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QUASI-ANALYTIC APPROXIMATION OF SHARMA-TASSO-OLVER EQUATION

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ABSTRACT

In this study, three quasi-analytical techniques; Variational Iteration Method (VIM), Reduced Differential Transform Method (RDTM), and Adomian Decomposition Method (ADM) were applied to the nonlinear Sharma-Tasso-Olver equation to deduce the performance of their convergence characteristics. A comparative analysis shows Variational Iteration Method (VIM) converge faster and reduced computational complexity relative to RDTM and ADM. In particular, VIM avoids the explicit construction of Adomian polynomials, which can become increasingly cumbersome for strongly nonlinear terms. Consequently, VIM proves to be computationally efficient approach for solving the Sharma–Tasso–Olver equation.

1 INTRODUCTION

Nonlinear partial differential equations (NPDEs) play a fundamental role in modeling complex phenomena across diverse scientific and engineering disciplines, including hydrodynamics, engineering systems, quantum field theory, nonlinear optics, and plasma physics. In most cases, these equations do not admit closed-form exact solutions and therefore require approximate analytical or numerical techniques for their solution [1]. Although numerical schemes are widely used, they are often associated with discretization errors and round-off inaccuracies arising from mesh-based approximations. In contrast, semi-analytical methods provide approximate solutions in series form without requiring spatial or temporal discretization, thereby reducing computational complexity and avoiding cumulative numerical errors.

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Moreover, when an exact solution exists, these methods can converge toward it under appropriate conditions. Several quasi-analytical approaches have been developed for solving nonlinear partial differential equations. These include the Adomian Decomposition Method (ADM) [2], the Differential Transform Method (DTM) [3], the Homotopy Perturbation Method (HPM) [4], the Variational Iteration Method (VIM) [5], and the Reduced Differential Transform Method (RDTM)[6], among others. The Variational Iteration Method (VIM) has been successfully applied to a broad class of nonlinear problems [7-10]. Similarly, the Reduced Differential Transform Method (RDTM) has demonstrated effectiveness in solving various nonlinear partial differential equations [6, 7, 11, 12], while the Adomian Decomposition Method (ADM) has been widely used to treat nonlinear systems through series decomposition techniques [1, 13]. In this paper, we apply the VIM, RDTM, and ADM to the nonlinear Sharma-Tasso-Olver equation (STO) equation, given by:

$$\frac{\partial}{\partial t}v + 3\frac{\partial}{\partial x}v + 3v^2\frac{\partial}{\partial x}v + 3v\frac{\partial^2}{\partial x^2}v + \frac{\partial^3}{\partial x^3}v = 0 \qquad (1)$$

The STO equation is a well-known doubly nonlinear dispersive model, consisting of the linear dispersive term $\frac{\partial^3}{\partial x^3}v$ together with two nonlinear terms $3v^2\frac{\partial}{\partial x}v$ and $3v\frac{\partial^2}{\partial x^2}v$, that account for its double non-linearity.

Subject to the initial condition:

$$v(x,0) = \frac{1}{2}\left[1 + tanh\frac{x}{2}\right] \qquad (2)$$

2 QUASI-ANALYTICAL METHODS

2.1 Variational Iteration Method

To explain the fundamental concept of the of variational iteration method [5], consider a differential equation expressed in the form:

$$L(v(x,t)) + N(v(x,t)) = g(x,t) \qquad (3)$$

where L denotes a linear operator, N represents a nonlinear operator and g is an inhomogeneous term. Based on this formulation. A correctional functional is constructed as follows:

$$v_{k+1} = v_k(x,t) + \int_0^t \lambda(\zeta)\ \{\mathcal{L}(v(x,\zeta))\} + N(\bar{v}(x,\zeta)) - g(x,\zeta)\, d\zeta \qquad (4)$$

where λ is a general Lagrange multiplier [14], which is optimally determined using the Variational theory. The second term on the right-hand side account for the non-linear correction and is treated as a restricted variation using

$$\delta v_{k+1} = \delta v_k(x,t) + \delta\int_0^t \lambda(\zeta)\ \frac{\partial v_k(x,\zeta)}{\partial \zeta}\, d\zeta \qquad (5)$$

Thus,

$$\delta v_{k+1} = \delta v_k(x,t) + \int_0^t \lambda(\zeta)\, \frac{\partial\,(\partial v_k(x,\zeta))}{\partial \zeta}\, d\zeta$$

The second term is expanded using integration by parts

$$\int_0^t \lambda\,(\mathrm{t})\, \frac{\partial\,(\partial v_k)}{\partial \zeta}\, d\zeta = \lambda(t)\,\delta v(t) - \lambda(0)\,\delta v(0) - \int_0^t \lambda'(\zeta)\,\delta v_k\, d\zeta \qquad (6)$$

Substituting equation (6) to equation (5) gives

$$\delta v_{k+1} = \delta v_k(x,t) + \lambda(t)\,\delta v_k(t) - \lambda(0)\,\delta v_k(0) - \int_0^t \lambda'(\mathrm{t})\,\delta v_k\, dt \qquad (7)$$

The determination of the approximations v_{k+1} follows immediately.

2.2 Reduced Differential Transform Method

The Reduced Differential Transform Method (RDTM) is defined as follows [6];

Assume that a function $v\,(x,t)$ is analytic and continuously differentiable with respect to time t and space x within the domain of interest. The transformed function, $v_{\mathrm{k}}(\mathrm{x})$ is then introduced such that,

$$v_k = \frac{1}{k!}\left[\frac{\partial^k\, v(x,t)}{\partial t^k}\right]_{t\,=\,0} \qquad (8)$$

the *k*-dimensional spectrum function, $v_{\mathrm{k}}(\mathrm{x})$ represents the transformed form of the original function, $v\,(x,t)$.

The inverse differential transform, $v_{\mathrm{k}}(\mathrm{x})$ is defined accordingly.

$$v\,(x,t) = \sum_{\mathrm{k}=0}^{\infty} t^{\mathrm{k}}\, v_{\mathrm{k}} \qquad (9)$$

By combining Eq. (8) and (9), we obtain

$$v\,(x,t) = \sum_{\mathrm{k}=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k\, v(x,t)}{\partial t^k}\right]_{t\,=\,0} t^{\mathrm{k}} \qquad (10)$$

From these definitions, it is evident that RDTM is based on the concept of power series expansion. Following the RDTM framework [6, 11, 12], an iterative scheme can be constructed to compute the transformed components, $v_1, v_2, v_3,\ v_4, v_5$. The first term of the transformed solution, $v_0(\mathrm{x})$ corresponds directly to the initial condition at *t = 0*. For second-order partial differential equations, the time derivative of the initial condition provides the subsequent term, $v_1(\mathrm{x})$. Applying the inverse transform to the resulting set of components yields an approximate solution,

$$\overline{v_n}\,(x,t) = \sum_{\mathrm{k}=0}^{\infty} t^{\mathrm{k}}\, v_{\mathrm{k}}(x) \qquad (11)$$

Where n denotes the order of the approximation. Consequently, the exact solution is obtained in the limit as the number of terms approaches infinity

$$v(x,t) = \lim_{n\,\to\,\infty} \overline{v_n}(x,t) \qquad (12)$$

2.3 Adomian Decomposition Method

Another type of quasi-analytical method is the Adomian Decomposition scheme was developed by George Adomian. It is often employed to obtain approximate solutions of nonlinear partial differential equations PDEs [2] . Consider the general form of the equation

$$\mathcal{L}v + Rv + Nv = g(t) \qquad (13)$$

Where L denotes the highest-order linear differential operator, R represent the remaining linear part, and N is the non-linear term. The equation can therefore be rewritten as

$$\mathcal{L}v = g(t) - Rv - Nv \qquad (14)$$

We define the inverse operator, $\mathcal{L}^{-1}$ associated with $\mathcal{L}$ as

$$\mathcal{L}^{-1} = \int dt^n \qquad (15)$$

Applying the inverse operator, $\mathcal{L}^{-1}$ in equation (15) to both sides of equation (14) yields

$$\mathcal{L}^{-1}\mathcal{L}v = \psi_0 + \mathcal{L}^{-1}(\mathrm{g(t)} - \mathrm{R}v - \mathrm{N}v)$$

$$v = \psi_0 + \mathcal{L}^{-1}(\mathrm{g(t)} - \mathrm{R}v - \mathrm{N}v) \qquad (16)$$

Where ψ_0 is a constant of integration representing the solution of the corresponding homogeneous equation, $\mathcal{L}v = 0$and is is determined from the given initial or boundary conditions, depending on whether the problem is an initial value or boundary value problem.

In ADM, the solution $v(\mathrm{x,t})$ is expressed as an infinite series of the form

$$v(x,t) = \sum_{k=0}^{\infty} \lambda^k v_k(x,t) \qquad (17)$$

and the non-linear term, Nv is decomposed into an infinite series of Adomian polynomials given by

$$Nv = \sum_{k=0}^{\infty} \lambda^k A_k \qquad (18)$$

A_k is the Adomian special polynomial [3] given by

$$A_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial\lambda^k} N\mathrm{v}\right]_{\lambda=0} \qquad k = 0, 1, 2, ... \qquad (19)$$

λ is an arbitrary parameter which aids in the grouping of the terms. The parameterized form of equation (16) is written as

$$v = \psi_0 + \lambda\mathcal{L}^{-1}(\mathrm{g(t)} - \mathrm{R}v - \mathrm{N}v) \qquad (20)$$

$$\sum_{k=0}^{\infty} \lambda^k v_k(x,t) = \psi_0 + \lambda L^{-1}(\mathrm{g(t)} - \mathrm{R}\sum_{k=0}^{\infty} \lambda^k v_k(x,t) - \sum_{k=0}^{\infty} \lambda^k A_k)$$

$$\sum_{k=0}^{\infty} \lambda^k v_k(x,t) = \psi_0 + \mathcal{L}^{-1}(g(t) - R\sum_{k=0}^{\infty} \lambda^{k+1} v_k(x,t) - \sum_{k=0}^{\infty} \lambda^{k+1} A_k)$$

Comparing powers of λ gives

$$v_0 = \psi_0(x)$$

$$v_1 = \mathcal{L}^{-1}(\mathrm{g(t)} - \mathrm{R}v_0 - \mathrm{A}_0)$$

$$v_2 = \mathcal{L}^{-1}(\mathrm{g(t)} - \mathrm{R}v_1 - \mathrm{A}_1)$$

$$v_3 = \mathcal{L}^{-1}(\mathrm{g(t)} - \mathrm{R}v_2 - \mathrm{A}_2)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$v_{k+1} = \mathcal{L}^{-1}(\mathrm{g(t)} - Rv_k - A_k)$$

Earlier studies by other researchers indicate that three non-zero terms are enough to get reliable solution.

$$v(x,t) = \sum_{k=0}^{\infty} v_k\,(x,t)$$

$$v = v_0 + v_1 + v_2 + v_3 + \ldots\ldots\ldots\ldots..$$

3 APPLICATIONS

3.1 Application to Variational Iteration Method

Recall the nonlinear Sharma-Tasso-Olver equation given by equation (1), this can be written in subscript notation as

$$v_t + 3v_x + 3v^2 v_x + 3v\, v_{xx} + v_{xxx} = 0$$

Subject to the initial condition:

$$v\,(x,0) = \frac{1}{2}\left[1 + tanh\frac{x}{2}\right]$$

According to VIM, we can construct the correction functional as:

$$v_{k+1} = v_k(x,t) + \int_0^t \lambda\, \{v_{kt} + 3\bar{v}_{kx} + 3\overline{v_k}^2 v_{kx} + 3\bar{v}_k \bar{v}_{kxx} + \bar{v}_{kxxx}\}dt \qquad (21)$$

Where $\bar{v}_k$ is considered as a restricted variation, i.e. $\delta\bar{v}_k = 0$ and λ is the general Lagrange multiplier. Making the above correction functional stationary, we yield the stationary conditions:

$$1 + \lambda = 0 \qquad \lambda' = 0$$

$$\lambda = -1$$

Substituting this value of the Lagrange multiplier into the correction functional gives the iteration formula

$$v_{k+1} = v_k(x,t) - \int_0^t \{v_{kt} + 3v_{kx} + 3v^2 v_{kx} + 3v_k v_{kxx} + v_{kxxx}\}\; d\xi \qquad (22)$$

We begin with the initial condition as the initial approximation v_0

$$v_0 = \frac{1}{2}\left[1 + tan\text{h}\frac{x}{2}\right]$$

Using the iteration formula and the initial approximation, we obtain successive approximations $v_1, v_2, v_3, v_4, \ldots$ with the aid MAPLE Software. The first few solutions are presented below.

$$v_1 = -t\left(\frac{1}{2} sech^4\left(\frac{x}{2}\right) cosh(x) + \frac{5}{16} sech^4\left(\frac{x}{2}\right)\right) + \frac{1}{2}\left[1 + tan\text{h}\frac{x}{2}\right]$$

3.2 Application to Reduced Differential Transform Method

From equation (1):

$$v_t + 3v_x + 3v^2 v_x + 3v\, v_{xx} + v_{xxx} = 0$$

Equation (20) can be written in standard operator form as (22), where $L = \frac{\partial}{\partial t}$, $R = 3\frac{\partial}{\partial x} + \frac{\partial^3}{\partial x^3}$ and $N = v^2 \frac{\partial}{\partial x}$, $M = v\frac{\partial^2}{\partial x^2}$
are the non-linear terms.

$$L(v\,(x,t)) + Rv\,(x,t) + Nv\,(x,t) + Mv\,(x,t) = 0 \qquad (23)$$

with initial condition

$$v\,(x,0) = \frac{1}{2}\left[1 + tanh\frac{x}{2}\right]$$

$$L(v\,(x,t)) = \frac{(k+1)!}{k!} v_k \qquad (24)$$

$$R\,(v\,(x,t)) = 3\frac{\partial v_k}{\partial x} + \frac{\partial^3 v_k}{\partial x^3} \qquad (25)$$

The first few non-linear terms using the Adomian Polynomials [3] becomes:

$$N_0 = {v_0}^2 \frac{\partial v_0}{\partial x}$$

$$N_1 = {v_0}^2\frac{\partial v_1}{\partial x} + 2v_0v_1\frac{\partial v_0}{\partial x}$$

$$N_2 = {v_0}^2\frac{\partial v_2}{\partial x} + 2v_0v_1\frac{\partial v_1}{\partial x} + 2\,v_0v_2\frac{\partial v_0}{\partial x} + {v_1}^2\frac{\partial v_0}{\partial x}$$

$$N_3 = {v_0}^2\frac{\partial v_3}{\partial x} + 2v_0v_1\frac{\partial v_2}{\partial x} + 2\,v_0v_2\frac{\partial v_1}{\partial x} + 2\,v_0v_3\frac{\partial v_0}{\partial x} + 2\,v_1v_2\frac{\partial v_0}{\partial x} + {v_1}^2\frac{\partial v_1}{\partial x}$$

$$M_0 = v_0\frac{\partial^2 v_0}{\partial x^2}$$

$$M_1 = v_1\frac{\partial^2 v_0}{\partial x^2} + v_0\frac{\partial^2 v_1}{\partial x^2}$$

$$M_3 = v_3\frac{\partial^2 v_0}{\partial x^2} + v_2\frac{\partial^2 v_1}{\partial x^2} + v_1\frac{\partial^2 v_2}{\partial x^2} + +v_0\frac{\partial^2 v_3}{\partial x^2}$$

Substituting equation (24), (25) and the nonlinear terms into equation (23), we get the transformed form of the STO equation.

$$\frac{(k+1)!}{k!}v_k = -3\frac{\partial v_k}{\partial x} - \frac{\partial^3 v_k}{\partial x^3} - 3N_k - 3M_k \qquad (26)$$

From the initial condition we can write

$$v_0 = \frac{1}{2}\left[1 + tanh\frac{x}{2}\right]$$

Subsequent v_k values are obtained using the iteration formula (26) and the initial condition.

The first few values presented below.

$$v_1 = -\frac{5}{16} sech^4(\frac{x}{2}) - \frac{1}{2} cosh(x) sech^4(\frac{x}{2})$$

$$v_2 = -\frac{1}{136} sech^7(\frac{x}{2}) \left[-28cosh\,(\frac{x}{2}) + 12cosh\,(\frac{3x}{2}) + 95sinh\,(\frac{x}{2}) - 7sinh\,(\frac{3x}{2}) + 12sinh\,(\frac{5x}{2})\right]$$

$$v_3 = -\frac{1}{4010}\left[sech^{10}(\frac{x}{2})\,(1321 - 20251cosh\,(x) + 4983cosh(2x) + 789cosh(3x) + 81\,cosh(4x)\right]$$

The solution is given by the inverse differential transform in equation (9):

$$\sum_{k=0}^{\infty} t^k\, v_k(x) = v_0 + v_1 t + v_2 t^2 + v_3 t^3 + v_4 t^4 + v_5 t^5 + v_6 t^6 + \ldots \qquad (27)$$

The implementation of the RDTM is done with the aid of MAPLE Software to obtain the v_k values and plug them into equation (27) to get the approximate solution

3.3. Application to Adomian Decomposition Method

The STO equation can be written in standard form as

$$\mathcal{L}_t v + \mathcal{L}_x v + 3Nv + 3Mv = 0 \qquad (28)$$

We define the operators $\mathcal{L}_t = \frac{\partial}{\partial t}$, $\mathcal{L}_x = 3\frac{\partial}{\partial x} + \frac{\partial^3}{\partial^3 x}$, $N = v^2 \frac{\partial}{\partial x}$ and $M = v\frac{\partial^2}{\partial x^2}$ where M and N are nonlinear operators.

From (28), we can write

$$\mathcal{L}_t v = -\mathcal{L}_x v - 3Nv - 3Mv \qquad (29)$$

We define the inverse operator to operator $\mathcal{L}_t$ and ${\mathcal{L}_t}^{-1}$ given by

$${\mathcal{L}_t}^{-1} = \int dt \qquad (30)$$

Applying the inverse operator in equation (30) on both sides of equation (29) gives

$${\mathcal{L}_t}^{-1}\mathcal{L}_t v = \psi_0 + {\mathcal{L}_t}^{-1}(-\mathcal{L}_x v - 3Nv - 3Mv)$$

$$u = \psi_0 + {\mathcal{L}_t}^{-1}(-\mathcal{L}_x v - 3Nv - 3Mv) \qquad (31)$$

Where ψ_0 which is a constant of integration is the solution of the equation $\mathcal{L}_t = 0$ and is just the initial condition which is purely a function of x. The solution $v(x, t)$ to the PDE can be expressed by an infinite series of the form

$$v(x,t) = \sum_{k=0}^{\infty} \lambda^k v_k (x,t) \qquad (32)$$

And the decomposed form of the non-linear operators Nv and Mv is an infinite series of polynomials given by

$$Nv = \sum_{k=0}^{\infty} \lambda^k A_k , Mv = \sum_{k=0}^{\infty} \lambda^k B_k \qquad (33)$$

A_k and B_k are the Adomian special polynomials and they are respectively equivalent to the N_k and M_k values used in RDTM.

The parameterized form of equation (31) is

$$u = \psi_0 + \lambda \mathcal{L}_t^{-1}(-\mathcal{L}_x v - 3Nv - 3Mv) \qquad (34)$$

$$\sum_{k=0}^{\infty} \lambda^k v_k = \psi_0 + \lambda \mathcal{L}_t^{-1}(-\mathcal{L}_x \sum_{k=0}^{\infty} \lambda^k v_k - 3 \sum_{k=0}^{\infty} \lambda^k A_k - 3 \sum_{k=0}^{\infty} \lambda^k B_k)$$

$$\sum_{k=0}^{\infty} \lambda^k v_k = \psi_0 + \mathcal{L}_t^{-1}(-\mathcal{L}_x \sum_{k=0}^{\infty} \lambda^{k+1} v_k - 3 \sum_{k=0}^{\infty} \lambda^{k+1} A_k - 3 \sum_{k=0}^{\infty} \lambda^{k+1} B_k)$$

Comparing powers of λ gives

$$v_0 = \psi_0$$

$$v_1 = \mathcal{L}_t^{-1}(-\mathcal{L}_x v_0 - 3\mathrm{A}_0 - 3B_0)$$
$$v_2 = \mathcal{L}_t^{-1}(-\mathcal{L}_x v_1 - 3\mathrm{A}_1 - 3B_1)$$
$$v_3 = \mathcal{L}_t^{-1}(-\mathcal{L}_x v_2 - 3\mathrm{A}_2 - 3B_2)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$v_{k+1} = \mathcal{L}_t^{-1}(-\mathcal{L}_x v_k - 3A_k - 3B_k)$$

The solution to the Sharma-Tasso-Olver equation with ADM is given by

$$v(x,t) = \sum_{k=0}^{\infty} v_k (x,t)$$

$$v = v_0 + v_1 + v_2 + v_3 + \ldots\ldots\ldots\ldots + v_k$$

The first few v_k values for the STO equation are

$$v_1 = -t\left(\frac{3}{7} sech^5 \left(\frac{x}{2}\right) + \frac{1}{2} cosh(x) sech^5 \left(\frac{x}{2}\right)\right)$$

$$v_2 = -\frac{2}{137} t^2 sech^5 \left(\frac{x}{2}\right)\left[-18 cosh\left(\frac{x}{2}\right) + 9 cosh\left(\frac{3x}{2}\right) + 91 sinh\left(\frac{x}{2}\right) - 27 sinh\left(\frac{3x}{2}\right) + 19 sinh\left(\frac{3x}{2}\right)\right]$$

$$v_3 = -\frac{3}{1037} t^3 \left[sech^7 \left(\frac{x}{2}\right)(12387 - 13925 cosh(x) + 3079 \cosh 2\mathrm{x} - 718 \cosh 3\mathrm{x} + 64 \cosh 4\mathrm{x} + + 418 \sinh \mathrm{x} - 9644 \sinh 2\mathrm{x} + 150 \sinh 3\mathrm{x}\right]$$

RESULTS AND DISCUSSION

The variational iteration method, reduced differential transform method and the Adomian decomposition method have been applied to the nonlinear Sharma-Tasso-Olver equation with results obtained for the VIM for k = 5 while RDTM and ADM results are obtained for k = 7. The absolute differences between the VIM and RDTM solutions and the absolute differences between the VIM and ADM solutions at selected positions and times are presented in Table 1. From Table 1, the absolute differences observed for the RDTM and ADM shows that the RDTM converges to the same results as the ADM for the STO equation. We also observe that the VIM gave comparable results to the RDTM and ADM results with a maximum absolute difference of order 1E-07 which implies that the VIM and RDTM results agree to at least 6 decimal places. In Conclusion, the VIM is simpler to implement and more efficient than the RDTM and ADM methods as it converges faster than the RDTM and ADM and it does not require the calculation of the cumbersome and difficult Adomian Polynomials associated with the RDTM and ADM.

Table 1: Absolute differences between VIM and RDTM for the STO equation

|VIM - RDTM|				|VIM - ADM|			
x	$t = 0.01$	$t = 0.02$	$t = 0.03$	x	$t = 0.01$	$t = 0.02$	$t = 0.03$
-10	5.841E-12	7.595E-11	3.815E-10	**-10**	4.784E-12	7.595E-11	3.815E-10
-8	4.542E-11	5.467E-10	2.747E-09	**-8**	3.442E-11	5.467E-10	2.747E-09
-6	3.089E-10	3.303E-09	1.665E-08	**-6**	2.074E-10	3.303E-09	1.665E-08
-4	2.397E-10	1.791E-09	8.927E-09	**-4**	1.139E-10	1.791E-09	8.927E-09
-2	2.995E-09	4.214E-08	2.273E-07	**-2**	2.495E-09	4.214E-08	2.273E-07
0	8.113E-09	9.320E-08	1.909E-07	**0**	8.113E-09	9.320E-08	1.909E-07
1	3.112E-09	4.801E-08	2.568E-07	**1**	3.112E-09	4.801E-08	2.568E-07
3	1.381E-09	2.186E-08	1.096E-07	**3**	1.381E-09	2.186E-08	1.096E-07
5	5.850E-10	9.410E-09	4.789E-08	**5**	5.850E-10	9.410E-09	4.789E-08
7	9.516E-11	1.534E-09	7.827E-09	**7**	9.516E-11	1.534E-09	7.827E-09
9	1.322E-11	2.132E-10	1.088E-09	**9**	1.322E-11	2.132E-10	1.088E-09
10	4.875E-12	7.862E-11	4.012E-10	**10**	4.875E-12	7.862E-11	4.012E-10

CONCLUSION

In this work, the variational iteration method, reduced differential transform method and the Adomian decomposition methods were applied to obtain solutions to the nonlinear Sharma-Tasso-Olver equation. The RDTM and ADM gave the similar results for the STO equation. Though VIM, RDTM and the ADM gave comparable solution with maximum absolute difference of order 1E-07, the VIM is proposed for solving the STO equation and other high order nonlinear partial differential equations due to its faster convergence rate and because it does not require calculating the cumbersome Adomian polynomials used in ADM and RDTM. A limitation of this study is the absence of detailed computational cost analysis and extended validation against exact solutions over larger time domains, which will be addressed in future work.

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