimate of $R(t_0;\lambda^*\beta)$	0.7251882
95% UBPI estimate of $R(t_0;\lambda_*\beta)$	0.7322152

LBPI: Lower Bayesian Probability Interval and UBPI: Upper Bayesian Probability Interval

As seen in Table 1.4, HPC-2 has an expected 72.98% reliability at 36 hours operating time, with a standard error of 0.001. The 95% lower and upper Bayesian prediction interval is [0.7251882, 0.7322152]. . Presented in Table 1.5 are the expected posterior MTTF and the 95% TBPI for the HPC- 2 using non-informative prior distribution on failure rate, under Weibull data fitting (an extraction of Microsoft spreadsheet).

Table 1.5: MTTF Estimation of HPC-2 (Non-informative Prior Distribution on η)

Noninformative Prior Case	
Parameter: $s = n = 42$, $w = 954.24$, $\alpha_0 = 0.003$, $\beta_0 = 0.009$, $\beta = 0.7100$	
	Bayesian Estimation
Posterior expected MTTF , E[MTTF/w]	105.8867594
Var[MTTF/w]	568.8150078
Standard Error (S.E) of MTTF	3.68011066

95% LBPI for MTTF	68.4229243
95% UBPI for MTTF	161.196193

LBPI: Lower Bayesian Probability Interval and UBPI: Upper Bayesian Probability Interval

As shown in Table 1.5, the estimated MTTF of HPC-2 in Weinbull under a non-informative prior distribution of failure rate is 105.89 hours with a standard error of 3.68 hours. Also, the estimated 95% lower and upper Bayesian probability interval is [68.4229243, 161.196193]. Presented in Table 1.6 are the expected posterior MTTF and the 95% TBPI for the HPC- 2 under Gamma prior distribution on failure rate (an extraction of Microsoft spreadsheet).

Table 1.6: MTTF Estimation of HPC-2 (Gamma Prior Distribution on η)

Gamma Prior Distribution on η for MTTF Estimation	
Parameter: $s = n = 42$, $w = 954.26$, $\alpha_0 = 2$, $\beta_0 = 0.003$, $\beta = 0.7100$	
	Bayesian Estimation
Posterior expected MTTF , E[MTTF/w; α_0, β_0]	150.9457796
Var[MTTF/w; α_0, β_0]	3662.876149
Standard Error (S.E) of MTTF	9.33870180
95% LBPI for MTTF	98.55764617
95% UBPI for MTTF	227.609431

LBPI: Lower Bayesian Probability Interval and UBPI: Upper Bayesian Probability Interval

As seen in Table 1.6, the estimated MTTF of HPC-2 in Weibull under a Gamma prior distribution of failure rate is 150.96 hours, with a standard error of 9.34 hours and the estimated 95% lower and upper Bayesian probability interval is [98.57764617, 227.609431]. Presented in Table 1.7 is the estimated cost of preventive and corrective maintenance of the high pressure compressor - 2 (HPC-2) per year (an extraction of Microsoft spreadsheet).

Table 1.7: HPC-2 Annual Maintenance Cost

Parameters: SOH = 8560hrs/yr; ST_{pm} = 0.5hrs; TT_{pm} = 0.25hrs; TT_{cm} = 0.25hrs and SI_{pm} = 720hrs		
Preventive Maintenance Cost (PMC)	**Corrective Maintenance Cost**	
PMC (USD/Year)	Model 1 (USD/Year)	Model 2 (USD/Year)
769.51	**39388.48**	**41185.14**

Notations: SIpm = schedule interval for PM, SOH = usage time of equipment/yr, STpm = schedule time for PM, TTcm = expected travel time for CM, and TTpm = expected travel time for PM.

As seen in Table 1.7, the annual preventive maintenance cost is 761.51USD and corrective maintenance cost for this same period is estimated to be 39,338.48 USD based on model 1 and 41,185.14 USD on model 2. Consequently, the expected annual maintenance cost is 41,954.65 USD.

CONCLUSION

The developed Bayesian reliability framework for the HPC-2 equipment, modeled under the Weibull distribution, demonstrates a strong capacity for accurately estimating critical reliability parameters under uncertainty. The analysis produced an expected failure rate of 0.008749 failures per hour with a standard error of 3.743×10^{-5}, and a 95% two-tailed Bayesian prediction interval (TBPI) of [0.0086578, 0.0089257]. The narrowness of this interval indicates a high level of precision in the posterior estimate, suggesting that the Weibull model effectively captures the stochastic behavior of the system's time-to-failure process. At 36 hours of operation, the estimated reliability of 72.98% (with a standard error of 9.912×10^{-4} and a 95% Bayesian reliability interval of [0.7252, 0.7322]) implies that approximately one-quarter of the equipment population may experience failure within this period. This finding aligns with established reliability theory, which recognizes the Weibull model as one of the most flexible and empirically validated models for analyzing complex mechanical systems with age-dependent failure mechanisms [11-13].

In terms of life expectancy, the Bayesian estimation of the mean time to failure (MTTF) under a non-informative prior yielded a value of 105.89 hours, with a standard error of 3.68 hours and a 95% probability interval of [68.42, 161.20] hours. Conversely, the Gamma prior resulted in a higher MTTF estimate of 150.95 hours with a standard error of 9.34 hours. The observed differences across priors emphasize the sensitivity of Bayesian reliability inference to prior selection - a phenomenon well-documented in the literature [14-15] - and highlight the importance of aligning prior beliefs with realistic operational knowledge or manufacturer specifications. The relatively wider Bayesian interval associated with the non-informative prior also reflects epistemic uncertainty, particularly when limited failure data are available.

From a maintenance and operational perspective, the Bayesian cost model estimated the annual maintenance expenditure for the HPC-2 equipment at approximately USD 41,954.65. This figure reflects the combined expected cost of preventive and corrective maintenance interventions and underscores the financial significance of reliability-driven decision-making in industrial operations. The results suggest that preventive or condition-based maintenance strategies could provide considerable cost savings by reducing the frequency and severity of unplanned failures - an approach consistent with the reliability-centered maintenance (RCM) philosophy advocated by Nowlan and Heap [16] and subsequent industrial maintenance optimization frameworks.

Overall, the developed Bayesian Weibull framework provides a comprehensive and probabilistically grounded approach for reliability evaluation, predictive assessment, and maintenance cost optimization of critical gas equipment. By integrating prior information, observed data, and uncertainty quantification within a unified model, the framework supports more informed engineering decisions, improved maintenance scheduling, and enhanced operational dependability of the HPC-2 system.

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Appendix 1: Historical Information of HPC-2

S/N	TTF (Hrs)	TTR (Hrs)	SPARES COST ($)	LABOUR ($)	TOTAL COST ($)
1		5.67	200	300	500
2	217.25	2.5		300	300
3	328.17	48.17	200	600	800
4	7.25	2.3	200	600	800
5	31.78	2.2	200	600	800
6	624.05	38.17		300	300
7	475.67	4.03	200	750	950
8	69.97	51.83		750	750
9	120.83	1	200	600	800
10	74.5	3.6		600	600
11	93.23	1.67		600	600
12	434.25	2.25	200	750	950
13	9.18	3.32	200	600	800
14	481.03	0.96	400	850	1250
15	46.9	0.63	200	750	950
16	77.03	0.47	200	600	800
17	41.07	0.6	200	600	800
18	1.7	2.13	200	600	800
19	90.07	2.17	300	750	1050
20	103.67	26	300	750	1050
21	45.63	1.05	400	850	1250
22	61.75	35.17	400	600	1000
23	65.22	48.17	350	750	1100
24	98.4	2	200	450	650
25	22.22	1.78	200	450	650
26	46.88	0.17	350	600	950

TTF: Time to Failure and TTR: Time to Repair

S/N	TTF (Hrs)	TTR (Hrs)	SPARES COST ($)	LABOUR ($)	TOTAL COST ($)
27	24.18	3.25	400	600	1000
28	44.57	1.83	450	600	1050
29	20.63	0.67	450	600	1050
30	24.3	1.33	400	850	1250
31	20.63	1	-	600	600
32	25.3	2	450	650	1100
33	119.63	1.83	450	600	1050
34	18.82	1	-	600	600
35	23.6	1	-	600	600
36	0.52	2.25	200	750	950
37	67.67	1.83	450	600	1050
38	23.15	1.33	400	850	1250
39	70.4	0.5	-	600	600
40	0.52	0.5	400	850	1250
41	0.47	1.33	450	600	1050
42	114.73	1.38	450	600	1050
43	3.47	5.87	450	850	1300

TTF: Time to Failure and TTR: Time to Repair

HPC-2 Maintenance Cost Estimation

***Computing the total Labor Cost (TLC)*:** The total labor cost, TLC for HPC-2 was computed by summing the labor cost. Therefore, TLC is $27,350.00.

***Computing the Total Repair time (TRT)*:**

Total Repair time, TRT is computed was computed by summing the TTR variable. Therefore, TRT is 316.91 hours

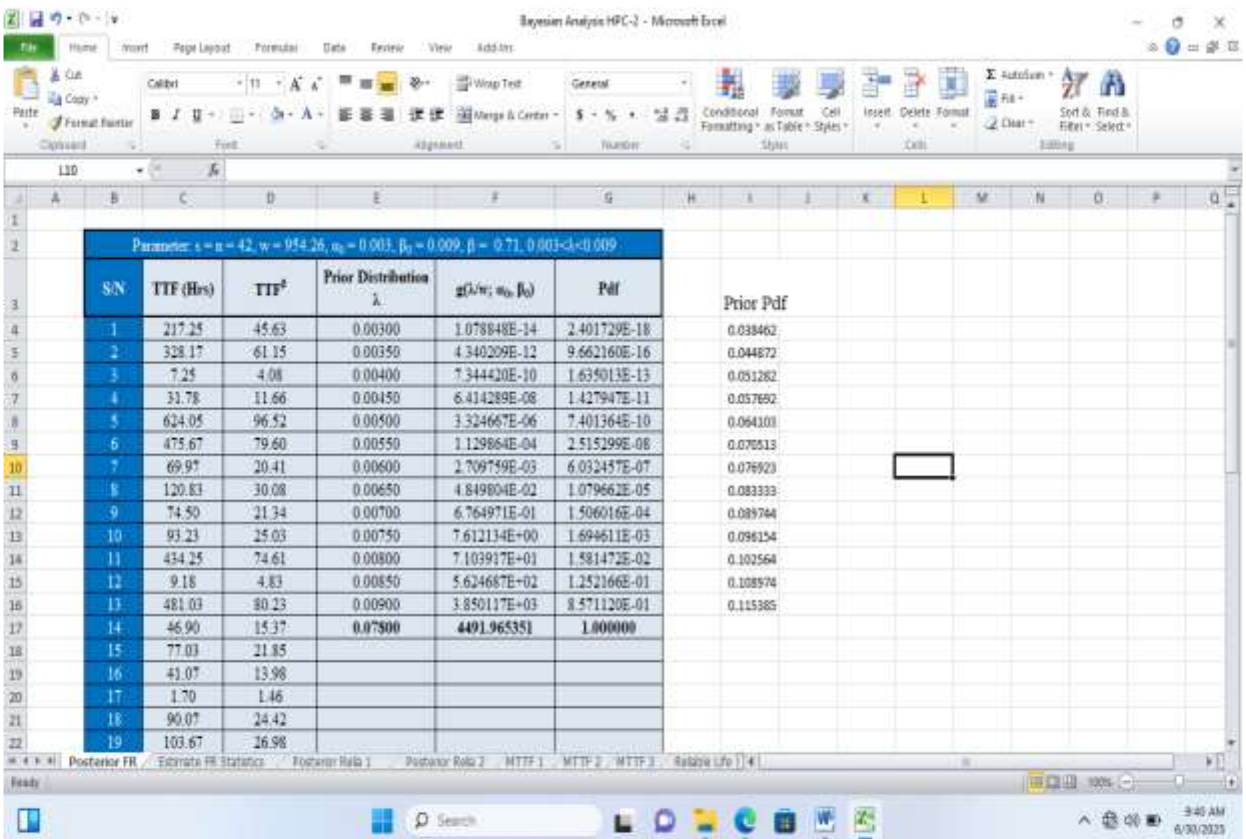
***Computing the Labor Cost per hour (LC)*:**

The labor cost per hour, LC was computed using the ratio of TLC to TRT. Therefore, LC = 27,350/316.91 = $86.30 per hour.

***Computing Spare Parts Cost (SPC)*:**

The spare parts cost, SPC was computed by summing the total costs of parts used in the studied period. Therefore, SPC is $10,700.00.

Appendix 2: Bayesian Analysis in Spreadsheet



Parameter: $s = n = 42$, $w = 954.26$, $\alpha_0 = 0.003$, $\beta_0 = 0.009$, $\beta = 0.71$, $0.003<\lambda<0.009$

S/N	TTF (Hrs)	TTF^β	Prior Distribution λ	$g(\lambda/w; \alpha_0, \beta_0)$	Pdf	Prior Pdf
1	217.25	45.63	0.00300	1.078848E-14	2.401729E-18	0.038462
2	328.17	61.15	0.00350	4.340209E-12	9.662160E-16	0.044872
3	7.25	4.08	0.00400	7.344420E-10	1.635013E-13	0.051282
4	31.78	11.66	0.00450	6.414289E-08	1.427947E-11	0.057692
5	624.05	96.52	0.00500	3.324667E-06	7.401364E-10	0.064103
6	475.67	79.60	0.00550	1.129864E-04	2.515299E-08	0.070513
7	69.97	20.41	0.00600	2.709759E-03	6.032457E-07	0.076923
8	120.83	30.08	0.00650	4.849804E-02	1.079662E-05	0.083333
9	74.50	21.34	0.00700	6.764971E-01	1.506016E-04	0.089744
10	93.23	25.03	0.00750	7.612134E+00	1.694611E-03	0.096154
11	434.25	74.61	0.00800	7.103917E+01	1.581472E-02	0.102564
12	9.18	4.83	0.00850	5.624687E+02	1.252166E-01	0.108974
13	481.03	80.23	0.00900	3.850117E+03	8.571120E-01	0.115385
14	46.90	15.37	**0.07800**	**4491.965351**	**1.000000**	
15	77.03	21.85				
16	41.07	13.98				
17	1.70	1.46				
18	90.07	24.42				
19	103.67	26.98				

All the Bayesian analysis and the estimated cost in this study are computed through a combination of the spreadsheet function(s) and the required formula(s) expression as presented in Equation 1.1 to 1.40 for each analysis.