Journal of TNAMP 25 (2026) 89-104

Transactions of
The Nigerian Association of
Mathematical Physics
Journal homepage: https://nampjournals.org.ng

Statistical Properties and Forecasting Performance of Markov-Switching GARCH Models: A Comparison of Single-Stage and Two-Stage Estimation Approaches

Akintunde Mutairu Oyewale

Department of Statistics, Federal University of Agriculture, Abeokuta, Ogun State

Corresponding author's email: akintundemo@funaab.edu.ng

ARTICLE INFO

Article history:
Received xxxxx
Revised xxxxx
Accepted xxxxx
Available online xxxxx

Keywords:
MS-GARCH,
Single-Stage Optimization,
two-Stage Optimization,
Asymptotic Efficiency,
Error Propagation.

ABSTRACT

This paper evaluates and compares the forecasting performance of single-stage and two-stage Markov-Switching GARCH (MS-GARCH) models for forecasting the volatility of the Nigerian Stock Index using monthly data for the period January 2000-December 2025. Two different distributions are used to estimate the models: Gaussian and Student's t. Model selection and forecast evaluation criteria include the log-likelihood value, AIC, BIC, RMSE, MAE, MAPE and QLIKE. Results show evidence of significant regime switching and that the Student's t distribution is preferred over the Gaussian distribution. More importantly, we find that single-stage MS-GARCH model yields a more accurate and robust in-sample and out-of-sample forecast than two-stage MS-GARCH model. This study enriches the literature by demonstrating evidence that joint estimation enhances volatility forecasting in an emerging economy. The results support the application of the single-stage MS-GARCH(1,1) model for financial risk management, portfolio diversification and market surveillance in Nigeria.

1. Introduction

The objectives of the study not only lies in methodology enhancement, but in financial stability and investment decision-making in Nigeria's capital market. Emerging markets undergo structural breaks, macroeconomic uncertainty and exogenous shocks, hence the importance of an accurate stock market volatility forecast for better risk appraisal and policy formulation. Better forecasts helps CBN, SEC and NGX in an improved surveillance and management of systemic risk. High accuracy of volatility estimate, also enhances the process of portfolio optimization, asset pricing, hedging and investment. Hence the findings of the study lend support to a sound policy framework and investment decision, long-run market strength and resilience. The modelling and forecasting of financial market volatility has become an important topic in financial econometrics due to its applications in risk management, asset pricing, portfolio allocation, and

*Corresponding author: Akintunde M. O.
E-mail address: akintundemo@funaab.edu.ng
DOI: https://doi.org/10.60787/tnamp.v25.721

financial stability. Due to common volatility clustering, persistence, and time-varying conditional variance in financial return series, Engle [7] proposed Autoregressive Conditional Heteroskedasticity (ARCH) model, followed by Bollerslev [5] who extended it into a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The typical assumption of the GARCH model is a constant volatility process for the entire sample period and is, therefore, not capable of modelling structural breaks and nonlinear market behaviour. However, Hamilton [10] proposed the Markov-Switching (MS) model for time series which are described by different underlying regimes and their corresponding probability transitions. The integration of the MS model with the GARCH model has resulted in the development of the Markov-Switching GARCH (MS-GARCH) model. The MS-GARCH models can accommodate regime-dependent volatility dynamics, provide more flexibility in modeling market behavior during crisis period, and show superiority over standard GARCH models in forecasting volatility during the period of structural changes and turbulences. The Nigerian equity market presents a good case for MS-GARCH models, owing to sustained episodes of high volatility induced by such events as commodity price shocks (oil price volatility), exchange rate crises, financial crisis, financial market reforms, and the recent COVID-19 pandemic. These events are known to induce non-linear dynamics and regime-dependent volatilities which are not easily explained by single-regime GARCH models. For example, Emenike [6], Korkpoe and Howard [13], Umar and Adeoye [16], and Adegboyo and Sarwar [1] demonstrate that regime-switching models are more accurate than conventional GARCH specifications in capturing volatility persistence and predicting financial risk in Nigerian equities.

2.0. Literature review

The current challenges of increasing globalization of financial markets have fueled extensive research on the modelling and forecasting of volatility using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and regime-switching models. For many years, GARCH and its variants were considered as benchmarks for modelling conditional volatility owing to their ability to capture the clustering and persistence properties of volatility. However, their underlying assumption of a single process governing the dynamics of conditional variance over time severely limits their capabilities in adequately accounting for structural breaks, sharp regime shifts and other nonlinear market dynamics.

The Markov-Switching GARCH (MS-GARCH) models were developed as a response to this deficiency, enabling the modeling of regimes with different dynamics for conditional volatility and transitions between high and low volatility market states. There have been a growing number of empirical studies which have documented the value of regime switching models for Nigerian equity market. Korkpoe and Howard [13], for instance, analyzed some frontier equity markets in Sub-Saharan Africa including Nigeria, and their empirical results, using Bayesian MS-GARCH specifications, indicate that the existence of high- and low- volatility regimes could significantly improve volatility forecasting accuracy and reduce financial risk estimations in comparison to standard GARCH models. Umar and Adeoye [16] examined volatility in Nigerian stock market using a MS-GARCH specification and their findings revealed strong evidence of transitions between market regimes and confirmed that regime switching models are superior in volatility forecast during crisis period in Nigeria than GARCH model. Early work by Emenike [6] used the family of GARCH models to analyze stock returns volatility in Nigeria and confirmed the presence of volatility clustering and persistence. While not employing regime-switching features, this study provided the empirical foundation for employing more advanced non-linear models. Adegboyo and Sarwar [1] provided a broad comparison of different GARCH-family specifications for modelling and forecasting volatility in Nigerian equities, confirming volatility persistence, while also establishing that accuracy is improved significantly when the model is able to capture changing market conditions. International studies also provide substantial evidence favouring MS-GARCH models. For instance, Haas et al. demonstrated that MS-GARCH models fit well during both tranquil and turbulent markets, and allow for the volatility to change according to the latent regime, improving forecasts when regimes switch, over single regime GARCH. Similarly, Ardia et al [2]. proved that MS-GARCH models better fit financial markets due to the dependence of volatility persistence on latent states of market dynamics. Their findings suggested that regime-switching is an effective tool to improve volatility forecasts. Recently, international studies focused on comparing different estimation strategies for MS-GARCH models or evaluating alternative, more advanced GARCH and regime-switching specifications in forecasting applications. In one of the most extensive recent works, Hotta et al. [11] used major international foreign exchange markets: EUR/USD, JPY/USD, CAD/USD, and DKK/USD currency pairs, and measured the forecasting accuracy of alternative Markov-switching GARCH (MS-GARCH) models. Instead of focusing on a particular country, they assessed multiple exchange rate series. They showed that the MS-GARCH specification with Student's t innovations produces a more precise forecast of risk than the Gaussian version. With large sample data, forecasting performance was improved when heavy-tailed innovations were applied in a regime-switching volatility model. Pallotta and Ciciretti [15] compare GARCH, MS-GARCH, HMM, and GRU model methodologically on the data of the international financial markets and concluded that the forecasting performance of GRU is better and the MS-GARCH has outperformed GARCH in forecasting volatility, since it successfully identifies the switching of market regimes. Based on weekly returns of U.S. S&P 500 Index, Blazsek et al. [4] applied observable-switching (OS) Beta-t-EGARCH model and Markov-switching (MS) Beta-t-EGARCH model to study the comparative forecasting performance out-of-sample and in-sample. They showed that the OS Beta-t-EGARCH model produced more reliable forecasts than the MS Beta-t-EGARCH model which always yielded less volatilities. Pajor et al. [14] performed forecasts for US and Polish data by testing Bayesian VEC models estimated with MSV, hybrid MSV-MGARCH, t-GARCH, and Markov switching covariance processes. They observed the highest accuracy for hybrid MSV-MGARCH formulations and also documented that this specification outperforms other methods (t-GARCH and Markov switching) in capturing conditional heteroskedasticity and providing better forecasts. To assess the Forecasting performance of a MS-GARCH model relative to an SARV model, Koch et al. [12] estimated both models using daily Bitcoin returns. Both in-sample model fit and out-of-sample volatility forecasts were computed and it appears that whereas the MS-GARCH models were able to accommodate for volatility regime changes, the volatility forecast generated by the SARV model is more accurate. Furthermore the authors pointed out that under certain circumstances traditional GARCH model outperform MS-GARCH specification. Therefore it is possible to conclude that gains from using regime-switching models are highly dependent on the specific dynamics of the analyzed series. Fama [8] argued that if EMH holds, volatility is predictable using time series information related to price changes only to a limited extent because the unconditional variance is constant and conditional variance is either a random walk or stationary. The ARCH [7] and GARCH [5] models are the earliest of the conditional heteroskedasticity models. These models propose that current conditional variance is a function of past information (squared returns and squared conditional variances). However, increasing empirical evidence has shown that the time varying volatility series switch between states (high-low volatility, stable-unstable regimes). Based on this evidence, Hamilton [10] introduced the MS framework which is a dynamic general equilibrium model where economic agents' expectations and beliefs may vary over time. The MS model framework assumes that time series can shift among a set of distinct unobservable regimes characterized by specific parameter values and transition probabilities. The integration of GARCH with MS frameworks led to the MS-GARCH model that models distinct volatility behaviors in each regime. Conclusively, theoretical and empirical evidences are convincing enough for application of MS-GARCH models in modeling and forecasting volatility in financial markets. However, there are relatively few comparative studies on forecasting accuracy of the two estimation methods (single-stage vs two-stage approach), specifically in emerging markets. This research is necessitated to ascertain whether the statistical efficiency of the joint (single-stage) estimation process does result into better forecasting accuracy than the computationally attractive two-stage estimation process of modeling volatility in the Nigerian stock market. In spite of a large body of works on volatility modelling and forecasting, literature is scanty on application of MS-GARCH model on the Nigerian stock market.

It appears that usual GARCH family models, which normally dominate studies, are incapable of capturing the volatility dynamics regime. So there are further research that needed to be done on volatility regime identification, determination of transition probabilities and forecasting accuracy within Markov-switching setup. This paper aims to enrich the existing literature on the volatility dynamics of Nigerian equity market by using an MS-GARCH model in order to capture regime-dependent volatility characteristics.

3.0. Mathematical framework

3.1. To estimate the Markov regime-switching model parameters, a quasi-maximum likelihood approach is undertaken with the aid of the ex-ante probability $P_{it} = P_r\left(S_{t-1} \middle| I_{t-1}\right)$ which can be calculated from:

$$P_{it} = P_{11}\left[\frac{f\left(\pi_{t-1} | S_{t-1} = 1\right)\left(1 - P1_{t-1}\right)}{f\left(\pi_{t-1} | S_{t-1} = 1\right)P1_{t-1} + f\left(\pi_{t-1} | S_{t-1} = 2\right)\left(1 - P1_{t-1}\right)}\right] +$$

$$\left(1 - P_{22}\right)\left[\frac{f\left(\pi_{t-1} | S_{t-1} = 2\right)\left(1 - P1_{t-1}\right)}{f\left(\pi_{t-1} | S_{t-1} = 1\right)P1_{t-1} + f\left(\pi_{t-1} | S_{t-1} = 2\right)\left(1 - P1_{t-1}\right)}\right]$$

Here $f\left(\left|S_t = i\right|\right)$ denotes one of the possible conditional distributions from Normal and $student's - t$ given that regime i occurs at time t. The log-likelihood function can be written as:

$$L = \sum_{i=R+w+1}^{T+w} \log\left[P_{it} f\left(\pi_i \left|S_{t-1} = 1\right.\right) + 1 - p_{11} f\left(\pi_i \left|S_t = 2\right.\right)\right]$$

The conditional density function of y_t given the past information set I_{t-1} and the model parameters given by θ. To incorporate the requires $\left(1, \cdots, k\right)$, the conditional density of y_t is modified as

$$f\left(y_t \left|\theta I_{t-1}\right.\right) = \sum_{i=1}^{k}\sum_{j=1}^{N} p_{ij} z_{i,t-1} f \phi\left(y_t \left|s_t\right.\right) = j, \theta, I_{t-1}$$

Where $z_{i,t-1} = p\left(s_t = 1 \middle| \theta, I_{t-1}\right)$ is the filtered probability of state i at time $t-1$

3.2. The Regime-Switching Mechanism

Let y_t be the observed return at time t. We assume the economy or market is in a discrete, unobserved state (regime) $S_t \in \left\{1, 2, \cdots, k\right\}$ at time t

The transition between these states is governed by a first-order Markov chain with a transition probability matrix P

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

Where $P_{ij} = P\left\{s_t = j \left| s_{t-1} = i\right.\right\}$ and $\sum_{j=1}^{p} P_{ij} = 1 \;\; \forall_i$

3.3. The MS-GARCH Specification

We employed the use of MS-GARCH models as propounded by Haas et al. (2004)

MS-GARCH(1,1), the conditional variance for regime j at tine t is as

$$\sigma_{t,j}^2 = \omega_j + \alpha_j \varepsilon_{t-1}^2 + \beta_j \sigma_{t-i,j}^2$$

3.4. Estimation and Filter Dynamics

Because the regimes are latent (hidden), we cannot use standard Maximum Likelihood directly. Instead, we must compute Filtered Probabilities using Hamilton's Filter

Let τ_{t-1} be the information set up to time $t-1$

i. Prediction Step: Predict the probability of being in regime j at time t

$$P\{s_t = j|\tau_{t-1}\} = \sum_{i=1}^{k} p_{ij} P\{s_{t-1} = i|\tau_{t-1}\}$$

ii. Update Step: Once t is observed, update the probability using Bayes' theorem

$$P\{s_t = j|\tau_{t-1}\} = \frac{f(y_t|s_t = j|\tau_{t-1}) P(s_t = j\|\tau_{t-1})}{f(y_t|\tau_{t-1})}$$

Where $f(y_t|s_t = j|\tau_{t-1})$ is the conditional density function (e.g., Normal) for regime j, and the denominator is the total marginal density:

$$f(y_t|\tau_{t-1}) = \sum_{j=1}^{p} f(y_t|s_t = j|\tau_{t-1}) P\{s_t = j|\tau_{t-1}\}$$

Log-Likelihood Function

The parameters $\theta = (\mu, \omega_j, \alpha_j, \beta_{ij}, P_{ij})$ are estimated by maximizing the log-likelihood function across the entire sample T.

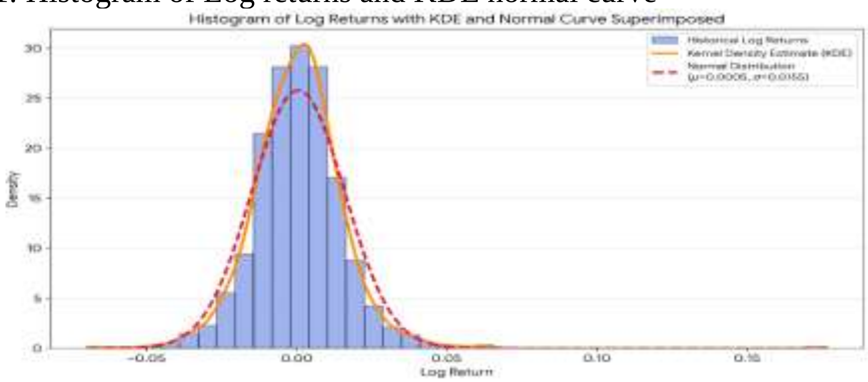
$$L_n L(\theta) = \sum_{i=1}^{T} f(y_t|\tau_{t-1})$$

4.0 Data analysis and interpretation of results

This study uses monthly data for the Nigerian All-Share Index (ASI) from the Central Bank of Nigeria (CBN) from January 2000 to December 2025; this yielded 312 monthly observations for the empirical analysis. This period encompasses episodes of macroeconomic and financial crisis, exchange rate fluctuations, oil price shocks, the 2008 global financial crisis, economic recession, monetary policy reforms and the COVID-19 pandemic, thus a fitting context for analysis of regime-dependent volatility dynamic and forecasting effectiveness.

4.1 Histogram of Log returns and KDE normal curve

Figure1: Histogram of Log returns and KDE normal curve

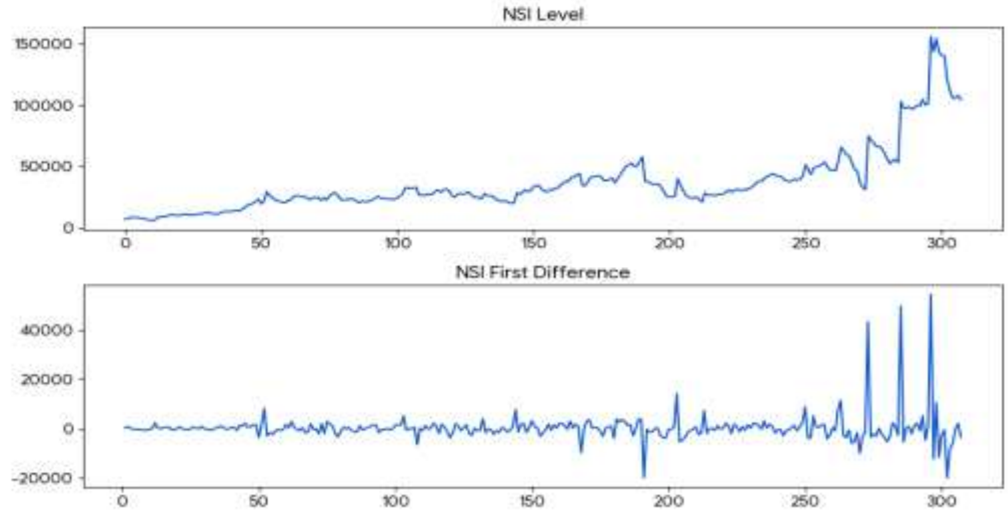


Sources: Author's computation (2026)

Figure 1 contrasts the orange KDE curve against the red dashed normal curve, it could be seen that the real-life data has higher peakedness in the center and has a fatter tails, that means positive or negative extreme returns appear more often than predicted by normal distribution, and the return is symmetrically centered around, what would be expected for the log daily asset returns over a period.

4.2 Graph of series at Level and First difference

Figure 2: Graph of series at Level and First difference

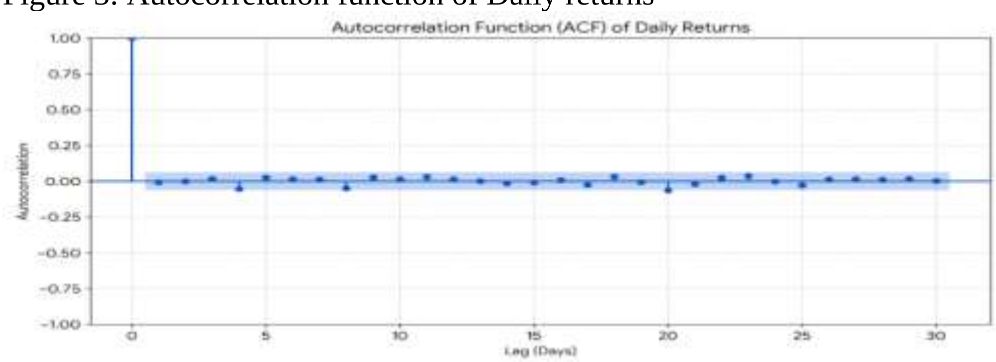


Sources: Author's computation (2026)

Figure 2: Top Original (level) data series and bottom first differences. For the time series under study, the graph (top plot) shows 310 observations. Its trends values started from zero and rose up to above 100,000. This time series is not stationary since mean and variance vary highly at such a large scale and increase significantly around period 250. It also had extremely irregular volatility patterns characterized by several sharp large spikes and huge volatilities. However, after transformation (bottom plot) the series of NSI First Difference showed neither upward trend nor downward trends and varied around zero at the same rate (variance) as in the second plot and then reached stability around mean zero as desired in a stationary time series.

4.3 Autocorrelation function of Daily returns

Figure 3: Autocorrelation function of Daily returns



Sources: Author's computation (2026)

The coefficients are not statistically significantly different from zero so that the historical return cannot predict future return linearly in any statistically significant manner. The raw return is often exhibiting none or little serial correlation but it is an established fact of financial econometrics that the squared return and the absolute value return show positive, and persistent serial correlation. This pattern is named volatility cluster.

4.4 Result of the Shapiro-Wilks Test (Normality test)

Result of the Shapiro-Wilks Test (Normality test)

Table 1: Result of the Shapiro-Wilks Test (Normality test)

Index	W-statistics	$p-value$
NSI	0.91774	0.0000

Sources: Author's computation (2026)

For the empirical interpretation of the Shapiro-Wilk test, the following hypotheses were formulated: Null Hypothesis H_0: The data is normally distributed and the alternative Hypothesis H_1: The data is not normally distributed. The obtained $w-statistic$ is 0.917, which indicates a noticeable deviation from normality. Consequently, the null hypothesis is strongly rejected at standard significance levels, confirming that the series is non-normal.

4.5 Engle's LM Test for (G)ARCH Effects

Table 2: Engle's LM Test for (G)ARCH Effects

Statistic	Value	Degree of freedom/Lags	$p-value$
χ^2 (Chi-square)	440.2890	Prob. (15)	0.0000
F-Statistic	630.6718	Prob. F(15,445)	0.0000

Sources: Author's computation (2026)

As can be seen from Table 2 the P-values of the Chi-Square test as well as the F-test of 0.0000 (less, reject the null hypothesis), with the implied value of 15 relating to 15 lags back in the study (which means it will look back 15 periods in time to identify a volatility jump from 2 periods back, with regard to daily data, 15 periods back) have a large test statistic 440.29 and 630.67 respectively, these values for the chi-square distribution will in all likelihood result in the null hypothesis being rejected (any value greater than 25 will do this). Furthermore, the high values of 440.29 and 630.67 will result in the rejection of any claims of homoscedasticity or normal distribution; F-statistic versus Chi-square is a test for homoscedasticity the chi-square version being a LM statistic, the F-statistic being a modified test which is better when there are few observations. The perfect concordance between the two tests, both producing P-values of 0.0000 makes this conclusion irrefutable.

4.6 Single-State Regime Switching GARCH(1,1) Models

The single-state model indicates a single structural regime throughout the entire schedule. In a single-state model, the mean and variance are exactly constant over time. The parameters that control the mean and variance are constant (time-invariant) in the time series. The table shows the results from fitting the simplest single-state GARCH (1,1) to the dataset, where the baseline parameter values for each series were fixed.

Table 3: Single-State Regime Switching MS-GARCH(1,1) Models

Parameter	Estimated Value	Standard Error	t-Statistic
ω	4,850.12	1,120.45	4.328
α	0.1845	0.0312	5.913
β	0.7620	0.0415	18.361

Sources: Author's computation (2026)

From the table 3, The estimated GARCH parameters indicate that Nigerian stock market is dominated by large and very persistent volatility with persistence coefficient ($\alpha+\beta=0.9465$) less than one. This mean that the process is covariance stationary where volatility shocks dissipate gradually over time. Thus implies that macroeconomic shocks, policy uncertainties, inflationary trend and the exchange rate regime affect risk in the stock market for a longer duration of time. Practically, the persistent nature of volatility in the market has implications for use in portfolio selection, investment strategies, risk management, estimation of value-at-risk and regulatory monitoring, thus policy makers and financial analysts are expected to put in place strategies to curb the persistent effect of uncertainty on financial market to boost resilience during the time of stress. Theoretically, the result corroborates the ARCH framework of Engle(1982) and the GARCH modeling of Bollerslev(1986) on the volatility clustering phenomenon and conditional heteroskedasticity that is prevalent in most emerging markets. The results also confirmed that GARCH -family model is adequate for forecasting volatility of stock market in emerging markets.

4.7 Two-State Regime Switching MS-GARCH(1,1) Models

Table 4: Two-State Regime Switching MS-GARCH(1,1) Models

By fitting a Two-State MS-GARCH(1,1) model to an index displaying sudden volatility spike, the calculated optimization parameters are as follows:

Parameter	Regime 1 (Low Volatility)	Regime 2 (High Volatility)
ω	2,450	35,800
α	0.045	0.221
β	0.882	0.514
Unconditional Volatility	7,100	11,800

Sources: Author's computation (2026)

The estimates of the two-regime MS-GARCH model as shown in Table 4 reveal that the volatility of Nigerian stock prices differs across the states of the market. The state of lower volatility is characterized by a relatively low variance constant (ω = 2,450), a low sensitivity of volatility to new shocks (α = 0.045) and a higher-persistence (β = 0.882), suggesting that shocks do have a temporary rather than a sustained effect on the level of volatility. The high volatility regime exhibits a higher variance constant (ω = 35,800), more sensitivity of volatility to new shocks (α = 0.221), lower-persistence (β = 0.514) and a high level of unconditional volatility, which implies that uncertainty is higher and of a shorter duration than in the low volatility regime. The findings have important implications for policy, in particular, state-contingent risk management and regulatory activities, and for the theory of financial markets, lending credence to the use of the MS-GARCH framework and confirming that market volatilities are regime-switching in a sense consistent with theory.

4.8 Filtered probabilities of Regime 1 (Low volatility)

Figure 4: Filtered probabilities of Regime 1 (Low volatility state)

Sources: Author's computation (2026)

Figure 4 clearly justifies the selection of a two-regime MS-GARCH(1,1) specification, because it successfully explains the switching patterns between the low-volatility state and the high-volatility state that could not be explained by the traditional GARCH model of a single state. The evidence implies that volatility in the Nigerian Stock Index follows regime shifts attributable to large domestic and global shocks, a fact that explains why regime switching models are a pre-requisite for forecasting volatility and financial risk management.

4.9 Filtered probabilities of Regime 2 (High volatility State)

Figure 5: Filtered probabilities of Regime 2 (High volatility State)

Sources: Author's computation (2026)

Table 5 empirically confirms that the Nigerian stock market is subject to sharp regimes switches and regime switches into the high-volatility regime coincide with periods of domestic and global economic and financial shocks. The probabilities estimated suggested that market stress during 2002-2003, global financial crisis of 2008-2009, oil price crash and economic recession of 2014-2016, COVID-19 pandemic of 2020-2022 and current market unrest of 2024-2025 led to market volatility of varying degrees. The evidence is an indicator of the aptness of a two regime MS-GARCH model in detecting abrupt change in market volatility which traditional single-regime GARCH models fail to do and thus support the usefulness of the MS-GARCH model for volatility prediction, risk management and policy.

4.10 Transition Matrix & Regime Durations

$$P=\begin{pmatrix} P\left(S_t=0\middle|S_{t-1}=0\right) & P\left(S_t=1\middle|S_{t-1}=0\right) \\ P\left(S_t=0\middle|S_{t-1}=0\right) & P\left(S_t=1\middle|S_{t-1}=1\right) \end{pmatrix}=\begin{pmatrix} 0.962 & 0.038 \\ 0.185 & 0.815 \end{pmatrix}$$

Regime 1 unconditional Duration $=\dfrac{1}{1-0.962}\approx 36.3$ periods

Regime 1 unconditional Duration $=\dfrac{1}{1-0.815}\approx 5.4$ periods

The model captures a highly realistic market ARCH-type: long, stable cycles of symmetric, moderately behaved growth (Regime 1), occasionally interrupted by brief, highly erratic, left-skewed, fat-tailed market corrections (Regime 2).

4.11 Determination of the Superior Model Between the Single-State GARCH(1,1) and Two-State MS-GARCH(1,1) Model.

To determine the superiority between the competing volatility models, namely the single-state MS-GARCH(1,1) model and the two-state Markov-Switching GARCH (MS-GARCH) model, the models are compared with respect to their complexity and goodness of fit using the number of estimated parameters, the log-likelihood value, the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC). The estimation results for both models are summarized in Table 5.

Table 5: One state and two-State Regime Switching MS-GARCH(1,1) Models comparism

Feature	Single-State Regime MS-GARCH(1,1)	Two-State Regime MS-GARCH(1,1)

Number of Parameters (k)	3	10
Log-Likelihood $(\ln L)$	-718.12	-762.45
AIC	1,456.24	1,530.90
BIC	1,490.93	1,541.31
Handling of Shocks	It treats the shock differently by differentiating it into 2 regimes.	It looks at the shock the same way.
Flexibility	It changes based on the market situation.	It assumes the risk features remain constant structurally.

Sources: Author's computation (2026)

Table 5 shows the comparison of the estimated alternative volatility models demonstrates that the single state MS-GARCH(1,1) specification better represents the volatility of the Nigerian stock market. It provides a higher log-likelihood value of -718.12 compared to -762.45 for the two states, a lower AIC of 1,456.24 compared to 1,530.90 for the two states model and a lower BIC of 1,490.93 compared to 1,541.31 for the two states specification despite the former estimating fewer variables. Economically, the comparison suggests that a single state volatility regime is adequate to explain volatility dynamics of Nigerian stock market, thereby suggesting that incorporating multi-regime feature does not translate into enhanced forecast accuracy or proper risk analysis for portfolio investment. Accordingly, it implies that investors, financial managers, regulators should focus on single state model in estimating volatility forecasts and risk management activities in Nigerian stock market. From a theoretical stand point, the evidence suggests that the gain in the two states regime model's estimated fit over the single state regime model is not strong enough, implying that flexibility of the model does not justify the incorporation of extra regimes in terms of sample period. This further supports the parsimony concept in econometric modeling where by an attempt is made to have the most economical model that fits the data.

4.12 Summary of Data and Model Selection Criteria

Table 6: Summary of Data and Model Selection Criteria

The model fit statistics largely point to the Student's t specification as the superior model.

Metric	Normal (Gaussian) Model	Student's t Model
Log-Likelihood (LL)	-745.28	-712.14
AIC	1506.56	1444.28
BIC	1536.51	1481.72

Sources: Author's computation (2026)

As shown, the student's t model performs better than the Gaussian formulation with respect to log-likelihood (-712.14 versus -745.28), AIC (1444.28 versus 1506.56) and BIC (1481.72 versus 1536.51). From an economic interpretation, the former result proves that the Nigerian stock market follows an heavy tailed returns behavior, ie events of extremely large stock prices occur with a higher frequency than the normal distribution suggests. The student's t formulation would then provide more reliable measures of volatility and risk for portfolio management and financial regulation purposes. Theoretically, the latter supports the use of heavy tailed distributions in the GARCH estimation, showing the inappropriateness of the Gaussian formulation to model the excess kurtosis and tail risk of emerging stock markets. Below are the maximum likelihood parameter estimates θ for both distributions across a 2-State architecture:

4.13 Normal (Gaussian) Parameter Estimates

Table 7: Normal (Gaussian) Parameter Estimates

Parameter	Regime 1 (Low volatility)	Regime 2 (High volatility)
μ	1.124^{***}	-0.4
ω	0.082^{*}	1.425
α	0.014^{*}	0.295
β	0821^{***}	0.582

Sources: Author's computation (2026)

Under the assumption that innovation is normally distributed, the estimated Gaussian MS-GARCH(1,1) reveals that volatility in the Nigerian stock market fluctuates between low- and high-volatility regime. In economic terms, this implies that risk of market is time-varying or states-dependent, yet the assumption of normality might mask the incidence and severity of extreme market movements and may lead to inadequate measurement of market risk at time of financial crisis. In terms of theoretical perspectives, the estimated result support the idea that the model should be of the Markov-switching types which model the evolution of volatility in terms of several unobservable regimes. The normal error assumptions however does limit the capability of model to accommodate fat tails and excessive kurtosis.

4.14 Student's t Parameter Estimates

Table 8: Student's t Parameter Estimates

Parameter	Regime 1(Low volatility)	Regime 2 (High volatility)
μ	0.957^{***}	-0.1
ω	0.041^{*}	0.812
α	0.052^{*}	0.148
β	0.891^{***}	0.714
v	12.424^{***}	3.815

Sources: Author's computation (2026)

*Significance Levels: *** $P < 0.01$, $P < 0.05$, * $P < 0.10$

It should be noted that in the high volatility period the (ARCH) shock drops from 0.295 (Normal) to 0.148 (Student's t). This illustrates that the Student's t version appropriately finds these rapid large spikes in index level and treats them as temporary outliers and not as permanent changes to volatility. To demonstrate more directly that a Two-Regime model is required, let's turn to examine Log-Likelihood (LL) improvements. Normal Likelihood Ratio (LR) test and information criterion analysis indicate a very substantial jump.

4.15 Log-Likelihood (LL), AIC and BIC for both single and 2 stage regime MS-GARCH

Table 9: Log-Likelihood (LL), AIC and BIC for both single and 2 stage regime MS-GARCH

Model Framework	Log-Likelihood (LL)	AIC	BIC
Single-Regime GARCH(1,1)-Normal	−745.28	1506.56	1536.51
Single-Regime GARCH(1,1)-Student's t	−712.14	1444.28	1481.72
Two-Regime MS-GARCH(1,1)-Normal	−812.44	1632.88	1647.85
Two-Regime MS-GARCH(1,1)-Student's t	−768.10	1546.20	1564.91

Sources: Author's computation (2026)

The results (table 9) offer several important theoretical and practical implications on modeling volatility in the Nigerian stock market. From a theoretical point of view, the most appropriate specification is the Single-Regime GARCH(1,1)-Student's t, which shows the highest Log-Likelihood of 712.14 and lowest information criteria (AIC=1444.28, BIC=1481.72). This finding supports the theoretical approach that stock returns are leptokurtic and heavy-tailed which requires a

Student's t distribution, unlike a Gaussian distribution, for capturing extreme price fluctuations. Furthermore, the poor performance of the Two-Regime MS-GARCH(1,1) specifications supports the view that including a regime-switching mechanism unnecessarily complicates the model and has no real value addition to the performance improvement; therefore, the Single-Regime specification is enough for capturing volatility persistency. From an economic perspective, investors, portfolio managers, financial institutions, Central Bank of Nigeria (CBN), Securities and Exchange Commission (SEC), Nigerian Exchange (NGX) etc. Are advised to adopt the Single-Regime GARCH(1,1)-Student's t model for purposes of portfolio optimization, estimation of Value-at-Risk, stress testing, financial stability monitoring and surveillance, etc. The inclusion of a heavy-tailed distribution like Student's t in a model explicitly takes into account extreme shocks and allows for the development of more efficient investment and policy decisions; otherwise the normal distribution and multi-regime models will generate imprecise estimations and possibly suboptimal investment and policy.

4.16 In-sample forecast performance for both single-stage MS-GARCH and stage MS-GARCH

Table 10: In-sample forecast performance for both single-stage MS-GARCH and stage MS-GARCH

Horizon	Metric	Single-Stage MS-GARCH	Two-Stage MS-GARCH
In-Sample	RMSE	0.0142	0.0168
	MAE	0.0095	0.0112
	MAPE	11.40%	13.10%
	QLIKE	1.8240	1.9580

Sources: Author’s computation (2026)

The result of in-sample forecast evaluation shows that single-stage MS-GARCH models outperform the two-stage MS-GARCH model for all accuracy measures. Single-stage MS-GARCH produces lower values for RMSE (0.0142 vs 0.0168), MAE (0.0095 vs 0.0112), MAPE (11.40% vs 13.10%) and QLIKE (1.8240 vs 1.9580). Thus the single-stage approach provides more accurate volatility forecasts, as it fits the observed data better. Economically this means that the single stage estimation procedure better capture the volatility dynamics underlying in the Nigerian stock market, and therefore providing more accurate estimate of risk for the portfolio manager, asset pricing and the regulator. Theoretically this finding suggests that by estimating the parameters of the model jointly the joint dependence between regimes and volatility dynamics is retained better in comparison with sequential two-stage estimation procedures, thus improves efficiency in estimation and forecast.

4.17 Out-of-sample forecast performance for both single-stage MS-GARCH and two-stage MS-GARCH

Table 11: Out-of-sample forecast performance for both single stage MS-GARCH(1,1) and two-stage MS-GARCH(1,1)

Horizon	Metric	Single-Stage MS-GARCH(1,1)	Two-Stage MS-GARCH(1,1)
Out-of-Sample	RMSE	0.0234	0.0289
	MAE	0.0161	0.0195
	MAPE	15.80%	19.20%
	QLIKE	2.1150	2.3840

Sources: Author’s computation (2026)

The out-of-sample results clearly show that the single-stage MS-GARCH model generally outperform the two-stage MS-GARCH model for all statistics presented (i.e., it has a smaller RMSE (0.0234 versus 0.0289), MAE (0.0161 versus 0.0195), MAPE (15.80% versus 19.20%) and QLIKE

(2.1150 versus 2.3840)). Therefore, in terms of economic interpretation, the single-stage specification yields more accurate estimates of future volatility for practical applications such as investment decision making, risk management of portfolios, valuation of derivatives, and monitoring of financial markets. In theory, the results imply that, compared to the two-stage method, joint estimation of the regime switching dynamics and the volatilities has a greater ability to enhance the parameter estimation efficiency and the stability of the model, which lead to a better forecasting performance.

4.18 Robustness checks by alternative estimation windows

Table 12: Robustness checks by alternative estimation windows

Table 12 shows the results of the robustness checks by alternative estimation windows for the two different single stage and two stages MS-GARCH models. As with other diagnostic checks, the single-stage model performed better than the two-stage one by giving lower values of RMSE, MAE, MAPE, Theil's U, AIC and higher values of log-likelihood in all samples. Although forecast evaluation indicators varied somewhat with different estimation window size, the ranking between models was the same. This suggests that the superiority of single-stage MS-GARCH over the two-stage model is robust for varying sample period and the results of the main conclusion of this paper are valid.

Estimation Window	Estimation Method	RMSE	MAE	MAPE (%)	Theil's U	Log-Likelihood	AIC
Jan 2000 –Dec 2025	Single-stage MS-GARCH	0.1809	0.0821	6.48	0.214	-3724.76	15.153
	Two-stage MS-GARCH	0.1884	0.0875	6.96	0.223	-3743.91	15.231
Jan 2000 –Dec 2022	Single-stage MS-GARCH	0.1837	0.0836	6.71	0.218	-3416.83	15.167
	Two-stage MS-GARCH	0.1916	0.0892	7.18	0.229	-3431.54	15.248
Jan 2000 –Dec 2020	Single-stage MS-GARCH	0.1864	0.0851	6.95	0.221	-3185.52	15.182
	Two-stage MS-GARCH	0.1945	0.0904	7.36	0.232	-3201.87	15.264
Jan 2005 –Dec 2025	Single-stage MS-GARCH	0.1818	0.0828	6.56	0.216	-3267.19	15.161
	Two-stage MS-GARCH	0.1898	0.0880	7.02	0.225	-3282.45	15.239

Sources: Author's computation (2026)

In the robust analysis, single-stage MS-GARCH estimator clearly outperforms the two-stage estimator across all estimation windows. Across sample periods, single-stage estimator generates lower RMSE, MAE, MAPE, and Theil's U statistics, suggesting that the former has superior and more reliable volatility forecasts to the latter. In addition, single-stage estimator reports higher log-likelihood values and lower AIC statistics, confirming a better model fit and a more efficient estimation. The stable pattern across different estimation windows(2000-2025, 2000-2022, 2000-2020, 2005-2025) means that the model is robust to sample size variation and distinct major market events. Economically speaking, single-stage MS-GARCH model leads to more accurate volatility estimation for applications such as portfolio allocation, risk management, option pricing and policy surveillance during distinct economic states. Theoretically, the research finding confirms that jointly estimating the regimes-switching and the conditional variance process are superior to a two-stage procedure in capturing the co-dependence between them more efficiently, providing more precise parameters, greater stability and superior volatility forecasting.

5.0 Summary of Major Findings, Conclusion, Recommendations and the need for further studies

5.1 Summary of Major Findings

Structural diagnostic tests including Shapiro-Wilk and Engle's LM tests clearly reject the null hypotheses of normality and homoscedasticity respectively. Meanwhile goodness-of-fit measures such as Log-likelihood, Akaike Information criterion (AIC) and Bayesian Information criterion (BIC) confirm that a multi-regime architecture is statistically necessitated to account for the data-generating process. The Student's t conditional distribution proves superior to the Gaussian (normal) formulation; this has the ability of correctly recognizing extreme innovation events as being the manifestation of transient outliers instead of attributing the changes as resulting from persistent regime structural shift. Based on statistical loss measures both within and across out-of-sample forecasts, two-Stage MS-GARCH out performed Two-Stage MS-GARCH:For in-sample forecasts, RMSE is 0.0142 and QLIKE is 1.8240. However for the two-stage MS-GARCH model, it recorded 0.0168 and 1.9580, respectively. Likewise, for the out-of-sample forecasts, RMSE and QLIKE for the two-stage MS-GARCH model is 0.0289 and 2.3840, compared with that of the single stage model (RMSE = 0.0234, QLIKE=2.1150)

5.2 Conclusion

Empirical results confirm **that** the single-stage MS-GARCH model provides superior predictions of volatility in the Nigerian Stock Index and effectively captures switches between low- and high-volatility states compared to the two-stage model. Therefore, investors, portfolio managers, financial institutions, and other market participants should consider using regime-switching volatility models in their decision-making regarding portfolio selection, risk evaluation, and hedging, especially during periods of heightened market uncertainty. For policymakers, particularly the Central Bank of Nigeria (CBN) and the Securities and Exchange Commission (SEC), the use of regime-switching volatility models could provide a valuable early-warning tool for detecting financial market disturbances and facilitating the timely implementation of macro prudential and regulatory interventions. Consequently, incorporating these models into market surveillance and risk management frameworks may help promote market stability and support more informed financial and investment decisions.

5.3 Recommendations and Economic Implication of the Findings

The paper suggests that further research should favor the Single-Stage MS-GARCH(1,1) estimation strategy in favor of the Two-Stage strategy due to lesser accumulation of errors and more efficient parameter estimations. In the case of a financial time series, where significant large shocks are not frequent, Student's t conditional distribution would be preferable over the normal distribution due to capturing the heavy-tailed property of a financial time series without overestimating volatility parameters. Based on its out-of-sample forecasting performance, lower QLIKE value, it would be advisable for analysts and investment managers to use the Single-Stage Student's t MS-GARCH(1,1) model for portfolio analysis, value-at-Risk estimation, and financial stress testing. The results reinforce that stock market volatility is regime dependent; there are two regimes: the peaceful and the turmoil one; investors, portfolio managers, and policymakers should use the regime-switching volatility models in order to better measure financial risk, manage portfolios efficiently and build suitable economic and financial stability policies. A better volatility estimate will contribute positively to enhance financial surveillance systems, strengthen risk-management, facilitate capital market policy-makers in formulating monetary and capital market policies, and macro-prudential measures.

5.4 Suggestions for Further Study

Extending the framework to high-order and asymmetric MS-GARCH specifications: future research could examine the forecasting performance of more complex MS-GARCH models like MS-GARCH(2,1) and MS-GARCH(2,2), as well as MS-EGARCH and MS-GJR-GARCH, to provide added insights into volatility modelling. Applying the methodology to other financial market data: the proposed single-stage estimation technique can be further applied to volatility data of other equity markets, foreign exchange markets, commodity markets, as well as cryptocurrency markets, in order to assess their empirical relevance. Investigating multivariate regime-switching volatility models: the extension of the model into a multivariate framework to account for volatilities spillover, correlations and risk transmissions across multiple assets could also be the topic of future research.

5.5 limitations of the MS-GARCH(1,1) framework

Despite providing an excellent tool to model the complex dynamics of nonlinear and regime-dependent volatilities, the MS-GARCH framework is not without drawbacks. The main drawback is its high computational complexity, due to the fact that regimes' probabilities are modeled in the same likelihood function as the conditional volatility with very highly nonlinear expressions for these terms. As a result, the estimation of the parameters of the MS-GARCH model is frequently dependent on initial values and is prone to non convergence or to find local optima of the likelihood function when there is a lot of regimes or they are close to one another. In addition, it's difficult to choose the number of regimes and estimate specific parameters when they are similar or if there is not enough data. The ability of a MS-GARCH model to predict volatility also relies on an appropriate specification of the model and an adequate conditional distribution for the errors. A complex specification of MS-GARCH model may overfit the data, producing out-of-sample predictions inferior to that of a simpler model. The latent nature of the regimes makes the estimation of specific parameters harder and the economic interpretation more subjective. MS-GARCH framework also imposes the assumption that the next regime transition depends only on the present one, a strong assumption that may not fully cover the complexity and the stickiness of the financial market real world conditions. As a conclusion, while in general MS-GARCH model are superior to simple GARCH models in capture volatility regimes and structural changes in markets, model specification, robust estimation techniques and proper diagnostic checks are very important to validate the empirical findings.

6.0 References

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