Transactions of NAMP 24, (2026) 155-168

Transactions of
The Nigerian Association of
Mathematical Physics
Journal homepage: https://nampjournals.org.ng

A STUDY ON ORBIT-ERROR IN PSEUDO-RANGE GNSS POSITIONING USING SIMULATED SATELLITE CONSTELLATIONS

[1] Joseph O. Odumosu,[1] Geoffery O. Nwodo [2] Temitope O. Oyadokun, [1] Adesina Ekundayo
[1]Department of Geomatics, University of Benin*
[2] Department of Surveying & Geoinformatics, Federal University of Technology, Minna.

ARTICLE INFO

Article history:
Received xxxxx
Revised xxxxx
Accepted xxxxx
Available online xxxxx

Keywords:
Satellite Constellation;
Global Navigation Satellite Systems;
Pseudorange;
Reference Frames;
Point Positioning.

ABSTRACT

This study investigates orbit error effects on pseudo-range GNSS positioning using simulated satellite constellations. The research methodology adopted in this study was basically empirical, wherein simulated satellites constellations, GPS Time, and pseudo-range positions were used to estimate the average positional errors obtainable from absolute GNSS point positioning technique using the GPS and GLONASS satellite constellations. By employing MATLAB-based simulations, the study analyzes the positional deviations due to orbital perturbations. From the findings of the study, it was identified that orbital errors in satellite constellations affect pseudo-range position determination by varying magnitude at different time epochs depending om the number of available satellites at the time of observation. Based on a simulated wobble value of ±20m, the effects of orbital error on pseud-orange positions range between -0.61m to +2.026m. The findings from this study have provided empirical evidence to assumed limitations of the single receiver GNSS receiver.

1 INTRODUCTION

Precise point positioning is one of the main tasks of the Surveyor; and in doing this, every other field of human endeavor rely heavily on the surveyor's decision exclusively. There is no doubt that the Global Positioning System (GPS) and its several other counterparts (generally referred to as Global Navigation Satellite System (GNSS)), right from inception had been a superb tool for precise positioning. Invariably, position engineers (in practice) believe that GNSS measurements are always precise and reliable. Nevertheless, the GNSS system is not without its own errors [1]. Being a satellite-based positioning system, position determination in the GNSS could be affected by a number of factors including but not limited to; the atmosphere, receiver hardware components, observation environment, satellite itself, or the satellite's orbit.

*Corresponding author: JOSEPH O. ODUMOSU
E-mail address: joseph.odumosu@uniben.edu
https://doi.org/10.60787/tnamp.v24.683

Orbit errors are errors that arise in positioning as calculated by the GNSS receiver due to shifts in the actual position of the satellite in orbit from the expected position at the time of signal observation [2].

In GNSS operations, there are two types of observables; being the codes and the carrier itself without the codes. The code solution provides a pseudorange. The pseudorange can serve efficiently in applications where virtually instantaneous point positions are required or relatively low accuracy will suffice; while, the carrier-phase measurements are used for very high-precision GPS surveys. Furthermore, it has been established that differential pseudo-range positioning is more accurate and preferable to absolute pseudo-range positioning [3]. Nevertheless, there are few occasions where absolute pseudo-range position determination seems to be an affordable option. Besides, even in relative pseudo-range positioning, despite that there are continuous broadcast corrections being sent from ground control stations to minimize satellite in-orbit perturbations, there are still small errors in the orbit that can result in up to ±2.5 meters of position error [4]. Previous studies have concluded that the best solution to pseudorange errors is relative (differential) positioning, notwithstanding, there is need to understand and characterize the nature of pseudorange errors in GNSS positioning.

According to [2] real-time Precise Orbit Determination (POD) is indispensable in the current dispensation and mainly includes two methods which are the ultra-rapid orbit prediction and the real-time filtering orbit determination. The study concludes that orbit error accounts for a large amount of error in GNSS positioning. Thereafter, [5] identified the main causes of deficiencies in GNSS satellite orbit modeling and also investigated the effects of such model errors on point positioning. [6] Presented a study to examine the influence of orbital perturbations on satellite coverage calculation. Also, [7] analyzed the ranging errors of GNSS crosslink and then performed high-orbit satellite orbit determination experiments on the crosslink ranging accuracy level using the geometric and kinematic methods respectively. Several other studies have been done on orbit errors in GNSS positioning, but not much has been done to examine and characterize the positional effects of orbit error in Pseudo-range positioning. This study therefore presents an examination of the effect of GNSS satellite orbit-related error on pseudo-range positioning in GNSS observation. The study presents an empirical investigation of the effects of orbit perturbations on pseudo-range positioning using simulated satellite constellations. The study further presents an analytical examination of the positional implication of improved GDOP for pseudo-range positioning using simulated satellites.

2. MATHEMATICAL PERSPECTIVE

Pseudorange positioning involves determination of the coordinates of a station using ephemeris information received from four or more satellites simultaneously. The general least squares procedure for Pseudorange positioning can be found in [8]. Consider the mathematical formulation for Pseudorange computation (equation 1), it is obvious that the orbital error remains one of the major challenges to accurate absolute positioning by pseudorange.

$$\boldsymbol{\rho} = \boldsymbol{\rho_T} + \boldsymbol{d_\rho} + \boldsymbol{c}(\boldsymbol{d_t} - \boldsymbol{d_T}) + \boldsymbol{d_{ion}} + \boldsymbol{d_{trop}} + \in_{\boldsymbol{mul}} + \in_{\boldsymbol{rec}} \qquad (1)$$

Where;

$\boldsymbol{\rho_T}$ = True range

$\boldsymbol{d_\rho}$ = satellite orbital error

c = speed of light

$\boldsymbol{d_t}$ = satellite clock offset from GPS time

$\boldsymbol{d_T}$ = receiver clock offset from GPS time

$\boldsymbol{d_{ion}}$ = Ionospheric delay

$\boldsymbol{d_{trop}}$ = Tropospheric delay

$\in_{\boldsymbol{mul}}$ = multipath error

$\in_{\boldsymbol{rec}}$ = receiver noise

For the purpose of numerical modelling (as performed in this study), the ionospheric and tropospheric errors are removed from the equation. Thus, equation (1) becomes (2);

$$\boldsymbol{\rho = \rho_T + c.\left(\delta_{t_R} - \delta_{t^s}\right) + \in} \qquad (2)$$

ρ = geometric distance between satellite and receiver in 3D

ρ_T = True range

c = speed of light

δ_{t_R} , δ_{t^s} = Receiver and Satellite clock errors respectively

$\in$ = orbit error

In as much as geometric distance between 2 points in 3D can be mathematically expressed as equation (3);

$$\boldsymbol{\rho = \sqrt{(x_s - x_r)^2 + (y_s - y_r)^2 + (z_s - z_r)^2}} \qquad (3)$$

Where;

x_s, y_s, z_s = 3D ECEF coordinates of the satellite

x_r, y_r, z_r = 3D ECEF coordinates of the receiver

The objective in GPS point positioning, is to solve for x_r, y_r, z_r, ΔT_r (clock bias). Since the unknowns (x_r, y_r, z_r) are not linearly related to the observables (x_s, y_s, z_s), the mathematical solution begins with an initial guess of the approximate position of receiver station, then the final coordinates of the receiver station will be equal to the approximate value plus a slight adjustment as given in equations 4(a) – (c)

$$x_r = x_0 + \Delta x_0 \qquad 4(a)$$

$$y_r = y_0 + \Delta y_0 \qquad 4(b)$$

$$z_r = z_0 + \Delta z_0 \qquad 4(c)$$

Therefore $(\Delta x_0, \Delta y_0, \Delta z_0)$, are now the new unknowns and they can be solved by method of least squares, so we now write the normal equation as Equation (5) which upon expansion by Taylors series yields equation (6).

$$f(x_r, y_r, z_r) = f(x_0 + \Delta x_0, y_0 + \Delta y_0, z_0 + \Delta z_0) \qquad (5)$$

$$f(x_r, y_r, z_r) = f\left(x_0, y_0, z_0\right) + \frac{\partial f\,(x_0, y_0, z_0)}{\partial x_0} \Delta x_1 + \frac{\partial f\,(x_0, y_0, z_0)}{\partial y_0} \Delta y_1 + \frac{\partial f\,(x_0, y_0, z_0)}{\partial z_0} \Delta z_1 \qquad (6)$$

Therefore, equation (3) can now be expressed as equation (7)

$$f(x_0, y_0, z_0) = \sqrt{(x_s - x_0)^2 + (y_s - y_0)^2 + (z_s - z_0)^2} = \rho_0(t)^s \qquad (7)$$

By receiving ephemeris information from four or more satellites, equation (7) could be achieved for multiple satellites leading to equation 7(a) – (b); with each equation representing the relationship between receiver position and each satellite.

$$f(x_0, y_0, z_0) = \sqrt{(x_s - x_0)^2 + (y_s - y_0)^2 + (z_s - z_0)^2} = \rho_0(t)^{s1} \qquad (7a)$$

. . . .

$$f(x_0, y_0, z_0) = \sqrt{(x_s - x_0)^2 + (y_s - y_0)^2 + (z_s - z_0)^2} = \rho_0(t)^{s4} \qquad (7b)$$

The partial derivatives in equation (6) are as given below;

$$\frac{\partial f\,(x_0, y_0, z_0)}{\partial x_0} = \frac{x^s(t) - x_0}{\rho_0(t)^s} \qquad 8(a)$$

$$\frac{\partial f\,(x_0, y_0, z_0)}{\partial y_0} = \frac{y^s(t) - y_0}{\rho_0(t)^s} \qquad 8(b)$$

$$\frac{\partial f\,(x_0, y_0, z_0)}{\partial z_0} = \frac{z^s(t) - z_0}{\rho_0(t)^s} \qquad 8(c)$$

Substituting equations 8(a) – (c) into equation (6), then;

$$f(x_r, y_r, z_r) = f\left(x_0, y_0, z_0\right) - \frac{x^s(t) - x_0}{\rho_0(t)^s} \Delta x_1 - \frac{y^s(t) - y_0}{\rho_0(t)^s} \Delta y_1 - \frac{z^s(t) - z_0}{\rho_0(t)^s} \Delta z_1 \qquad (9)$$

Again, depending on the number of visible of satellites, equation (9) could be generated for multiple satellites. Putting all into perspective, equation (3) can be re-written as equation (10)

$$\rho_r = {\rho_0}^{s1} - \frac{x^s(t) - x_0}{\rho_0(t)^s} \Delta x_1 - \frac{y^s(t) - y_0}{\rho_0(t)^s} \Delta y_1 - \frac{z^s(t) - z_0}{\rho_0(t)^s} \Delta z_1 \qquad (10)$$

Rearranging and replacing the derivatives with "a", "b" and "c" yields

$$\rho_r - {\rho_0}^{s1} = a{x_r}^1 \Delta x_1 + a{y_r}^1 \Delta y_1 + a{z_r}^1 \Delta z_1 - cdT = l_1 \qquad 11(a)$$

$$\rho_r - {\rho_0}^{2} = a{x_r}^2 \Delta x_1 + a{y_r}^2 \Delta y_1 + a{z_r}^2 \Delta z_1 - cdT = l_2 \qquad 11(b)$$

$$\rho_r - {\rho_0}^{4} = a{x_r}^4 \Delta x_1 + a{y_r}^4 \Delta y_1 + a{z_r}^4 \Delta z_1 - cdT = l_4 \qquad 11(c)$$

Equations 11(a) to (c) can now be solved via ordinary least squares. To do this, we re-arrange equation (11) to bring out the least squares terms as follows;

$$A = \begin{bmatrix} ax_r{}^1 & ay_r{}^1 & az_r{}^1 & -c \\ ax_r{}^2 & ay_r{}^2 & az_r{}^2 & -c \\ ax_r{}^3 & ay_r{}^3 & az_r{}^3 & -c \\ ax_r{}^4 & ay_r{}^4 & az_r{}^4 & -c \end{bmatrix} \qquad 12(a)$$

$$\hat{X} = \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \\ dT \end{bmatrix} \qquad 12(b)$$

$$\hat{L} = \begin{bmatrix} \rho_{r_1} \\ \rho_{r_2} \\ \rho_{r_3} \\ \rho_{r_4} \end{bmatrix} \qquad 12(c)$$

Where:

A = Design matrix (matrix of linear functions of the unknown)

X = Vector of unknowns

$\hat{L}$ = Vector of observations. Must have at least four elements (i.e., four satellites), but in reality will have from 4 to 12 elements depending on the satellite visibility at the time of observation. The excess satellite coverage (where it exceeds 4) provides redundant data for the least squares.

X ca be determined using the ordinary least squares (OLS) formulation as given in equation (13) as follows;

$$X = (A^T P A)^{-1} A^T P L \qquad (13)$$

Where

$P = \sigma_0{}^2 . I$ = Weight

$\sigma_0{}^2$ = Apriori variance of unit weight

I = Identity Matrix

Propagating for errors, then

$$\sum x = (A^T\ A) = \begin{bmatrix} \sigma_x{}^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{x\Delta t} \\ \sigma_{yx} & \sigma_y{}^2 & \sigma_{yz} & \sigma_{y\Delta t} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z{}^2 & \sigma_{z\Delta t} \\ \sigma_{\Delta tx} & \sigma_{\Delta ty} & \sigma_{\Delta tz} & \sigma_{\Delta t}{}^2 \end{bmatrix} \qquad 14(a)$$

14(a) can be converted from the ECEF frame to the Local Topocentric Frame by reducing the dimensionality of the matrix as follows;

$$\sum T = R \sum X\ R^T = \begin{bmatrix} \sigma_x{}^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y{}^2 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z{}^2 \end{bmatrix} \qquad 14(b)$$

Where

$$R = Rotation\ matrix = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{bmatrix}$$

φ = geodetic latitude of GNSS receiver position

λ = geodetic longitude of GNSS receiver position

Then the dilution of precision values (DOP) can be calculated as follows;

$$VDOP = \sigma_z \quad 15(a)$$

$$HDOP = \sqrt{{\sigma_x}^2 + {\sigma_y}^2} \quad 15(b)$$

$$PDOP = \sqrt{{\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2} \quad 15(c)$$

$$TDOP = \sigma_t \quad 15(d)$$

$$GDOP = \sqrt{{\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2 + {\sigma_t}^2} \quad 15(e)$$

3. MATERIALS AND METHODS

3.1 Materials

Since the study is basically a simulation study, the basic material used was the MATLAB (Matric Laboratory) programming software along with some basic add-ons. The MATLAB 2022 version, release "B" was used for this study and the appropriate licenses (student's license) to allow for on-line simulation and processing obtained from "mathworks". The add-ons used along with the MATLAB software as well as other tools used for this study are;

(i) Mapping toolbox
(ii) Aerospace toolbox
(iii) Navigation toolbox
(iv) Simulation Block
(v) Excel software for analysis of differences

3.2 Methods

The following research steps were performed (further described by the work schematics presented in Fig 1) in this study;

1. Simulate the Galileo and GLONASS satellite constellation using the orbit propagator block model in MATLAB

2. Simulate real-time positioning over a specific period of time (24hours) wherein the simulated satellites would interact with a hypothetical receiver and positions determined using the Pseudorange positioning technique using fictitious orbital perturbation values.

3. From the simulations in step 2 above, determine the number of available satellites that can be tracked at a specific location at a time and determine the overall HDOP and VDOP.

4. Statistical analysis to determine the effects of orbital errors on GNSS pseudorange positioning under varying satellite visibility (Number of available satellites) and satellite constellation arrangement (The Galileo and GLONASS constellations used here)

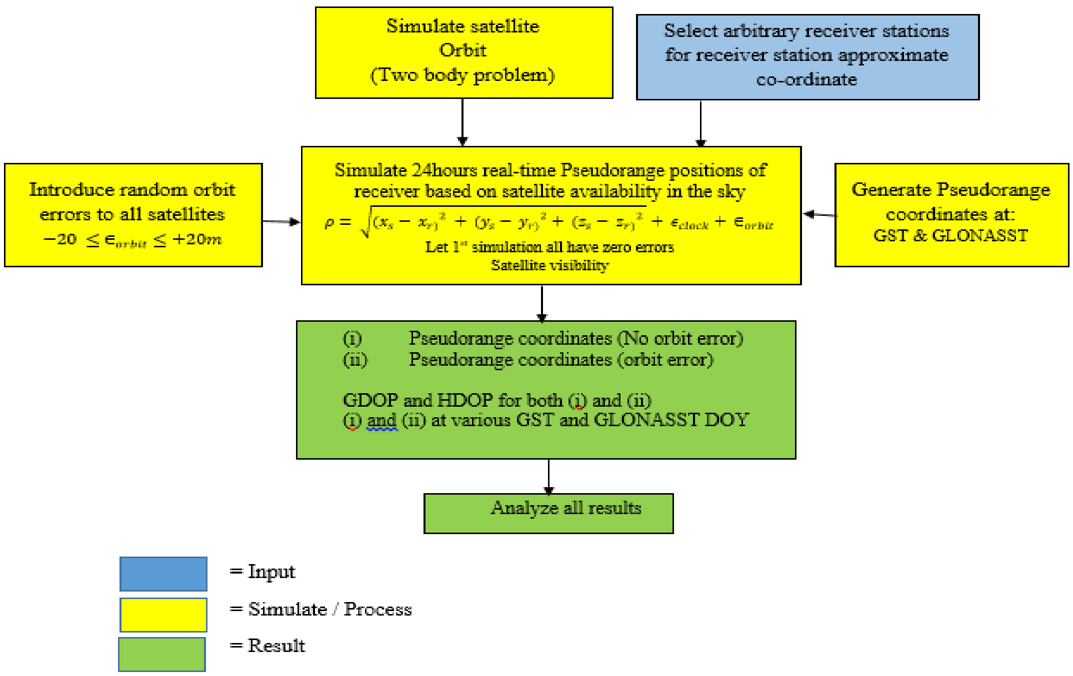


Fig 1: Work schematic

Simulation of the Galileo and GLONASS satellite orbits (constellation modelling) was implemented in MATLAB using the "satellite scenario" function in the aerospace toolbox. The orbital parameters used for the constellation modelling are as given in Table 1.

Table 1: Orbital parameters for the Galileo and GLONASS satellite constellations

S/No	Satellite / Orbital element	Galileo	GLONASS
1	Semi-major axis	23,000km	19,000km
2	Eccentricity	0.0005	0.0019
3	Inclination	56°	65°
4	RAAN (Ω)	350°	357°
5	Argument of Perigee (ω)	320°	233.3819°
6	True Anomaly (t_p)	38.9740°	121.2470°
7	Orbital planes	3	3
8	Time (satellite clock)	GST	GLONASST
9	No of Satellites	26	25

Source: [9]

It should be noted that the satellite position determination from simulated constellations requires three basic coordinate frames. This cross conversion from one frame to the other by MATLAB is one of the essential purposes of purchasing the Navigation and Mapping toolbox add-ons. The coordinate frame conversions involved are;

(i) From Earth-Centered Inertial Frame (ECI) to Earth Centered Earth Fixed (ECEF)

(ii) From ECEF to Tolocentric Local Frame (TLF)

Also, the need for accurate conversion from local time (UT + 1) to GLONASS Time (GLONASST) and Galileo System Time (GST) cannot be overlooked. With these conversions kept in mind, the pseudorange positions as well as the HDOP were simulated using equations 11 – 15. Random orbit error values ranging from -20m to +20m were deliberately introduced. The choice of ±20m range was used based on findings from the works of [10].

RESULTS AND DISCUSSION

4.1 Line of Access (LOA) Analysis

Using the simulated satellite constellations (Fig 2a and b), line of access (LOA) analysis was performed to examine the satellite visibility of the GLONASS and Galileo constellation across Nigeria.

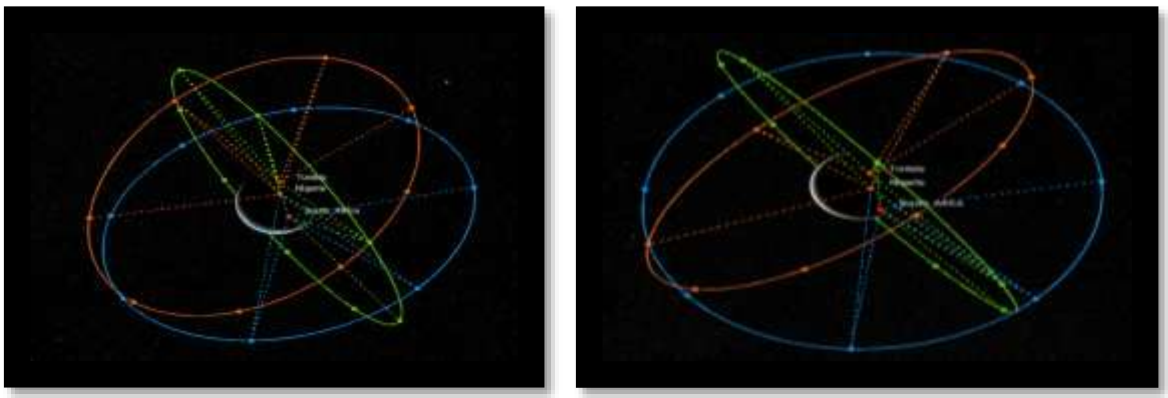


Fig 2: (a) Galileo (b) GLONASS

According to the LOS analysis, the entire country (Nigeria) had sufficient satellite visibility / access for the entire 24hours of the positioning simulation which began from DOY 30th November, 2020 – 1st December, 2020 from 22:23:24hours.

Table 2: LOA across Nigeria (a) Galileo (b) GLONASS

Parameter	Nigeria
1 Total # of intervals	30
2 Total interval time (hrs)	191.1
3 Mean interval length (min)	382.2
4 Mean # of satellites in view	7.9778
5 Min # of satellites in view	6
6 Max # of satellites in view	9

Parameter	Nigeria
1 Total # of intervals	30
2 Total interval time (hrs)	192.25
3 Mean interval length (min)	385.2
4 Mean # of satellites in view	7.9778
5 Min # of satellites in view	6
6 Max # of satellites in view	9

[9]

As seen from Tables 2(a) and (b) for the Galileo and GLONASS satellites, the minimum number of satellites from both constellations available to GNSS users in Nigeria is 6 satellites. Hence in a combined GNSS system, a GNSS receiver within Nigeria will have access to 12 satellites. This is a good situation for GNSS users in Nigeria. It means that receiver units all over the country will always have access to 6 satellites from both the Galileo and GLONASS satellite constellation. A possible reason for this is because of the location of Nigeria and its closeness to the equator. At maximum satellite visibility periods, some locations across the country could have access to 9 satellites from both satellites; making a total of 18 satellites from both Galileo and GLONASS.

4.2 Pseudo-range positioning at various DOY from the Galileo and GLONASS

Based on the simulated Galileo and GLONASS constellations, pseudorange technique was used to determine positions of receiver location in Nigeria at different time epochs. A total of eight position (in NED coordinate system) simulation results were performed for each of the constellations in the order specified below;

(i) NED position of point A at epoch 11/30/22 22:23:24 without orbit error

(ii) NED position of point A at epoch 11/30/22 22:23:24 with orbit error

(iii) NED position of point A at epoch 12/01/22 06:30:32 without orbit error

(iv) NED position of point A at epoch 12/01/22 06:30:32 with orbit error

(v) NED position of point A at epoch 12/01/22 15:30:24 without orbit error

(vi) NED position of point A at epoch 12/01/22 15:30:24 with orbit error

(vii) NED position of point A at epoch 12/01/22 21:00:12 without orbit error

(viii) NED position of point A at epoch 12/01/22 21:00:12 with orbit error

At each determination, the positions and Dilution of precision values were extracted and examined. Table 3 presents a summary of the results obtained for both constellations in the order above.

General assessment of Table 3 indicates slight differences between the results obtained from both constellations for station A and for all the epochs even when no orbit error was introduced. This is presented in Table 4.

Table 3: Summary of position simulation for both constellations

	Position (NED) in Topocentric system(m)								Residual (Error)m			
S/No	North	East	Down	Constellation	DOY / Time	No_satellites	HDOP	max sate. Wobble (m)	North	East	Down	Total Horz error (m)
1	6236.786	950.330	999.678	Galileo	11/30/2022 22:23:24	8	0.9816	0	Base data			

2	6237.489	950.438	999.793	Galileo	11/30/2022 22:23:24	8	0.9816	20	0.702	0.108	0.115	0.711
3	6237.120	951.907	1001.349	Galileo	1/12/2022 6:30:32	8	0.9876	0	Base data			
4	6236.511	951.815	1001.252	Galileo	1/12/2022 6:30:32	8	0.9876	20	-0.609	-0.093	-0.097	0.616
5	6235.674	951.686	1001.115	Galileo	1/12/2022 15:30:24	9	0.88	0	Base data			
6	6235.606	951.675	1001.102	Galileo	1/12/2022 15:30:24	9	0.88	20	-0.068	-0.012	-0.012	0.069
7	6236.131	951.755	1001.189	Galileo	1/12/2022 21:00:12	7	0.9934	0	Base data			
8	6238.157	952.065	1001.518	Galileo	1/12/2022 21:00:12	7	0.9934	20	2.026	0.310	0.330	2.050
9	6236.353	951.789	1001.225	GLONASS	11/30/2022 22:23:24	8	0.8736	0	Base data			
10	6237.279	951.931	1001.375	GLONASS	11/30/2022 22:23:24	8	0.8736	20	0.925	0.142	0.151	0.936
11	6237.120	951.907	1001.349	GLONASS	1/12/2022 6:30:32	8	0.9318	0	Base data			
12	6236.510	951.815	1001.252	GLONASS	1/12/2022 6:30:32	8	0.9318	20	-0.610	-0.093	-0.096	0.617
13	6235.674	951.686	1001.115	GLONASS	1/12/2022 15:30:24	9	0.9121	0	Base data			
14	6235.621	951.700	1001.102	GLONASS	1/12/2022 15:30:24	9	0.9121	20	-0.053	0.014	-0.012	0.055
15	6236.411	951.799	1001.234	GLONASS	1/12/2022 21:00:12	7	0.9724	0	Base data			
16	6238.321	952.409	1001.563	GLONASS	1/12/2022 21:00:12	7	0.9724	20	1.910	0.610	0.330	2.005

Table 4: Differences in position obtained at different epochs when no orbit error is introduced

GPS Time and DOY	Constellation	Analysis	North	East	Down	Remarks
11/30/22 10:23 PM		Base				Within same constellation
1/12/22 6:30 AM	Galileo	2nd sim - 1st sim	0.334	1.577	1.670	

1/12/22 3:30 PM	Galileo	3rd sim - 1st sim	-1.112	1.356	1.436	
1/12/2022 21:00:12 PM	Galileo	4th sim - 1st sim	-0.655	1.425	1.510	
11/30/22 10:23 PM	GLONASS - Galileo	1st Galileo - 1st GLONASS	-0.433	1.459	1.546	Across constellations
1/12/22 6:30 AM	GLONASS	2nd sim - 1st sim	0.767	0.118	0.124	
1/12/22 3:30 PM	GLONASS	3rd sim - 1st sim	-0.679	-0.103	-0.110	Within same constellation
1/12/2022 21:00:12 PM	GLONASS	4th sim - 1st sim	0.057	0.010	0.009	

From Table 4, it is observed that the simulated positional differences range between -1.112m to +1.510m across for the East, North and Up even when no orbit error is induced. It should be noted that each simulation for pseudorange position gives a single value for that instant of time specified. Therefore, this observed differences confirm existing known claims that the GNSS absolute positioning (if not subjected to any kind of refinement) is accurate to within only 2m [11]. For this reason, it is often recommended that longer occupation time should be adopted in GNSS absolute point positioning i.e the longer the occupation time, the better the expected results.

4.3 Discussion of Results

The study identified that introduction of orbital errors into the pseudorange position determination resulted in coordinate residual values (positional errors) ranging from -0.61m to +2.026m in the East, North and Down directions. This value as obtained is consistent with existing literature which states that maximum orbital errors in GNSS positioning is 2.5m [4]. This is further illustrated by Tables 5(a) and (b) where it is seen that orbital errors affect pseudorange positioning. The facts presented in Tables 5(a) and (b) are further buttressed by Figures 3(a) – (c)

Table 5: Orbital effect on positioning

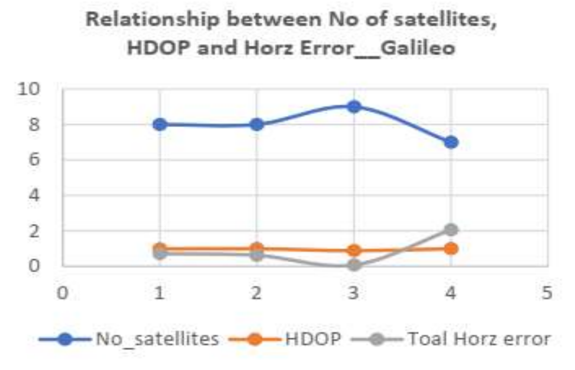
(a) Galileo

Constellation	#_satellites	HDOP	Toal Horz error
Galileo	8	0.9816	0.711
Galileo	8	0.9876	0.616
Galileo	9	0.88	0.069
Galileo	7	0.9934	2.050

(b) GLONASS

Constellation	#_satellite	HDOP	Toal Horz error
GLONASS	8	0.8736	0.936
GLONASS	8	0.9318	0.617
GLONASS	9	0.9121	0.055
GLONASS	7	0.9724	2.005

On the effect of orbital errors on pseudorange positioning in relation to satellite availability (GDOP) and (HDOP); it was discovered that more the number of available satellites, the higher the HDOP value and by extension the lower the observed horizontal error. Graphical plot of the total residual in horizontal position determination as obtained from both the Galileo and GLONASS constellations is presented in Fig 3(a) - (d)



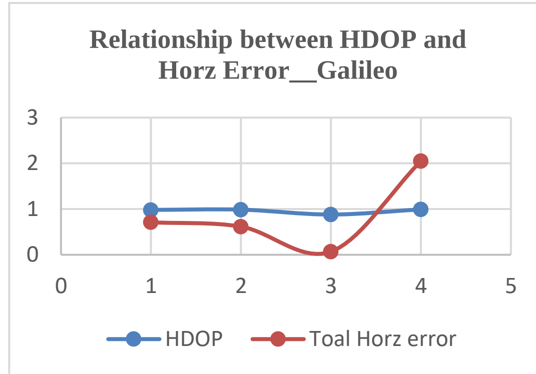
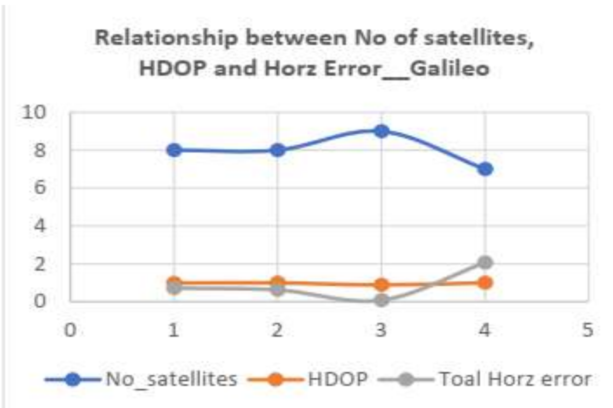


Fig 3: Overlay of (a) No of satellites, positional accuracy and DOP (b) positional accuracy and DOP



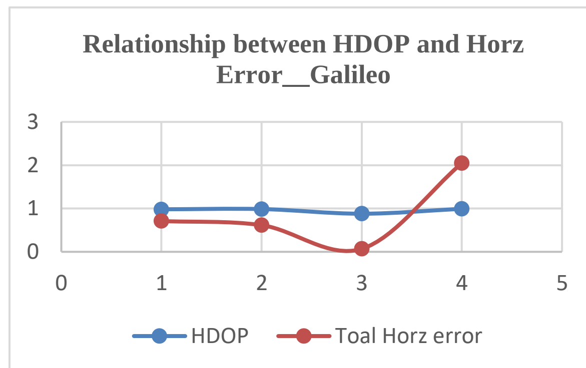


Fig 3: Overlay of (c) No of satellites, positional accuracy and DOP (d) positional accuracy and DOP

This means that the effect of orbital errors on pseudorange positioning decreases with increasing satellite availability and also higher HDOP. Although this trend is encouraging for single point absolute GNSS position determination users, it should be noted that since the pseudorange least squares solution is more like an average, the overall effect of orbital errors on positions can only be minimized substantially if an automatic filtering algorithm is embedded in the GNSS receiver [12]. This is because even when large number of satellites are tracked, the satellites with larger orbit errors at the time of observation will not be excluded by the pseudorange positioning. Now, since the fundamental theoretical assumption of a least squares estimate is that there are only random errors in the observation, such outlier observation (positional noise larger than others due to large orbital error) will similarly be averaged in the solution. This therefore may cancel out the cleansing effect that the large number of satellites might produce.

CONCLUSION

The effects of orbital errors in GNSS pseudorange positioning has been critically examined in this study using simulated satellite constellations. The MATLAB software was used to implement the simulation of both the satellite orbit, the pseudorange positioning without orbital errors and the same with orbital errors at different epochs. Conventional (established) methods for all the involved methods were used and implemented in the MATLAB software environment. The results obtained have been presented and analysed and the following inferences drawn as tenable conclusions based on the study outcomes;

That orbital errors in satellite constellation affect pseudorange position determination by varying magnitude at different time epochs depending on the number of available satellites at the time of observation. Based on a simulated wobble amount of ±20m, the effects of orbital error on pseudorange positions range between -0.61m to +2.026m. This value is consistent with earlier literature and can be found in many geodetic textbooks [13]

An increased Geometric Dilution of Precision (GDOP), which is inversely related to the spatial distribution of satellites relative to the receiver, can significantly reduce the impact of orbital errors in pseudorange positioning. This is because a higher GDOP value indicates poor satellite geometry, leading to greater sensitivity to measurement errors, including those from orbital inaccuracies [14].

In as much as pseudorange positioning technique relies on least-squares (LS) estimation, the influence of orbital errors varies among satellites within and across constellations. The LS estimator, designed to minimize the sum of squared residuals, does not inherently distinguish between satellites with varying error magnitudes. Consequently, satellites with larger orbital errors are not specifically excluded; instead, their errors are averaged with observations from other satellites, potentially degrading the overall positioning accuracy.

FUNDING

There was no funding for this research

CREDIT- AUTHOR CONTRIBUTION

JOO:	Designed the research concept
JOO & TOO:	Wrote the MATLAB codes and performed the position simulations and analysis in MATLAB
VCN & SOB:	Contributed substantially to the discussion of the results; plotting the graphs and writing the discussions
JOO & AOS:	Proofread the final manuscript and edited it into the Journal’s template

DECLARATION OF COMPETING INTERESTS

The authors have no conflict of interest to declare that are relevant to this article

DATA AVAILABILITY

In as much as the data was based on satellite constellation simulations, the data used can be replicated using the publicly assessible satellite constellation parameters for the Galileo and GLONASS GNSS satellite constellations. In variably, no additional field data collection was done in this study.

Use of Generative AI and AI-Assisted Technologies

No generative AI or AI-assisted technologies were employed in the preparation of this manuscript.

REFERENCES

[1] Gallon, E., Joerger, M., & Pervan, B: *Robust modeling of GNSS orbit and clock error dynamics*. NAVIGATION: Journal of the Institute of Navigation, 69 (4) 2022. https://doi.org/10.33012/navi.539

[2] Yidong, L., Xiaolei, D., Xiaopeng, G., Chenglong, L., Yun, Q., Yang, L., Shengfeng, G.: *A review of real-time multi-GNSS precise orbit determination based on the filter method*. Satellite Navigation 3 (15), 2022. https://doi.org/10.1186/s43020-022-00075-1.

[3] Van Sickle, J: *GPS for Land Surveyors*, 4th Ed. CRC Press, USA 2015. (ISBN: 978-1-4665-8310-8).

[4] Jeffery, C: An introduction to GNSS: GPS, GLONASS, BeiDou, Galileo and other Global Navigation Satellite Systems. Calgary Alberta, Canada: NovAtel, 2015.

[5] Maciej, K., Rolf, D., Arturo, V., & Adrian, J.: *Propagation of satellite orbit modelling deficiencies into the global GNSS solutions – simulation-based study*. EGU General Assembly 2021. EGU General assembly, https://doi.org/10.5194/egusphere-egu21-6341.

[6] Dai, G., Chen, X., Zuo, M., Peng, L., Wang, M., & Song, Z: *The Influence of Orbital Element Error on Satellite Coverage Calculation*. Hindawi, International Journal of Aerospace Engineering, Volume 2018, Article ID 7547128, 13 pages.

[7] Meng, Y., Fan, S., Song, X., Lu, J., & Su, C: *Orbit Determination and Error Analysis Based on GNSS Crosslink Ranging Observations*. In J. Sun, W. Jiao, H. Wu, & M. Lu, *China Satellite Navigation Conference* (CSNC) 2014 Proceedings: Volume III. Lecture Notes in Electrical Engineering, vol 305. https://doi.org/10.1007/978-3-642-54740-9_31. Berlin, Heidelberg.: Springers.

[8] Ogaja, C: Applied GPS for Engineers and Project Managers. Califonia, 2011. American Society of Civil Engineers.

[9] NASA. www.heavens-above.com. Retrieved from www.heavens-above.com: https://www.heavens-above.com/orbit.aspx?satid=41175. 20th Dec. 2022

[10] Sośnica, K., Bury, G., Zajdel, R., Ventura Traveset, J., & Mendes, L.: *GPS, GLONASS, and Galileo orbit geometry variations caused by general relativity focusing on Galileo in eccentric orbits*. GPS Solutions 26 (5), 2022.

[11] GPS.Gov. (2022). The Global Positionng System. GPS.gov. A journal of the National Coordination Office for Space-Based Positioning, Navigation, and Timing.

[12] Jgouta, M., & Nsiri, B: *Statistical estimation of GNSS pseudo-range errors*. Procedia Computer Science, 73 (2015) - Journal of the International Conference on Advanced Wireless, Information, and Communication, 258 – 265.

[13] FAA. (2022, December). www.faa.gov/GPS. Retrieved from www.faa.gov: https://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/techops/navservices/gnss/gps

[14] Langley, R. B: *Dilution of precision*. GPS World, 1999. Retrieved from https://en.wikipedia.org/wiki/Dilution_of_precision_%28navigation%29